JP Journal of Heat and Mass Transfer
Volume 2, Number 2, 2008, Pages 117-133
Published Online: September 10, 2008
This paper is available online at http://www.pphmj.com

INSTABILITY ANALYSIS OF A REACTIVE BOUNDARY LAYER FLOW IN THE PRESENCE OF LARGE BUOYANCY

S. SHATEYI, P. SIBANDA and S. S. MOTSA

Department of Mathematics
University of Venda
Private Bag X5050
Thohoyandou 0950
South Africa

School of Mathematics
University of KwaZulu-Natal
Private Bag X01, Scottsville
Pietermaritzburg 3209
South Africa
e-mail: sibandap@ukzn.ac.za

Mathematics Department
University of Swaziland
Private Bag 4, Kwaluseni
Swaziland

Abstract

In this paper we investigate the effects of the Damkohler and Schmidt numbers on the inviscid instability of reactive boundary layer flow over a horizontal rigid flat surface with heat and mass transfer. The fluid buoyancy caused simultaneously by thermal and species diffusion is assumed large. This then leads to a two-layered flow structure with the disturbances governed by the Taylor-Goldstein equation. A Chebyshev

Keywords and phrases: buoyancy, Damkohler number, inviscid wave modes, growth rate, Taylor-Goldstein equations.

Communicated by Chen-I Hung

Received March 19, 2008; Revised May 21, 2008

collocation spectral method was used to find the eigen-solution of the Taylor-Goldstein equation. The growth rate and phase speeds of the inviscid wave modes are obtained for varying buoyancy, Damkohler and Schmidt numbers.

1. Introduction

There are many transport processes which occur in nature and in artificial devices in which the fluid flow is driven or modified by density gradients caused by temperature and species composition differences. Atmospheric flows are driven appreciably by both temperature and water concentration differences. Flows in large water bodies such as lakes and oceans are driven by density and temperature differences and are also subject to the effects of differential concentration of dissolved salts and suspended particulate matter.

One of the earliest studies on the combined effects of thermal and concentration species diffusion driven flows was done by Somers [13]. This study was subsequently followed up by, among other researchers, Mathers et al. [7]. In 1971, Gebhart and Pera [4] investigated flows resulting from buoyancy forces which arise from a combination of temperature and species concentration effects of comparable magnitudes. Their study showed that in the case of vertical flows, the Boussinesq approximations yield a set of equations which have a similarity form solution for the combined buoyancy effects.

Among the more recent studies, Magyari and Keller [6] systematically investigated the mechanical and thermal properties of self-similar boundary layer flows induced by stretching surfaces with rapidly decreasing power-law and exponential velocities. The study observed that the flow problem admitted solutions only if a lateral suction was applied and, except for an isolated value of the suction velocity, the problem admitted a non-denumerable infinity of solutions for any given suction velocity less than the maximum suction velocity of the surface.

The effect of heat transfer on the upper-branch stability of Tollmien waves in accelerating boundary layer over a rigid surface in incompressible flow was investigated by Mureithi et al. [11]. It was observed in that study that buoyancy has a destabilizing effect on rigid bodies. Motsa et al. [9] showed that in the case of flow over a compliant boundary there are cases where large buoyancy leads to modes which are more stable than the instability modes which arise in the absence of buoyancy.

Motsa and Sibanda [10] gave a detailed investigation of the effects of surface flexibility on the inviscid instability boundary layer flow over a horizontal flat plate with heat transfer when the fluid buoyancy is large. Together, the Mureithi et al. [11] and Motsa et al. [9] studies have established that in the limit of large buoyancy, the boundary layer develops a non-constant pressure gradient and that the disturbances that arise are inviscid in nature and are governed by the Taylor-Goldstein (TG) equation. In Motsa and Sibanda [10], the analysis of the TG equation by means of a Chebyshev spectral collocation method showed that the influence of the plate flexibility on the inviscid instability is most important when the perturbation wave number α is small and that plate compliance destabilizes the Tollmien-Schlichting (TS) waves that are governed by the Taylor-Goldstein equation. While the Motsa and Sibanda [10] study was fundamentally about the dynamics of fluid-surface coupling behaviour, the current study seeks to establish what the impact of varying the ratio of the hydrodynamic time scale to the chemical reaction kinematics time scale as measured by the flow Damkohler number would be in the case of a reactive boundary layer flow. These perturbations are known to take the form of TS instability waves that are destabilized by viscous mechanisms.

Shateyi et al. [12] considered the effect of a chemical reaction between a chemical species and the fluid on the linear stability of two-dimensional disturbances wave modes. The effect of the Damkohler number was found to have destabilizing effects on the Tollmien-Schlichting waves.

Two complementary approaches are adopted for this study. In the first part a similarity approach is used to transform the governing Navier-Stokes equations to first order form which are then solved to find the effects of the fluid buoyancy, Damkohler and Schmidt numbers on the fluid velocity and temperature profiles. The Damkohler number is an important parameter that gives a quantification of whether the chemical reaction mechanism is diffusion or kinematically controlled. The system is characterized as diffusion controlled if the mixing rate of the reactants is slow compared to the reaction rate while fast mixing and slow reaction rates give rise to a kinematically controlled system.

The Schmidt number arises in mixing flows as a ratio of momentum diffusivity to mass diffusivity. It is a measure of the relative influences of the diffusion of momentum and the diffusion of species in the mixing boundary layer fluid. Most of the earlier work on the importance of the influence of the Schmidt number in species mixing have been experimental rather than theoretical, see for example, Broadwell and Breidenthal [2] and Dimotakis [3]. For an experimental study of the role played by high Schmidt numbers in jets and other mixing boundary layer flows the reader is referred to Miller [8] and the references therein. This study hopes to provide a theoretical basis for studying the influence of the Damkohler and Schmidt numbers in boundary layer flows.

In the second approach a Taylor-Goldstein equation that is derived from the governing Navier-Stokes equations is solved using a Chebychev spectral collocation method to determine how the growth rates and the phase speed of small perturbations to the flow are influenced by buoyancy, Damkohler and Schmidt numbers.

2. Mathematical Formulation

The equations governing a two-dimensional incompressible fluid flowing over a horizontal plate which is composed of a chemical species expressed in dimensionless form under a Boussinesq type approximation are given by (Shateyi et al. [12]):

$$u_x + v_y = 0, \tag{1a}$$

$$uu_x + uv_y = -p_x + \frac{1}{Re}\nabla^2 u, \tag{1b}$$

$$uv_x + vv_y = -p_y + Re^{-1}\nabla^2 v + G_c C + G_t T, \tag{1c}$$

$$uC_x + vC_y = Re^{-1} Sc^{-1}\nabla^2 C - DaC, \tag{1d}$$

$$uT_x + vT_y = Pr^{-1} Re^{-1}\nabla^2 T, \tag{1e}$$

where the subscripts x and y denote differentiation of the fluid property with respect to the streamwise and normal coordinates, respectively, u and v are the streamwise and normal velocity components, respectively, p is the pressure, C is the species chemical concentration and T is the fluid temperature.

The parameters of particular interest in this study are the Schmidt number $Sc = \nu/D$, where ν is the kinematic viscosity of the fluid and D is the molecular diffusivity of the chemical species in the mixing boundary layer. The Schmidt number is analogous to the Prandtl number $Pr = \nu/\kappa$ with D replaced by the thermal diffusion coefficient κ. The second important parameter is the Damkohler number Da. The Damkohler number is defined as the ratio of the flow time scale to the chemical time scale. Buoyancy is another important consideration in this study since the species fluid mixing usually gives off or absorbs heat. We denote by G_t and G_c the buoyancy terms induced by the temperature and concentration differences, respectively. The buoyancy terms are defined by $G_{c,t} = Gr/Re^2$, where Gr is the Grashof number and Re is the Reynolds number of the flow.

In the absence of any disturbances, the flow velocity, temperature, chemical concentration and pressure are given by:

$$u = U_B(x, Y) + \cdots, \quad v = Re^{-1/2}V_B(x, Y) + \cdots, \quad T = T_B(x, Y) + \cdots,$$
$$C = C_B(x, Y) + \cdots, \quad p = p_e(x) + p_b(x, Y), \tag{2}$$

where p_e $(= u_e u_{ex}$, where u_e is the external streaming flow) is the pressure induced by the streaming flow, p_b is the pressure induced by the buoyancy and $Y = Re^{1/2}y$ is the boundary layer coordinate. A simple analysis shows that when the buoyancy terms G_t, $G_c \sim O(1)$, the temperature, chemical and velocity fields are decoupled. It is only when G_t, $G_c \sim O(Re^{1/2})$ that these equations become fully coupled. Our main focus is on the modified upper-branch structure that obtains for large buoyancy, and to that end we define $G_t = G_{0t}\varepsilon^{-6}$, $G_c = G_{0c}\varepsilon^{-6}$ and $Da = D_0\varepsilon^{-6}$, where the small parameter ε is defined by $\varepsilon = Re^{-\frac{1}{12}}$ and

$(G_{0t}, G_{0c}, D_0) \sim O(1)$. Substituting equations (2) into the governing Navier-Stokes and taking the limit $(G_{0t}, G_{0c}) \to \infty$, $Re \to \infty$, we obtain the following basic steady state boundary layer equations:

$$\frac{\partial U_B}{\partial x} + \frac{\partial V_B}{\partial Y} = 0, \tag{3a}$$

$$U_B \frac{\partial U_B}{\partial x} + V_B \frac{\partial U_B}{\partial Y} = -\frac{\partial p_b}{\partial x} - u_e \frac{du_e}{dx} + \frac{\partial^2 U_B}{\partial Y^2}, \tag{3b}$$

$$U_B \frac{\partial C_B}{\partial x} + V_B \frac{\partial C_B}{\partial Y} = \frac{1}{Sc} \frac{\partial^2 C_B}{\partial Y^2} - D_0 C_B, \tag{3c}$$

$$U_B \frac{\partial T_B}{\partial x} + V_B \frac{\partial T_B}{\partial Y} = \frac{1}{Pr} \frac{\partial^2 T_B}{\partial Y^2}, \tag{3d}$$

$$\frac{\partial p_b}{\partial Y} = G_{0t} T_B + G_{0c} C_B, \tag{3e}$$

with boundary conditions

$$U_B = V_B = 0, \quad C_B = T_B = 1 \quad \text{at} \quad Y = 0,$$
$$U_B \to u_e(x), \quad C_B, T_B, p_b \to 0 \quad \text{as} \quad Y \to \infty. \tag{4}$$

We assume that the boundary layer profile is the prototype of the self-similar flow, (see Mureithi et al. [11] for details) given by:

$$U_B = x^{1/3} f'(\eta), \quad V_B = \frac{1}{3} x^{-1/3} (\eta f' - 2f), \quad T_B = x^{1/3} g(\eta), \tag{5a}$$

$$C_B = x^{1/3} h(\eta), \quad p_b = x^{2/3} q(\eta), \quad u_e = x^{1/3}, \tag{5b}$$

where the similarity variable is given by $\eta = Yx^{-1/3}$. Using the transformations (5) in equations (3) we obtain

$$f''' = \frac{1}{3}(f'^2 - 2ff'') - \frac{1}{3}(1 - 2q + q'\eta), \tag{6a}$$

$$h'' = \frac{1}{3} Sc(hf' - 2fh' + 3D_0 h), \tag{6b}$$

$$g'' = \frac{1}{3} Pr(gf' - 2fg'), \tag{6c}$$

$$q' = G_{0t} g + G_{0c} h, \tag{6d}$$

with boundary conditions

$$f(0) = 0, \quad f'(0) = 0, \quad f'(\infty) = 1, \quad g(0) = 1, \tag{7a}$$

$$g(\infty) = 0, \quad h(0) = 1, \quad h(\infty) = 0, \quad q(\infty) = 0. \tag{7b}$$

Equations (6) were first reduced into a system of ordinary differential equations and then solved using a combination of fourth and fifth order Runge-Kutta methods. The numerical results that illustrate the effects of the buoyancy, Damkohler and Schmidt numbers on the velocity and species concentration distributions are shown in Figures 2-3.

3. Analysis of the Growth Rate of Disturbances

The asymptotic structure for the upper-branch stability of boundary layer flows is now well known. However, in the limit $(G_{0t}, G_{0c}, D_0) \to +\infty$, the structure of the flow is modified as in Motsa and Sibanda [10] with the neutral wave number and wave speed of order $O(\varepsilon^{-1})$. We thus set $\alpha = \varepsilon^{-1}\alpha_0$ and $c = \varepsilon^{-1}c_0$, where α_0 and c_0 are the scaled real wave number and wave speed of the travelling wave disturbance, respectively. The disturbances are taken to be in the form of a modulated wave-train periodic in X, and we replace the derivatives by

$$\frac{\partial}{\partial x} \sim \varepsilon^{-6}\alpha_0 \frac{\partial}{\partial X} \quad \text{and} \quad \frac{\partial}{\partial t} \sim \varepsilon^{-6}\alpha_0 c_0 \frac{\partial}{\partial X}. \tag{8}$$

We now perturb the flow by writing

$$(u, v, T, C, p) = (U_B, 0, T_B, C_B, P_B) + \delta(u_0, v_0, T_0, C_0, p_0) + \cdots, \tag{9}$$

where $\delta \ll 1$ is a measure of the size of the disturbance, $G_t = G_{0t}\varepsilon^{-6}$, $G_c = G_{0c}\varepsilon^{-6}$ and $Da = D_0\varepsilon^{-6}$. Taking the limit $Re \to \infty$ in equations (1) we obtain

$$\alpha_0 u_{0X} + v_{0Y} = 0, \tag{10a}$$

$$\alpha_0(U_B - c_0)u_{0X} + v_0 U_{BY} = -\alpha_0 p_{0X}, \tag{10b}$$

$$\alpha_0(U_B - c_0)v_{0X} = -\alpha_0 p_{0Y} + G_{0t}T_0 + G_{0c}C_0, \tag{10c}$$

$$\alpha_0(U_B - c_0)T_{0X} + v_0 T_{BY} = 0, \tag{10d}$$

$$\alpha_0(U_B - c_0)C_{0X} + v_0 C_{BY} = -D_0 C_0. \tag{10e}$$

Setting $u_0 = \overline{u}_0 e^{iX} + c.c,$ $v_0 = \overline{v}_0 e^{iX} + c.c,$ etc., where $c.c$ denotes the complex conjugate and eliminating u_0, T_0, C_0 and p_0 gives the Taylor-Goldstein (TG) equation

$$(U_B - c_0)^2(v_0'' - \alpha_0^2 v_0) - [(U_B - c_0)U_B'' - G_{0t}T_B']v_0 + \frac{\alpha_0^2(U_B - c_0)^2 G_{0c}C_B' v_0}{D_0^2 + \alpha_0^2(U_B - c_0)^2} = 0, \tag{11}$$

with the boundary conditions given by

$$v_0 = 0 \text{ at } Y = 0, \infty \quad \text{and} \quad p_0 = 0 \text{ at } Y = 0, \tag{12}$$

where the primes denote differentiation with respect to Y. The TG equation can be transformed into a similarity form by setting

$$\eta = Yx^{-\frac{1}{3}}, \quad c_0 = x^{\frac{1}{3}}c, \quad \alpha_0 = x^{-\frac{1}{3}}\alpha,$$

$$U_B = x^{\frac{1}{3}}f'(\eta), \quad T_B = x^{\frac{1}{3}}g(\eta), \quad C_B = x^{\frac{1}{3}}h(\eta),$$

to give

$$(f' - c)^2(v_0'' - \alpha^2 v_0) - [(f' - c)f''' - G_{0t}g']v_0 + \frac{\alpha^2(f' - c)^2 G_{0c}h' v_0}{D_0^2 + \alpha^2(f' - c)^2} = 0, \tag{13}$$

where the primes now denote differentiation with respect to η. The appropriate boundary condition is

$$v_0 = 0 \quad \text{at } \eta = \infty. \tag{14}$$

Equations (13) and (14) constitute an eigenvalue problem of the

$$\mathcal{F}(\alpha, c, Pr, Sc, G, D_0) = 0, \tag{15}$$

which is solved for the complex eigenvalue $c = Re(c) + iIm(c)$ for fixed wave number α, effective buoyancy G, the Prandtl number Pr, the Schmidt number Sc and the scaled Damkohler number D_0. The real part $Re(c)$ gives the phase speed of the perturbation and the imaginary part $Im(c)$ gives the growth rate in time, $\alpha Im(c)$. The flow is linearly stable (unstable) when $Im(c)$ is positive (negative).

To solve equation (15) we used a spectral collocation method of the Chebyshev type. The Chebyshev Spectral Collocation method was used by

Motsa and Sibanda [10] in the case of flow over compliant surfaces. We look for an approximate solution,

$$v_0(z) \equiv \phi(z) = \sum_{k=0}^{N} \hat{\phi}_k T_k(z), \quad -1 \le z \le 1,$$

where $T_k(z)$ is the kth Chebychev polynomial of the first kind and $\hat{\phi}_k$ are the Chebyshev coefficients. The physical region $[0, \infty)$ was mapped into the region $[-1, 1]$ using the domain truncation technique proposed by Boyd [1] and the mapping

$$\frac{\eta}{L} = \frac{z+1}{2}, \quad -1 \le z \le 1, \tag{16}$$

where L is a scaling parameter which is used to invoke the boundary condition at ∞. For convenience, we used the Chebychev-Gauss-Lobatto points where

$$z_j = \cos\frac{\pi j}{N}, \quad -1 \le z \le 1, \ \ j = 0, 1, \ldots, N. \tag{17}$$

The derivatives of the functions $\phi_N(z)$ at the collocation points are represented by

$$\frac{d}{dz}(\phi_N(z_j)) = \sum_{k=0}^{N} \mathbf{D}_{jk}\tilde{\phi}_k, \tag{18}$$

where the derivative matrix $\mathbf{D}$ is given by

$$\mathbf{D}_{jk} = \frac{c_j}{c_k}\frac{(-1)^{j+k}}{z_j - z_k} \quad j \ne k;\ j, k = 0, 1, \ldots N,$$

$$\mathbf{D}_{kk} = -\frac{z_k}{2(1-z_k^2)} \quad k = 1, 2, \ldots, N-1,$$

$$\mathbf{D}_{00} = \frac{2N^2+1}{6} = -D_{NN},$$

where c_j and c_k are equal to 1 for $j, k = 1, 2, \ldots, N-1$ and $c_0 = c_N = 2$. High order derivatives are computed as simply multiple powers of $\mathbf{D}$, that is,

$$\frac{d^n\phi_N(z_j)}{dz^n} = \sum_{k=0}^{N} \mathbf{D}_{jk}^n\tilde{\phi}_k, \quad k = 0, 1, \ldots N, \tag{19}$$

where n is the order of the derivatives. Now substituting into equations (13)-(14) yields

$$(\mathbf{f}'-c)^2\left(\frac{4}{L^2}\mathbf{D}^2-\alpha^2\mathbf{I}\right)\Phi-[(\mathbf{f}'-c)\mathbf{f}'''-\hat{G}_{0t}\mathbf{g}']\Phi$$

$$+\frac{\alpha^2(\mathbf{f}'-c)^2+\hat{G}_{0c}\mathbf{h}'}{D_0^2+\alpha^2(\mathbf{f}'-\mathbf{c})^2}\Phi=0, \tag{20}$$

$$\widetilde{\phi}=0 \quad \text{at} \quad z=1, \tag{21}$$

where $\mathbf{I}$ is an $(N+1)\times(N+1)$ identity matrix, $\Phi=[\widetilde{\phi}_0, \widetilde{\phi}_1, \widetilde{\phi}_2, \ldots, \widetilde{\phi}_N]^T$ and

$$\mathbf{g}'=\operatorname{diag}(g'(z_j)), \qquad \mathbf{h}'=\operatorname{diag}(h'(z_j)),$$

$$\mathbf{f}'=\operatorname{diag}(f'(z_j)), \qquad \mathbf{f}'''=\operatorname{diag}(f'''(z_j)).$$

Here $\operatorname{diag}()$ signifies that the entries of equation (20) are placed on the main diagonal of an $(N+1)\times(N+1)$ matrix with the rest of the entries of the matrix being zero. By applying the Taylor series expansion to the last two terms of the equation about $(\mathbf{f}'-c)=0$, we can rewrite equation (20) as the nonlinear eigenvalue problem

$$M(c)\Phi=0, \tag{22}$$

where $M(c)=\mathbf{A}_0c^2+\mathbf{A}_1c+\mathbf{A}_2$, with,

$$\mathbf{A}_0=\frac{4}{L^2}\mathbf{D}^2-\alpha^2\mathbf{I}+\frac{\mathbf{2}\alpha^2G_{c0}}{LD_0^2}\left(1-\frac{24\alpha^2}{L^2D_0^2}\right)\mathbf{h}',$$

$$\mathbf{A}_1=\frac{8}{L^3}\mathbf{f}'''-\frac{4}{L}\mathbf{f}'\left(\frac{4}{L^2}\mathbf{D}^2-\alpha^2\mathbf{I}\right)-\frac{8\alpha^2G_{c0}}{L^2D_0^2}\mathbf{f}'\mathbf{h}'+\frac{64\alpha^4G_{c0}}{L^4D_0^4}\mathbf{f}'^3\mathbf{h}',$$

$$\mathbf{A}_2=\frac{4}{L^2}\mathbf{f}'^2\left(\frac{4}{L^2}\mathbf{D}^2-\alpha^2\mathbf{I}\right)-\frac{16}{L^4}\mathbf{f}'\mathbf{f}'''+\frac{2}{L}G_{t0}\mathbf{g}'$$

$$+\frac{8\alpha^2G_{c0}}{D_0^2L^3}\mathbf{f}'^2\mathbf{h}'-\frac{32\alpha^4G_{c0}}{L^5D_0^2}\mathbf{h}',$$

where the matrices $\mathbf{A}_0$, $\mathbf{A}_1$, $\mathbf{A}_2$ all have dimensions $(N+1)\times(N+1)$ and the boundary conditions have been explicitly incorporated in the first and the $(N+1)$th rows of $\mathbf{A}_0$, $\mathbf{A}_1$ and $\mathbf{A}_2$. By defining $\mathbf{\Phi}_1=c\mathbf{\Phi}$, we can transform equation (22) into the linear generalized eigenvalue problem

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{\Phi}_1 \\ \mathbf{\Phi} \end{bmatrix} = c \begin{bmatrix} -\mathbf{A}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{\Phi}_1 \\ \mathbf{\Phi} \end{bmatrix}, \tag{23}$$

where $\mathbf{0}$ is an $(N+1) \times (N+1)$ matrix of zeros. Equation (23) is a generalized eigenvalue problem that can be solved using any standard globally convergent numerical scheme.

4. Results and Discussion

Whereas in Motsa and Sibanda [10] the primary objective was to determine the impact of the flexible surface dynamics on the stability of the boundary layer flow, and to that end a *surface* parameter sensitivity analysis was carried out, in this study our primary focus is on the effect of reaction kinematics on the growth of the instability waves. The numerical results thus seek to show the effect of fluid buoyancy, the Damkohler and the Schmidt numbers on the fluid properties and the growth of the instability waves. The effects of buoyancy and Damkohler numbers on the velocity distribution are shown in Figure 1. Figure 1(a) depicts the velocity distribution for different values of the effective buoyancy parameter when the Damkohler number is held constant.

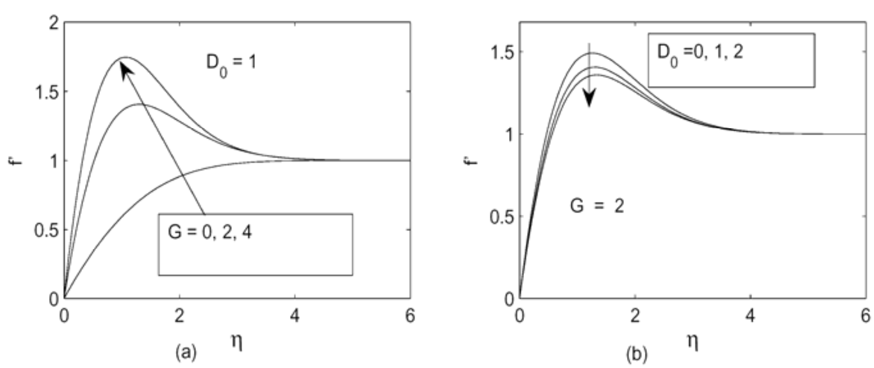


Figure 1. The effect of (a) buoyancy, and (b) the Damkohler number on the basic velocity $f'(\eta)$ when $Pr = Sc = 0.5$.

Increasing buoyancy results in significant increases in the velocity of the flow near the surface. The velocity reaches a distinguished maximum value near the boundary surface. Beyond this maximum the velocity asymptotically reduces to the free stream value.

Figure 1(b) shows the effect of the Damkohler number on the velocity distribution for fixed buoyancy. The Damkohler number is a very important parameter that gives a characterization of whether the reaction mechanism is diffusion controlled (when $D_0 \gg 1$) or kinematically controlled $(D_0 \ll 1)$. A Damkohler number, $D_0 \sim O(1)$ would therefore indicate parity between the characteristic mixing and chemical reaction time scales. High Damkohler numbers tend to promote low velocities and therefore high mixing of the fluids. Figure 2(a) shows the effect of increasing the buoyancy force on the species concentration profiles for fixed Damkohler numbers. We observe a reduction in concentration levels of the fluid as buoyancy is increased. A similar trend was also observed in the study by Motsa and Sibanda [10] for hydrodynamic and thermal boundary layers.

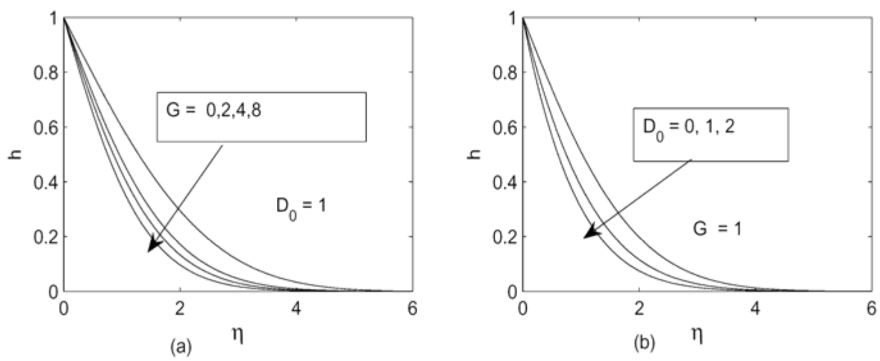


Figure 2. Plot to show the effect of (a) buoyancy, and (b) the Damkohler number on the concentration profiles when $Pr = Sc = 0.5$.

Figure 2(b) shows the effect of the Damkohler number on the species concentration distribution for fixed buoyancy. As can be seen on these plots, increasing the Damkohler number when other fluid and thermodynamic parameters are fixed reduces the concentration levels in the boundary layer flow. In Figure 3 we show the effect of the Schmidt numbers on the velocity and concentration distributions. The Schmidt number is the ratio of momentum diffusivity to mass diffusivity. It represents the relative ease of molecular momentum and mass transfer and is a very important parameter in calculations of binary mass transfer in multi-phase flows. The computations here have been carried out for low Schmidt numbers since for most gases $Sc \sim O(1)$. However, in many practical problems of interest, such as in the case of oxygen and carbon dioxide in oceans and lakes, Schmidt numbers may be very high, for example, $Sc \approx 700$ for CO_2 in seawater (Hasegawa and Kasagi [5]). The mixing of fluids however reduces as $Sc \to \infty$.

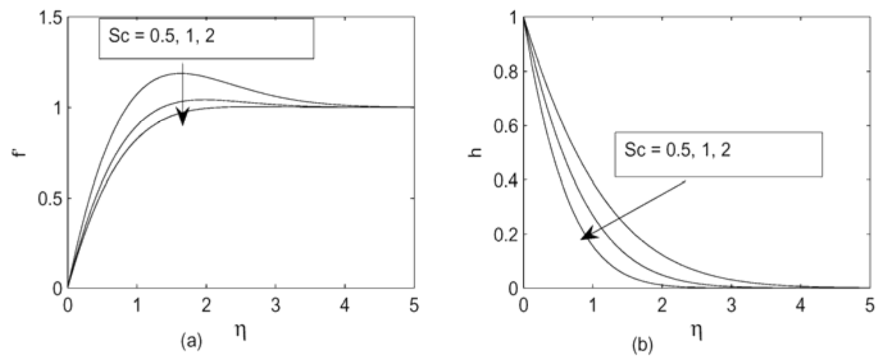
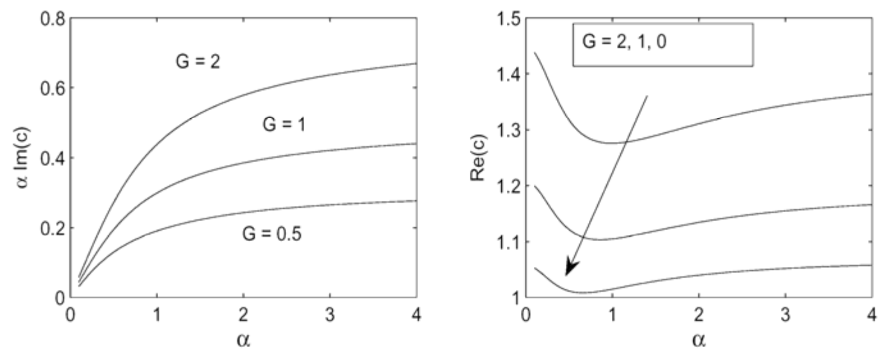


Figure 3. The effect of the Schmidt numbers on (a) the velocity flow, and (b) the concentration when $G = D_0 = 1$.

The effect of an increase in the Schmidt numbers is to reduce the momentum boundary layer and lead to a thinning of the species diffusion layer.

Figures 4 through 6 show the growth rate $\alpha Im(c)$ and the phase speed $Re(c)$ as functions of the wave number α. In Figure 4 we show the effect of increasing the buoyancy term on the growth rate and the phase speed. Increasing the buoyancy force increases both the growth rate of the disturbances and the phase speed. The effect of Damkohler number on the growth rate of the perturbations when wave numbers and speeds are low is marginal, see Figure 5(a). The growth rate however increases with increase in the Damkohler number as can be more clearly seen in Table 1 with increases becoming quite significant for high Damkohler numbers. In Table 1, the growth rate $\alpha Im(c)$ increases in each column with the wave number α. Further, simulations show that a change from a kinematically controlled reaction, $D_0 \ll 1$ to a diffusion controlled system, $D_0 \gg 1$ has no significant impact on the growth rate of the perturbations. However, increasing Damkohler numbers initially reduces the magnitude of the wave speed as is shown in Figure 5(b). Thus, overall, increasing the Damkohler number has only weakly destabilizing effects on the boundary layer flow that is governed by the TG equation. This result confirms the earlier tentative findings in Shateyi et al. [12] that increasing the Damkohler number has a destabilizing effect on Tollmien-Schlichting waves.



(a) (b)

Figure 4. The effect of increasing the buoyancy on (a) the disturbance growth rate $\alpha Im(c)$, and (b) the phase speed $Re(c)$.

Table 1. Comparison of the growth rate of the disturbance for various Damkohler numbers when $Pr = Sc = 1$ and $G = 4$

	$D_0 = 1 \times 10^{-5} \ll 1$	$D_0 = 1$	$D_0 = 2$	$D_0 = 5$
α	$\alpha Im(c)$	$\alpha Im(c)$	$\alpha Im(c)$	$\alpha Im(c)$
0.10	0.0873	0.0878	0.0882	0.0903
0.28	0.2415	0.2429	0.2439	0.2524
0.48	0.4054	0.4077	0.4091	0.4307
0.68	0.5647	0.5682	0.5695	0.6054
0.88	0.7375	0.7446	0.7475	0.7739
1.08	0.8977	0.9054	0.9090	0.9329
1.28	1.0329	1.0406	1.0443	1.0776
1.48	1.1485	1.1562	1.1601	1.2032
1.68	1.2487	1.2563	1.2604	1.3076
1.88	1.3366	1.3440	1.3482	1.3961

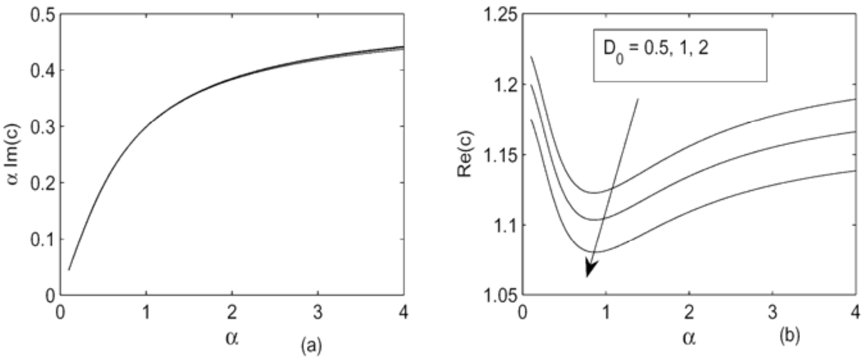


Figure 5. The effect of the Damkohler number on (a) the growth rate, and (b) phase speed.

Lastly, Figure 6 shows the effect of the Schmidt numbers on (a) the growth rate, and (b) the wave speed of the flow perturbations. For low Schmidt numbers (corresponding to fast mixing and chemical reactions) and $\alpha > 1$, we observe an increase in the growth rate of the disturbances suggesting a destabilization of the boundary layer flow with increasing Schmidt numbers. For $Sc > 0.2$, a steady growth of the disturbances is observed. However, for $Sc \leq 0.2$ a significant damping of the disturbances occurs when $\alpha \geq 1$ leading to a rapid decline in the growth rate of the disturbances. Figure 6(b) shows that the phase speed is reduced by increases in Schmidt numbers. At each Schmidt number the phase speed initially decreases for small wave numbers before increasing to a maximum for large wave numbers.

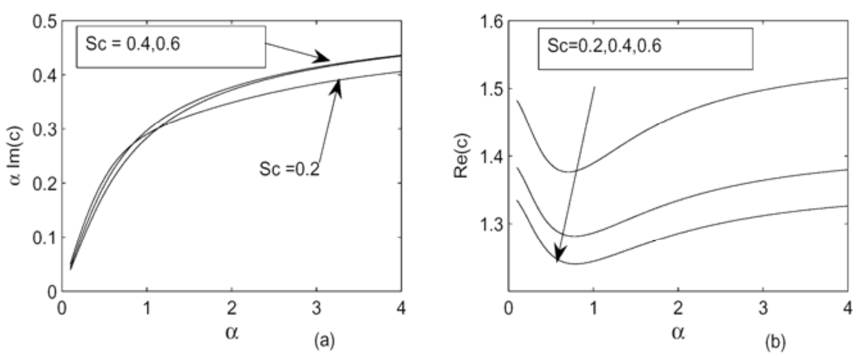


Figure 6. The (a) growth rate, and (b) phase speed for varying Schmidt numbers.

5. Conclusions

In this paper we have considered the stability of inviscid wave modes generated over a horizontal flat surface in the presence of a large buoyancy force and reactive kinematics. The governing boundary layer equations were first cast into a similarity form and solved to determine the effects of the fluid buoyancy, the Damkohler and the Schmidt numbers on the momentum and thermal boundary layer profiles. A Taylor-Goldstein equation that governs the evolution of the inviscid disturbance modes was derived and solved using a Chebyshev spectral collocation method to find the eigenvalues and the growth rates of the wave modes. This study has shown, *inter alia*, that (i) the effect of increasing fluid buoyancy is to increase the growth rate and the phase speed of the disturbance waves and thereby destabilizing the boundary layer flow, and (ii) increasing the Damkohler number has only a marginal effect on the growth rate of the inviscid wave modes for small wave numbers although the phase speed decreases before increasing in tandem with increase in the wave number. The effect of the Damkohler numbers on the growth rate of the inviscid wave modes is thus only significant for large wave numbers. The study has also shown that increasing the Schmidt number reduces the boundary layer velocity and the growth rate of the disturbances.

Acknowledgement

The authors wish to acknowledge financial support from the National Research Foundation (NRF) and the Norwegian Universities Aid Programme (NUFU).

References

[1] J. P. Boyd, The optimization of convergence for Chebyshev polynomial methods in unbounded domains, J. Comp. Phys. 45 (1982), 43-79.

[2] J. E. Broadwell and R. E. Breidenthal, A simple model of mixing and chemical reaction in a turbulent shear layer, J. Fluid Mech. 125 (1982), 397-410.

[3] P. E. Dimotakis, Turbulent shear layer mixing with fast chemical reactions, US-France Workshop on Turbulent Reactive Flows, 7-10 July, Rouen, France, 1987.

[4] B. Gebhart and L. Pera, The nature of vertical natural convection flows resulting

from the combined buoyancy effects of thermal and mass diffusion, Int. J. Heat Mass Transfer 14 (1971), 2025-2050.

[5] Y. Hasegawa and N. Kasagi, Proceedings of the 4th International Conference on Multiphase Flow, May 27 - June 1, New Orleans, USA, 2001.

[6] E. Magyari and B. Keller, Heat and mass transfer in the boundary layers on exponentially stretching continuous surface, J. Phys. D: Appl. Phys. 32 (2001), 577-585.

[7] W. G. Mathers, A. J. Maden and E. L. Piret, Simultaneous heat and mass transfer in free convection, Ind. Eng. Chem. 49 (1957), 961-968.

[8] P. L. Miller, Mixing in high Schmidt number turbulent jets, Ph.D. Thesis, California Institute of Technology, Pasadena, California, 1991.

[9] S. S. Motsa, J. S. B. Gajjar and P. Sibanda, On the upper-branch stability of boundary-layer flows over compliant surfaces with heating/cooling, Comp. Assist. Mech. Eng. Sci. 9 (2002), 163-181.

[10] S. S. Motsa and P. Sibanda, Investigation of compliancy effects on the inviscid instability in fluid flow over a flat plate with heat transfer, Int. J. Num. Methods Heat & Fluid Flow 15 (2005), 504-516.

[11] E. W. Mureithi, J. P. Denier and A. K. Stott, The effect of buoyancy on the upper-branch Tollmien-Schlichting waves, IMA J. Appl. Math. 58 (1997), 19-50.

[12] S. Shateyi, P. Sibanda, and S. S. Motsa, Asymptotic and numerical analysis of convection in boundary-layer flows in the presence of chemical reaction, Arch. Mech. 57 (2005), 25-41.

[13] E. V. Somers, Theoretical considerations of combined thermal and mass transfer from a vertical flat plate, J. Appl. Mech. 23 (1956), 295- 301.

■