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STRESS DISTRIBUTION IN A WEDGE-SHAPED ELASTIC SOLID INDENTED BY TWO RIGID PUNCHES

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Abstract

In this paper, we consider within the framework of classical elasticity under assumptions pertinent to plane strain, the problem of analytically finding the distribution of stress in a wedge-shaped homogeneous isotropic elastic solid, when the plane faces are indented by rigid punches of unequal size away from the apex of the wedge.

The solution of the three resulting parts mixed boundary value problem is reduced to the solution of triple integral equations involving Mellin transforms. Closed form solutions of the triple integral equations are obtained and the displacement component and resultant pressure on the faces are expressed in closed form. Numerical values for the resultant pressure are given in the form of a table. The results may be applicable to certain foundation problems.

1. Introduction

Problems concerning contact between deformable solids are of considerable theoretical and practical importance since contact is the

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commonest way to transmit loads from one structural member to another. This is why contact mechanics continues to be one of the most important branches of theoretical elasticity. Extensive accounts of this progress of contact problems are given by Gladwell [3], Hills et al. [4] and Jonson [5]. The problem of distribution of stress due to rigid punches in the wedge-shaped region has been discussed by Matczynski [7], Srivastav and Narain [8], Srivastav and Parihar [9]. Matczynski [7] reduced the problem to a Wiener-Hopf integral equation which is solved by an approximate method suggested by Koiter [6].

Srivastav and Narain [8] solved the contact problem when the plane faces of the wedges are indented normally and symmetrically by a rigid punch. They reduced the solution of the problem into dual integral equations by the use of Mellin transforms and finally reduced the problem into solving a Fredholm integral equation of the second kind. The Fredholm integral equation was solved numerically to find the physical quantities. Srivastav and Parihar [9] discussed the problem of a wedge with its plane faces indented by rigid punches of unequal size. They reduced the solution of the problem into two simultaneous Fredholm integral equations of the second kind, which are solved numerically to find the physical quantities. The three-part mixed boundary value problem for contact and crack problems at the middle of the wedge-shaped region has been solved by Erdogan and Gupta [2] and approximate results have been obtained.

This work is further extension of the work of Matczynski [7] and Srivastav and Narain [8], who considered the two-parts mixed boundary value problem.

As we know, an analytic solution in closed form has some advantages over numerical and approximate solutions, so that in many cases analytical solutions in closed form are desired for accurate analysis and design. Moreover, analytical solutions serve as a benchmark for the purpose of judging the accuracy and efficiency of various numerical and approximate methods. However, owing to the mathematical complexity, certain practical problems of complicated configurations are only solved with recourse to numerical schemes and it is difficult to obtain their analytic solution in closed form.

In this paper, we consider a wedge with plane faces indented by rigid punches of unequal size. The boundary value problem is reduced into the three-part mixed boundary value problem. The solution of the problem is reduced to the triple integral equations by using Mellin transforms. The closed form solution of the triple integral equations and the closed form expressions for shear stress and displacement component and the resultant pressures under the punches are obtained. The numerical results for the resultant pressure under the punch are given. With the application in foundation engineering in mind, the main interest in these problems is in the evaluation of the contact pressure.

The analysis throughout the paper is formal. As is customary for dealing with problems of this nature, we make no attempt to justify the change of order of integrations.

2. Solutions of Equations of Elastic Equilibrium

Let the wedge occupy the region defined in plane polar coordinates by $0 \leq r < \infty, -\alpha \leq \theta \leq \alpha.$ The equations of equilibrium:

$$\frac{\partial}{\partial r}\sigma_{rr} + \frac{1}{r}\frac{\partial}{\partial \theta}\sigma_{r\theta} + \frac{1}{r}(\sigma_r - \sigma_\theta) = 0, \tag{1}$$

$$\frac{\partial}{\partial r}\sigma_{r\theta} + \frac{1}{r}\frac{\partial}{\partial \theta}\sigma_{\theta\theta} + \frac{2}{r}\sigma_{r\theta} = 0, \tag{2}$$

are satisfied, if we assume that

$$\left.\begin{aligned} \sigma_{rr} &= \left(\frac{1}{r}\frac{\partial \chi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \chi}{\partial \theta^2}\right), \\ \sigma_{\theta\theta} &= \frac{\partial^2 \chi}{\partial r^2}, \\ \sigma_{r\theta} &= \left(\frac{1}{r^2}\frac{\partial \chi}{\partial \theta} - \frac{1}{r}\frac{\partial^2 \chi}{\partial \theta \partial r}\right), \end{aligned}\right\} \tag{3}$$

where χ is the airy stress function, while the strain components in terms of displacement and stresses are expressed as follows:

$$\left.\begin{aligned} \varepsilon_r &= \frac{\partial u_r}{\partial r} = \frac{1}{E}(\sigma_{rr} - \nu\sigma_{\theta\theta}), \\ \varepsilon_\theta &= \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} = \frac{1}{E}(\sigma_{\theta\theta} - \nu\sigma_{rr}), \\ \varepsilon_{r\theta} &= \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right) = \frac{-(1-\nu)}{E}\sigma_{r\theta}. \end{aligned}\right\} \tag{4}$$

The condition of compatibility imposed on χ yields a biharmonic equation

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)^2 \chi = 0. \tag{5}$$

Considering (3) and (4), we find that

$$\varepsilon_r = \frac{1}{E}\left[\left(\frac{1}{r^2}\frac{\partial^2 \chi}{\partial \theta^2} + \frac{1}{r}\frac{\partial \chi}{\partial r}\right) - \nu\frac{\partial^2 \chi}{\partial r^2}\right], \tag{6}$$

$$\varepsilon_\theta = \frac{1}{E}\left[\frac{\partial^2 \chi}{\partial r^2} - \nu\left(\frac{1}{r^2}\frac{\partial^2 \chi}{\partial \theta^2} + \frac{1}{r}\frac{\partial \chi}{\partial r}\right)\right], \tag{7}$$

$$\varepsilon_{r\theta} = -\frac{2(1+\nu)}{E}\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial \chi}{\partial \theta}\right]. \tag{8}$$

For plane strain, we replace

$$E \text{ by } \frac{E}{1-\nu^2}, \ \nu \text{ by } \frac{\nu}{1-\nu} \text{ and } E \text{ by } 2\mu(1+\nu). \tag{9}$$

In the above equations, E and ν represent the shear modulus and the Poisson's ratio for the material, respectively and μ represents the shear modulus.

With the help of equations (4), (6), (7) and (9), we get the displacement components:

$$2\mu u_r(r, \theta) = (1-\nu)\int (\nabla^2 \chi) dr - \frac{\partial \chi}{\partial r}, \tag{10}$$

$$2\mu u_\theta(r, \theta) = (1-\nu)\left[r\int (\nabla^2 \chi) d\theta - \iint (\nabla^2 \chi) dr d\theta\right] - \frac{\partial \chi}{r\partial \theta}, \tag{11}$$

where we have assumed that the constants of integration are zero as $r \to \infty$.

3. Statement of the Problem

Let us consider, the problem of distribution of stresses and displacements on the boundary of an infinite homogeneous isotropic plate having the form of a wedge with the vertex angle 2δ. We consider that the wedge with plane faces $\theta = \pm\delta$ is indented by punches of unequal size. Furthermore, the boundaries of the plate $\theta = \pm\delta$ are free from shear stress as shown in Figure 1. The line OX bisects the interior angle of 2δ of the wedge. The sign of θ is positive, when measured in the counter clockwise direction from OX to r.

The displacement and stress components for the upper part of the wedge $0 < \theta < \alpha$ are defined by $[u_\theta(r, \theta)]_1$, $[u_r(r, \theta)]_1$, $[\sigma_{rr}(r, \theta)]_1$, $[\sigma_{\theta\theta}(r, \theta)]_1$ and $[\sigma_{r\theta}(r, \theta)]_1$. For the lower part $-\alpha < \theta < 0$, the displacement and stress components are defined by $[u_\theta(r, \theta)]_2$, $[u_r(r, \theta)]_2$, $[\sigma_{rr}(r, \theta)]_2$, $[\sigma_{\theta\theta}(r, \theta)]_2$ and $[\sigma_{r\theta}(r, \theta)]_2$.

The boundary value problem is reduced to the mixed boundary value problem at the planes $\theta = \delta$ and $\theta = -\delta$. The boundary conditions are

$$[u_\theta(r, \delta)]_1 = f_1(r), \;\; a < r < b, \tag{12}$$

$$[\sigma_{\theta\theta}(r, \delta)]_1 = 0, \;\; 0 < r < a, \; r > b, \tag{13}$$

$$[\sigma_{r\theta}(r, \delta)]_1 = 0, \;\; 0 < r < \infty \tag{14}$$

and

$$[u_\theta(r, -\delta)]_2 = -f_2(r), \;\; a_1 < r < b_1, \tag{15}$$

$$[\sigma_{\theta\theta}(r, -\delta)]_2 = 0, \;\; 0 < r < a_1, \; r > b_1, \tag{16}$$

$$[\sigma_{r\theta}(r, -\delta)]_2 = 0, \;\; 0 < r < \infty. \tag{17}$$

We assume that along the line OX two pieces of same materials are joined to make one wedge. At the upper surface due to the frictionless joints along OX, we assume the following boundary conditions:

$$[\sigma_{\theta\theta}(r, 0)]_1 = [\sigma_{\theta\theta}(r, 0)]_2, \tag{18}$$

$$[\sigma_{rr}(r, 0)]_1 = [\sigma_{rr}(r, 0)]_2. \tag{19}$$

We take the stress function χ satisfying equation (5) in the form (see [10, 12]):

$$\chi(r,\ \theta) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{s^{-1}A(s)r^{1-s}}{2(s-1)\cos\left(\frac{\pi s}{2}\right)}\left[(s+1)\cos\left[(s-1)\left(\theta-\delta-\frac{\pi}{2}\right)\right]\right.$$

$$\left. + (s-1)\cos\left[(s+1)\left(\theta-\delta-\frac{\pi}{2}\right)\right]\right]ds,\ \ \theta \geq 0,\ -1 < c < 1 \tag{20}$$

and

$$\chi(r,\ \theta) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{s^{-1}A(s)r^{1-s}}{2(s-1)\cos\left(\frac{\pi s}{2}\right)}\left[(s+1)\cos\left[(s-1)\left(\theta+\delta+\frac{\pi}{2}\right)\right]\right.$$

$$\left. + (s-1)\cos\left[(s+1)\left(\theta+\delta+\frac{\pi}{2}\right)\right]\right]ds,\ \ \theta \leq 0,\ -1 < c < 1. \tag{21}$$

Equations (3), (10), (11) and (20) lead to

$$[\sigma_{rr}]_1 = -\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{A(s)r^{-s-1}}{2\cos\left(\frac{\pi s}{2}\right)}\left[(s+3)\cos(s-1)\left(\theta-\delta-\frac{\pi}{2}\right)\right.$$

$$\left. + (s+1)\cos(s-1)\left(\theta-\delta-\frac{\pi}{2}\right)\right]ds, \tag{22}$$

$$[\sigma_{\theta\theta}]_1 = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{A(s)r^{-s-1}}{2\cos\left(\frac{\pi s}{2}\right)}\left[(s-1)\cos(s+1)\left(\theta-\delta-\frac{\pi}{2}\right)\right.$$

$$\left. + (s+1)\cos(s-1)\left(\theta-\delta-\frac{\pi}{2}\right)\right]ds, \tag{23}$$

$$[\sigma_{r\theta}]_1 = -\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{(s+1)A(s)r^{-s-1}}{2\cos\left(\frac{\pi s}{2}\right)}\left[\sin(s+1)\left(\theta-\delta-\frac{\pi}{2}\right)\right.$$

$$\left. + \sin(s-1)\left(\theta-\delta-\frac{\pi}{2}\right)\right]ds, \tag{24}$$

$$[u_r]_1 = \left(\frac{1+\nu}{E}\right)\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{A(s)r^{-s}}{2s\cos\left(\frac{\pi s}{2}\right)}\left[(s+1)\cos(s-1)\left(\theta-\delta-\frac{\pi}{2}\right)\right.$$

$$\left.+(s+3-4\nu)\cos(s+1)\left(\theta-\delta-\frac{\pi}{2}\right)\right]ds, \tag{25}$$

$$[u_\theta]_1 = \left(\frac{1+\nu}{E}\right)\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{A(s)r^{-s}}{2s\cos\left(\frac{\pi s}{2}\right)}\left[(s+1)\sin(s-1)\left(\theta-\delta-\frac{\pi}{2}\right)\right.$$

$$\left.+(s-3+4\nu)\sin(s+1)\left(\theta-\delta-\frac{\pi}{2}\right)\right]ds. \tag{26}$$

From equations (3), (10), (11) and (21), we obtain

$$[\sigma_{rr}]_2 = -\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{A(s)r^{-s-1}}{2\cos\left(\frac{\pi s}{2}\right)}\left[(s+3)\cos(s+1)\left(\theta+\delta+\frac{\pi}{2}\right)\right.$$

$$\left.+(s+1)\cos(s-1)\left(\theta+\delta+\frac{\pi}{2}\right)\right]ds, \tag{27}$$

$$[\sigma_{\theta\theta}]_2 = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{A(s)r^{-s-1}}{2\cos\left(\frac{\pi s}{2}\right)}\left[(s-1)\cos(s+1)\left(\theta+\delta+\frac{\pi}{2}\right)\right.$$

$$\left.+(s+1)\cos(s-1)\left(\theta+\delta+\frac{\pi}{2}\right)\right]ds, \tag{28}$$

$$[\sigma_{r\theta}]_2 = -\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{(s+1)A(s)r^{-s-1}}{2\cos\left(\frac{\pi s}{2}\right)}\left[\sin(s+1)\left(\theta+\delta+\frac{\pi}{2}\right)\right.$$

$$\left.+\sin(s-1)\left(\theta+\delta+\frac{\pi}{2}\right)\right]ds, \tag{29}$$

$$[u_r]_2 = \left(\frac{1+\nu}{E}\right)\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{A(s)r^{-s}}{2s\cos\left(\frac{\pi s}{2}\right)}\left[(s+1)\cos(s-1)\left(\theta+\delta+\frac{\pi}{2}\right)\right.$$

$$\left.+(s+3-4\nu)\cos(s+1)\left(\theta+\delta+\frac{\pi}{2}\right)\right]ds, \tag{30}$$

$$[u_\theta]_2 = \left(\frac{1+\nu}{E}\right)\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{A(s)r^{-s}}{2s\cos\left(\frac{\pi s}{2}\right)}\left[(s+1)\sin(s-1)\left(\theta+\delta+\frac{\pi}{2}\right)\right.$$

$$\left. + (s-3+4\nu)\sin(s+1)\left(\theta+\delta+\frac{\pi}{2}\right)\right]ds. \tag{31}$$

From equations (22)-(26) and (27)-(31), the boundary conditions (14), (17)-(19) are satisfied identically and the boundary conditions (12), (13), (15) and (16) reduce to the following two sets of triple integral equations:

$$\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} s\tan\left(\frac{\pi s}{2}\right)A_1(s)r^{-s}ds = 0, \quad 0 < r < a, \tag{32}$$

$$\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} A_1(s)r^{-s}ds = \frac{-Ef_1(r)}{2(1+\nu)(\nu-1)}, \quad a < r < b, \tag{33}$$

$$\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} s\tan\left(\frac{\pi s}{2}\right)A_1(s)r^{-s}ds = 0, \quad r > b \tag{34}$$

and

$$\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} s\tan\left(\frac{\pi s}{2}\right)A_1(s)r^{-s}ds = 0, \quad 0 < r < a_1, \tag{35}$$

$$\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} A_1(s)r^{-s}ds = \frac{-Ef_2(r)}{(1+\nu)(\nu-1)}, \quad a_1 < r < b_1, \tag{36}$$

$$\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} s\tan\left(\frac{\pi s}{2}\right)A_1(s)r^{-s}ds = 0, \quad r > b_1, \tag{37}$$

where

$$A(s) = sA_1(s). \tag{38}$$

To find the solution of the problem, we shall consider the triple integral equations (32), (33) and (34).

Let us suppose that

$$\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} s\tan\left(\frac{\pi s}{2}\right)A_1(s)r^{-s}ds = rg(r), \quad a < r < b. \tag{39}$$

The inversion theorem for Mellin transforms and equations (32), (34) and (39) lead to

$$sA_1(s)\tan\left(\frac{\pi s}{2}\right) = \int_a^b g(t)t^s dt, \quad c > 0. \tag{40}$$

Substituting the value of $A_1(s)$ from equation (40) into equation (33) and interchanging the order of integration and using the result from Erde‘lyi [1 (18), p. 315]:

$$\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{1}{s}\cot\left(\frac{\pi s}{2}\right)t^s r^{-s}ds = \frac{1}{\pi}\log\left|\frac{r^2 - t^2}{r^2}\right|, \quad -2 < c < 0, \tag{41}$$

we find that

$$\frac{1}{\pi}\int_a^b g(t)\log\left|1 - \frac{t^2}{r^2}\right|dt = F(r), \quad a < r < b, \tag{42}$$

where

$$F(r) = \frac{-Ef_1(r)}{2(1+\nu)(\nu - 1)}. \tag{43}$$

Differentiating both sides of equation (42) with respect to r, we find that

$$\frac{1}{\pi}\int_a^b \frac{2t^2 g(t)dt}{(t^2 - r^2)} = -rF'(r), \quad a < r < b, \tag{44}$$

where the prime denotes derivative with respect to r. With the Hilbert transform theorem discussed by Tricomi [11], we obtain

$$tg(t) = \frac{2}{\pi}\left(\frac{t^2 - a^2}{b^2 - t^2}\right)^{\frac{1}{2}}\int_a^b\left(\frac{b^2 - r^2}{r^2 - a^2}\right)^{\frac{1}{2}}\frac{r^2 F'(r)dr}{r^2 - t^2}$$

$$+ \frac{C}{[(t^2 - a^2)(b^2 - t^2)]^{\frac{1}{2}}}, \quad a < t < b, \tag{45}$$

where C is an arbitrary constant.

If

$$f_1(r) = f_0 \quad \text{(a constant)}, \tag{46}$$

then

$$F(r) = F_0 = \frac{-Ef_0}{2(1+\nu)(\nu-1)}, \tag{47}$$

$$F'(r) = 0. \tag{48}$$

With equation (45), we find that

$$g(t) = \frac{C}{t\sqrt{(t^2-a^2)(b^2-t^2)}}, \quad a < t < b, \tag{49}$$

when $F(r)$ is constant. Substituting equation (49) into equation (42) and making use of the equation

$$\int_a^b \frac{\log\left|1-\frac{t^2}{r^2}\right| dt}{t\sqrt{(t^2-a^2)(b^2-t^2)}}$$

$$= \begin{cases} \dfrac{\pi}{ab}\left[\log\left|\dfrac{a\sqrt{(b^2-r^2)}+b\sqrt{(a^2-r^2)}}{r(a+b)}\right|\right], & 0 < r < a, \\ \dfrac{\pi}{2ab}\log\left|\dfrac{b-a}{a+b}\right|, & a < r < b, \\ \dfrac{\pi}{ab}\left[\log\left|\dfrac{a\sqrt{(b^2-r^2)}+b\sqrt{(a^2-r^2)}}{r(a+b)}\right|\right], & r > b, \end{cases} \tag{50}$$

we find that

$$C = \frac{F_0}{\frac{1}{2ab}\log\left|\frac{b-a}{a+b}\right|}. \tag{51}$$

Now, equation (49) may be written in the following form:

$$g(t) = \frac{F_0}{\frac{1}{2ab}\log\left|\frac{b-a}{a+b}\right| t\sqrt{(t^2-a^2)(b^2-t^2)}}. \tag{52}$$

We can easily find that

$$\sigma_{\theta\theta}(r, \delta) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} sA_1(s)r^{-s-1}\tan\left(\frac{\pi s}{2}\right)ds, \quad a < r < b, \tag{53}$$

$$= g(r), \quad a < r < b. \tag{54}$$

The total pressure P_1 under the stamp is given by

$$P_1 = -\int_a^b \sigma_{\theta\theta}(r, \delta)\, dr = -\int_a^b g(r) dr. \tag{55}$$

Substituting equation (52) into equation (55) and making use of the following integral:

$$\int_a^b \frac{dt}{t\sqrt{(t^2 - a^2)(b^2 - t^2)}} = \frac{\pi}{2ab}, \tag{56}$$

we find that

$$P_1 = \frac{-\pi F_0}{\log\left|\dfrac{b-a}{a+b}\right|}. \tag{57}$$

Making use of equations (43) and (9), we can write expression (57) in the following form:

$$\frac{(\nu - 1)P_1}{\mu f_0} = \frac{\pi}{\log\left|\dfrac{b-a}{a+b}\right|}. \tag{58}$$

The expression for the displacement component can be written in the following form:

$$[u_\theta]_1 = \frac{-1}{\pi}\left(\frac{\nu - 1}{\mu}\right)\int_a^b g(t) \log\left|1 - \frac{t^2}{r^2}\right| dt. \tag{59}$$

Substituting the value of $g(t)$ from equation (52) into equation (59) and making use of equations (47) and (50), we get

$$[u_\theta]_1 = \begin{cases} \dfrac{2f_0}{\log\left|\dfrac{b-a}{a+b}\right|}\left\{\log\left|\dfrac{a\sqrt{b^2 - r^2} + b\sqrt{a^2 - r^2}}{r(a+b)}\right|\right\}, & 0 < r < a, \\[2ex] \dfrac{2f_0}{\log\left|\dfrac{b-a}{a+b}\right|}\left\{\log\left|\dfrac{a\sqrt{r^2 - b^2} + b\sqrt{r^2 - a^2}}{r(a+b)}\right|\right\}, & r > b. \end{cases} \tag{60}$$

Solution of the triple integral equations (35)-(37) may be obtained by replacing $a, b, c, f_1(r)$ by a_1, b_1, c_1 and $f_2(r)$, respectively in the above solution.

Numerical values of the total pressure P_1 are given in Table 1.

It is interesting to note that the physical quantities P_1 and $[u_\theta]_1$ at $\theta = \delta$ are independent of the angle δ.

In the same way, the total pressure under the punch $a_1 < r < b_1$, $\theta = -\delta$ can be obtained.

Table 1. Values of $\frac{(\nu - 1)P_1}{\mu f_0}$ for $b = 1$

a	$\frac{(\nu - 1)P_1}{\mu f_0}$
0.3	−5.0750
0.4	−3.7078
0.5	−2.8596
0.6	−2.2662
0.7	−1.8111
0.8	−1.4298

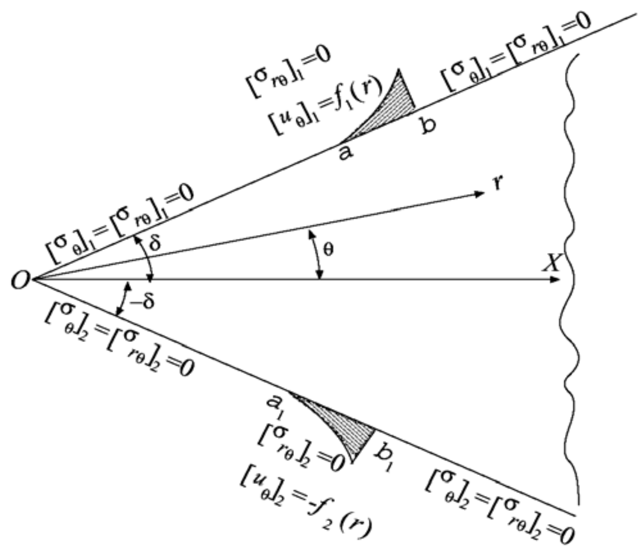

Figure 1. Wedge-shaped elastic solid indentation by rigid punches.

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