Far East Journal of Applied Mathematics
Volume 33, Number 1, October 2008, Pages 89-96
Published online: September 19, 2008
This paper is available online at http://www.pphmj.com

SINGULAR VECTOR BAROTROPIC ENSEMBLE FORECAST FOR TROPICAL CYCLONE TRACK

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Abstract

Singular vector is a method to generate several initial conditions for weather prediction model from the same observation data. In this paper, a singular vector associated with a barotropic model is applied to tropical cyclone track forecasts by creating 50 initial conditions. These initial conditions are used to create 50 track predictions with slightly different initial conditions, called ensemble prediction system.

1. Introduction

A Numerical Weather Prediction (NWP) model is a computer program used to predict the future state of the weather by solving a set of equations that govern the behavior of the atmosphere [8, 9]. At the present time, forecasting of the atmosphere is always uncertain especially the movement of tropical cyclones. This uncertainity happens because the atmosphere is sensitive to initial condition. Although, there are now highly complex models available as weather forecast models, the results have not been satisfied. A part of this problem is because tropical

2000 Mathematics Subject Classification: 15A18, 34A34, 65Z05, 93A30.

Keywords and phrases: singular vector, barotropic model, ensemble prediction system.

Received April 28, 2008

cyclones interact with the environment. However, to use a high resolution weather prediction model for operational forecast, a very large, fast and expensive computer will be required. Another option to improve accuracy of weather forecast is to reduce uncertainity in the output from numerical models by performing several forecasts of the same event from several initial conditions. This is the concept of ensemble prediction system (EPS).

From the fact that the atmosphere is sensitive to initial condition, using only single initial state value means that the result will be only one value in many possible values. So, if there are many initial conditions, there are also many possible solutions which represent the possibility of the future states of the atmosphere. One way to get more initial condition values is to generate initial conditions based on the observed initial condition by using singular vector technique.

2. Barotropic Model

The barotropic model is a simple model for the atmosphere. This model is based on the assumptions: (1) horizontal motion, (2) non-divergent, (3) density is constant and (4) no friction. The barotropic model is based on one equation and one unknown, the streamfunction ψ

$$\frac{\partial}{\partial t}\nabla^2\psi = -J(\psi, \nabla^2\psi) - \beta\frac{\partial\psi}{\partial x}, \tag{1}$$

where J is the Jacobian operator, $\beta = \frac{\partial f}{\partial y}$ is the beta parameter and f is the coriolis parameter $(f = 2\Omega\sin\phi)$. Equation 1 forms the basis of the non-divergent barotropic model [4].

3. Singular Vectors (SVs) Method

Singular vector is a method to generate several initial conditions for weather prediction models from the same observed data. Technically, this model is used with the tangent linear model and the adjoint model is also needed in the singular vector calculation.

3.1. Tangent linear model

Consider a nonlinear model

$$x(t) = M[x(t_0)], \tag{2}$$

where $x(t_0)$ is the state at initial time, $x(t)$ is the state at time t and M is the time integration of the numerical scheme from the initial condition to time t. The tangent linear model (TLM) is defined such that [5]

$$L = \frac{\partial M}{\partial x}, \tag{3}$$

where L is a tangent linear model of M.

3.2. Adjoint model

The adjoint tangent linear model is the transpose of the tangent linear model. It is defined with respect to the inner product of two arbitrary vectors [5]

$$\langle Lu, v\rangle = \langle u, L^T v\rangle, \tag{4}$$

where u and v are arbitrary vectors and L^T is the adjoint TLM.

3.3. Calculation of singular vectors

Singular value decomposition theory indicates that for any matrix L there exist two orthogonal matrices U, V such that

$$U^T LV = S, \tag{5}$$

where

$$S = \begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \sigma_n \end{bmatrix} \text{ and } UU^T = I, VV^T = I, \tag{6}$$

S is a diagonal matrix whose elements are the singular values of L and I

is the identity matrix. Left multiply equation (5) by U, to get

$$LV = US, \text{ i.e., } L(v_1, \ldots, v_n) = (\sigma_1 u_1, \ldots, \sigma_n u_n), \tag{7}$$

where v_i are the columns of V and u_i are the columns of U. This implies that

$$Lv_i = \sigma_i u_i. \tag{8}$$

Equation (8) defines $(v_1, \ldots, v_n)$ as the *right singular vector* of L and will be revered to as *initial singular vectors*. Right multiply equation (5) by V^T, to obtain

$$U^T L = SV^T. \tag{9}$$

Transposing equation (9), so that

$$L^T u_i = \sigma_i v_i. \tag{10}$$

Equation (10) defines $(u_1, \ldots, u_n)$ as the *left singular vector* of L and will be revered to as *final singular vectors*. Multiplying equation (8) by L^T and from equation (10), to obtain

$$L^T L v_i = \sigma_i^2 v_i. \tag{11}$$

Therefore, the initial singular vectors can be obtained as the eigenvectors of $L^T L$, a normal matrix whose eigenvalues are the squares of the singular values [5].

4. Experiment and Results

This case (backtracking) is used to perform the experiment as shown in Figure 1. It was run with the barotropic model for 72 hours and the time step is half an hour.

The reason for choosing a barotropic model is that it is a single-level model. It also contains less variables and equation.

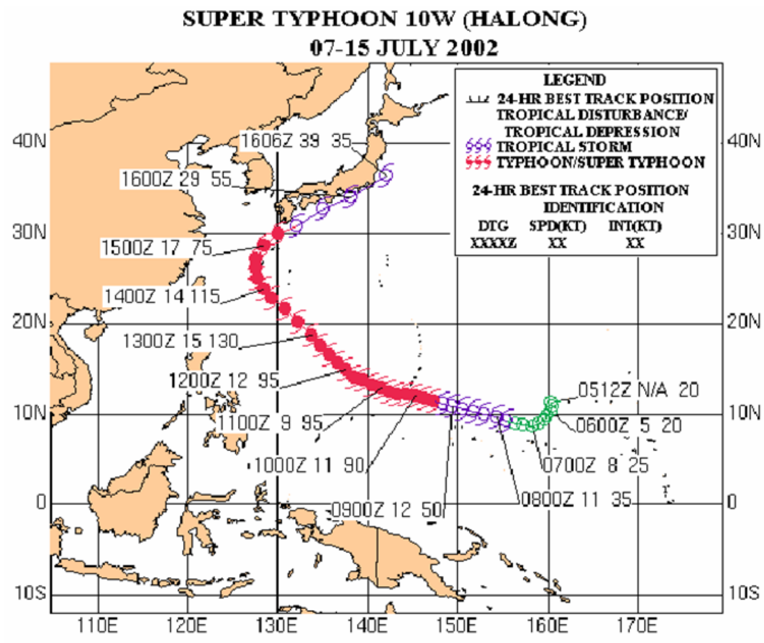

Figure 1. Backtracking of super typhoon HALONG, July 7th, 2002.

The data for the model are prepared using data from the European Center for Medium Range Weather Forecasting (ECMWF). The downloaded data can be classified in Table 1.

Table 1. Initial data from the ECMWF

	Backtracking of super typhoon HALONG
Type	Pressure levels
Date	10-07-2002
Time	00:00
Parameter	u, v velocity
Level	500 hPa
Resolution	2.5 degree

Since the downloaded data are low-resolution (2.5 degree), the center of tropical cyclone of initial streamfunction from running the barotropic model for this case has different positions from analysis data. So, it is important to modify initial streamfunction of this case as follows: (1) interpolating input data from 2.5 degree to 0.5 degree by the linear interpolation method and (2) relocated the storm center to the same positions of the observed data and increase the storm intensity by using Rankine vortex technique [10].

4.1. The results of backtracking of super typhoon HALONG

The output of running the model for backtracking of super typhoon HALONG on July 10th, 2002 and singular vector ensemble forecast are shown in this section. A singular vector associated with a barotropic model is used to generate 50 initial conditions. The 50 initial conditions are then used by the barotropic model to create an ensemble of 50 track predictions.

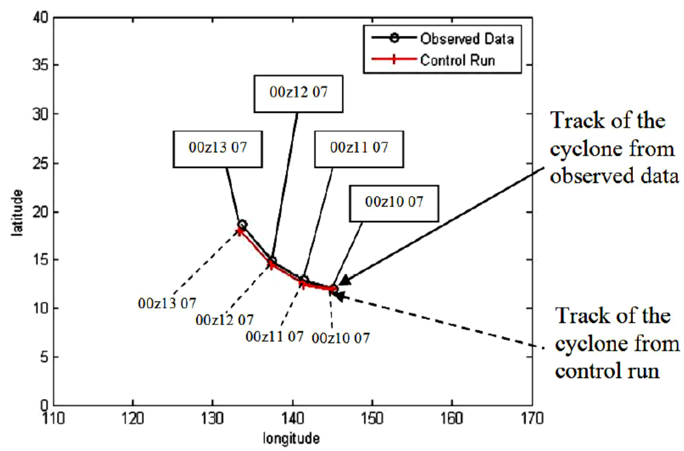

Figure 2. The comparison between the observed track and the track of the control run of super typhoon HALONG.

Figure 2 shows the comparison between the observed track and the track of the control run. From the figure, the control run gives the forecast track which is in agreement with the observed track.

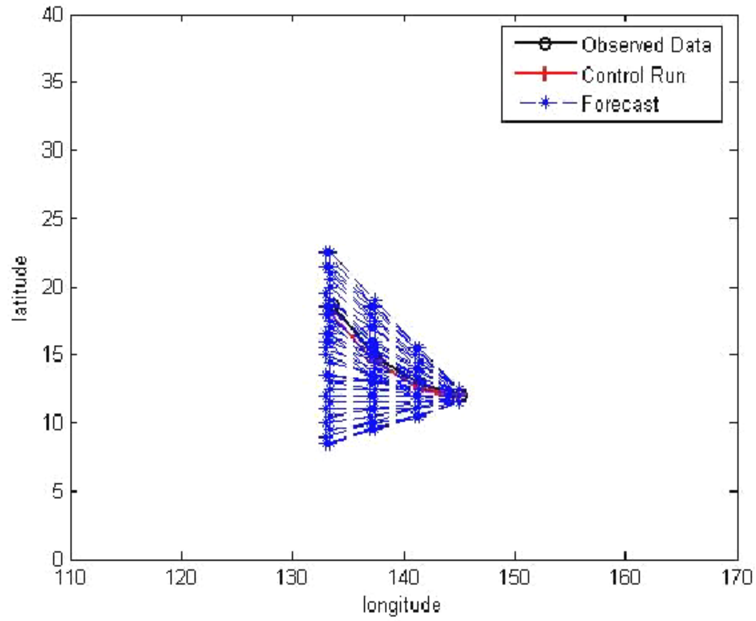

Figure 3. Singular vector ensemble forecast of 50 members of super typhoon HALONG.

Figure 3 shows that distribution of the ensemble forecast of 50 members of super typhoon HALONG. The ensemble track forecast is divergence of the track of the control run and observed track. This is because of the rotation and expansion of singular vector calculation process.

5. Conclusions

In the paper, this case of ensemble tropical cyclone track forecasts have been performed by generating initial conditions using singular vector method. The results show that initial conditions generated by the singular vector method result in different forecast tracks. This could provide a better guidance for tropical cyclone track forecast and warming. The ensemble forecasts of this case give the results that agree with the observed track.

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