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FREE CONVECTIVE NON-NEWTONIAN MHD FLOW AND HEAT TRANSFER THROUGH POROUS MEDIUM BETWEEN TWO LONG VERTICAL WAVY WALLS

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Abstract

The two dimensional problem of free convective MHD flow of a non-Newtonian electrically conducting fluid in porous medium confined between two long vertical wavy walls has been investigated under the assumption that the wavelengths of the wavy walls are large. The amplitude of the wavy walls considered are different. A uniform magnetic field is assumed to be applied perpendicular to the walls in the absence of waviness. Regular perturbation technique is used to solve the problem, where perturbation parameter is inversely proportional to the wavelength. Expressions for dimensionless velocity, temperature and shearing stress at both the walls have been obtained and numerically worked out for different values of the parameters involved in the solution. The shearing stress has been presented graphically for various non-Newtonian parameters.

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1. Introduction

Analysis of non-Newtonian fluid through porous media over a wavy wall has received special attention as a result of increasing practical interest in different areas of modern technology and in industrial applications. The interest in MHD flow stems from the fact that the liquid metals that occur in nature and industry are mathematically interesting and physically useful but the dynamical study of such flow problems is quite complicated. However, these problems are usually investigated under various simplifying assumptions.

Benjamin [2] was probably the first to consider the problem of the flow over a wavy wall. His analysis is based on the assumption of parallel flow in absence of waviness. The steady streaming generated by an oscillatory viscous flow over a wavy wall under the assumption that the amplitude of the wave is smaller than the Stoke's boundary layer thickness has been investigated by Lyne [6]. Lekoudis et al. [4] have presented a linear analysis of compressible boundary layer flow over a wavy wall. Shankar and Sinha [8] have made the detailed study of the Rayleigh problem for a wavy wall. It was found that at the low Reynolds number, the waviness of the wall quickly ceases to be of importance as the liquid is dragged along the wall, while at large Reynolds number, the effect of viscosity are confined to a thin layer close to the wall. Lessen and Gangwani [5] made a very interesting analysis of the effect of small amplitude wall waviness upon the stability of laminar boundary layer. Vajravelu and Sastri [9] have studied the problem of free convective heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall. The free convection of a viscous incompressible fluid in porous medium between two long vertical wavy walls has been investigated by Patidar and Purohit [7]. Ahmed et al. [1] have extended the problem to MHD case. Choudhury and Das [3] have studied this problem for visco-elastic fluid.

In this paper, the steady free convective MHD flow of a non-Newtonian fluid characterized by Walters liquid (Model B') confined between two long vertical wavy walls in porous medium has been investigated, when the amplitude of the waviness of both the walls are different. The boundary conditions at the surfaces are obtained by applying Taylor's series expansions for the variable quantities. The shearing stress at the wavy walls has also been presented graphically for various non-Newtonian parameters.

The constitutive equation for Walters liquid (Model B') is

$$\sigma^{ik} = -pg_{ik} + \sigma'_{ik}$$

$$\sigma'_{ik} = 2\eta_0 e^{ik} - 2k_0 e'^{ik}, \tag{1.1}$$

where σ^{ik} is the stress tensor, p is isotropic pressure, g_{ik} is the metric tensor of a fixed co-ordinate system x^i, v_i is the velocity vector, the contravariant form of e'^{ik} is given by

$$e'^{ik} = \frac{\partial e^{ik}}{\partial t} + v^m e^{ik}_{,m} - v^k_{,m} e^{im} - v^i_{,m} e^{mk}. \tag{1.2}$$

It is the convected derivative of the deformation rate tensor e^{ik} defined by

$$2e^{ik} = v_{i,k} + v_{k,i}. \tag{1.3}$$

Here η_0 is the limiting viscosity at the small rate of shear which is given by

$$\eta_0 = \int_0^\infty N(\tau)d\tau \quad \text{and} \quad k_0 = \int_0^\infty \tau N(\tau)d\tau, \tag{1.4}$$

$N(\tau)$ being the relaxation spectrum as introduced by Walters [11, 12]. This idealized model is a valid approximation of Walters liquid (Model B') taking very short memories into account so that terms involving

$$\int_0^\infty t^n N(\tau)d\tau, \; n \geq 2, \tag{1.5}$$

have been neglected.

2. Formulation of the Problem

Consider the free convective hydromagnetic flow of a Walters liquid (Model B') between two long vertical wavy non-electrically conducting walls in porous medium. We consider a set of Cartesian coordinates so that the $\bar{x}$ -axis is taken parallel to the walls, if there were no waviness in the walls. $\bar{y}$ -axis is taken perpendicular to it. Let the equations of the two wavy walls are given by $\bar{y} = \bar{\varepsilon} \cos \bar{\lambda}\bar{x}$ and $\bar{y} = d(1 + h\bar{\varepsilon} \cos \bar{\lambda}\bar{x})$, where $\bar{\varepsilon}$ and $dh\bar{\varepsilon}$ are the amplitudes of respective walls. Both the walls are maintained at constant but different temperatures T_1 and T_2. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected.

The boundary conditions relevant to the problem are taken as:

$$\bar{y} = \bar{\varepsilon} \cos \bar{\lambda}\bar{x};\ \bar{u} = 0,\ \bar{v} = 0,\ \bar{T} = T_1$$

$$\bar{y} = d(1 + h\bar{\varepsilon} \cos \bar{\lambda}\bar{x});\ \bar{u} = 0,\ \bar{v} = 0,\ \bar{T} = T_2, \tag{2.1}$$

where d is the distance between the two walls, if there were no waviness in the walls and h is the amplitude parameter for the second wavy wall.

We introduce the following non-dimensional parameters:

$$x = \frac{\bar{x}}{d},\ y = \frac{\bar{y}}{d},\ u = \frac{\bar{u}d}{\nu},\ v = \frac{\bar{v}d}{\nu},\ p = \frac{\bar{p}}{\rho\left(\frac{\nu}{d}\right)^2},$$

$$\theta = \frac{\bar{T} - T_s}{T_1 - T_s},\ M = \frac{gB_0^2 d^2}{\rho\nu}, \tag{2.2}$$

where $\bar{u}, \bar{v}$ are the velocity components, $\bar{p}$ the pressure, $g\beta(\bar{T} - T_s)$ the buoyancy force, k the permeability parameter, B_0 the uniform magnetic induction, T_s is the fluid temperature in static conditions and the other symbols have their usual meanings.

Introducing the non-dimensional parameters (2.2) in the governing equations for velocity and temperature, we obtain the equation of continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial \nu}{\partial y} = 0 \tag{2.3}$$

the momentum equations:

$$\begin{aligned}
u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= -\frac{\partial p}{\partial x} + \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \\
&- k_1\left[u\frac{\partial^3 u}{\partial x^3} + u\frac{\partial^3 u}{\partial x \partial y^2} + v\frac{\partial^3 u}{\partial x^2 \partial y}\right. \\
&+ v\frac{\partial^3 u}{\partial y^3} - 3\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} \\
&\left. - 2\frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial x^2}\right] + G_r\theta - \alpha^2 u - Mu,
\end{aligned} \tag{2.4}$$

$$\begin{aligned}
u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} &= -\frac{\partial p}{\partial y} + \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \\
&- k_1\left[u\frac{\partial^3 v}{\partial x^3} + u\frac{\partial^3 v}{\partial x \partial y^2} + v\frac{\partial^3 v}{\partial x^2 \partial y}\right. \\
&+ v\frac{\partial^3 v}{\partial y^3} - 3\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial y^2} - \frac{\partial v}{\partial x}\frac{\partial^2 v}{\partial x \partial y} \\
&\left. - 2\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial x \partial y} + \frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial x^2} - \frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial y^2}\right] - \alpha^2 v,
\end{aligned} \tag{2.5}$$

and the energy equation:

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{P_r}\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) \tag{2.6}$$

subject to the boundary conditions:

$$\begin{aligned}
&y = \varepsilon \cos \lambda x;\ u = 0,\ v = 0,\ \theta = 1, \\
&y = 1 + \alpha_1 \varepsilon \cos \lambda x;\ u = 0,\ v = 0,\ \theta = m,
\end{aligned} \tag{2.7}$$

where

$\alpha = \frac{d}{\sqrt{k}}$, the dimensionless porosity parameter

$G_r = \frac{g\beta d^3 (T_1 - T_s)}{\nu^2}$, the Grashoff number

$P_r = \frac{\nu}{a}$, the Prandtl number

$\varepsilon = \frac{\bar{\varepsilon}}{d}$, dimensionless amplitude

$\lambda = \bar{\lambda} d$, modified frequency

$m = \frac{(T_2 - T_s)}{(T_1 - T_s)}$, the wall temperature ratio

$\alpha_1 = hd$, the amplitude parameter

$k_1 = \frac{k_0}{\rho d^2}$, the non-Newtonian parameter.

3. Solution of the Problem

To solve the equations (2.3) to (2.6), subject to the boundary conditions (2.7), let us assume that the solutions consist of two parts, a mean part and a perturbed part (which is contributed from the waviness of two walls) as given below:

$$u(x, y) = u_0(y) + \varepsilon u_1(x, y),\ v(x, y) = \varepsilon v_1(x, y),$$

$$p(x, y) = p_0(y) + \varepsilon p_1(x, y),\ \theta(x, y) = \theta_0(y) + \varepsilon \theta_1(x, y), \tag{3.1}$$

where u_0, p_0, θ_0 and u_1, v_1, p_1, θ_1 are the mean and perturbed parts of the velocity, pressure and temperature, respectively. On substituting (3.1) in the equations (2.3) to (2.6) and equating the coefficients of like powers of ε, we obtain the following set of differential equations:

The zeroth-order equations:

$$u_0'' - (\alpha^2 + M)u_0 = -G_r\theta_0, \tag{3.2}$$

$$p_0' = 0, \tag{3.3}$$

$$\theta_0'' = 0, \tag{3.4}$$

where primes denote the differentiation with respect to y.

The corresponding boundary conditions are

$$\begin{aligned} &y = 0;\ u_0 = 0,\ \theta_0 = 1, \\ &y = 1;\ u_0 = 0,\ \theta_0 = m. \end{aligned} \tag{3.5}$$

Zeroth-order solutions are:

$$\theta_0 = 1 + (m-1)y, \tag{3.6}$$

$$u_0 = C_1 e^{-\lambda_1 y} + C_2 e^{\lambda_1 y} + \frac{G_r}{\lambda_1^2}[1 + (m-1)y], \tag{3.7}$$

where C_1, C_2 and λ_1 are constants and are given by

$$C_1 = \frac{G_r(m - e^{\lambda_1})}{2\lambda_1^2 \sin h\lambda_1},\ C_2 = -C_1 - \frac{G_r}{\lambda_1^2} \text{ and } \lambda_1^2 = \alpha^2 + M.$$

First-order equations:

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0, \tag{3.8}$$

$$\begin{aligned} &\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} - u_0\frac{\partial u_1}{\partial x} - \lambda_1^2 u_1 \\ &= v_1 u_0' + \frac{\partial p_1}{\partial x} - G_r\theta_1 \\ &\quad + k_1\left[u_0\frac{\partial^3 u_1}{\partial x^3} + u_0\frac{\partial^3 u_1}{\partial x \partial y^2} + v_1 u_0''' - u_0'\frac{\partial^2 u_1}{\partial x \partial y} + u_0''\frac{\partial u_1}{\partial x} - u_0'\frac{\partial^2 v_1}{\partial x^2}\right], \end{aligned} \tag{3.9}$$

$$\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2} - u_0 \frac{\partial v_1}{\partial x} - \alpha^2 v_1$$

$$= \frac{\partial p_1}{\partial y} + k_1 \left[u_0 \frac{\partial^3 v_1}{\partial x^3} + u_0 \frac{\partial^3 v_1}{\partial x \partial y^2} - 2u_0' \frac{\partial^2 v_1}{\partial x \partial y} - u_0'' \frac{\partial v_1}{\partial x} \right], \tag{3.10}$$

$$\frac{\partial^2 \theta_1}{\partial x^2} + \frac{\partial^2 \theta_1}{\partial y^2} = P_r \left(u_0 \frac{\partial \theta_1}{\partial x} + v_1 \theta_0' \right). \tag{3.11}$$

Corresponding boundary conditions are:

$$y = 0;\ u_1 = -u_0' \cos \lambda x,\ v_1 = 0,\ \theta_1 = -\theta_0' \cos \lambda x,$$

$$y = 1;\ u_1 = -\alpha_1 u_0' \cos \lambda x,\ v_1 = 0,\ \theta_1 = -\alpha_1 \theta_0' \cos \lambda x. \tag{3.12}$$

In order to solve the equations (3.8) to (3.11), we introduce stream function $\overline{\psi}_1(x, y)$ as:

$$u_1 = -\frac{\partial \overline{\psi}_1}{\partial y},\ v_1 = \frac{\partial \overline{\psi}_1}{\partial x}. \tag{3.13}$$

This stream function identically satisfies continuity equation.

Eliminating the pressure term from the equations (3.9) and (3.10), we obtain

$$\overline{\psi}_{1,xxxx} - u_0 \overline{\psi}_{1,xxx} - \alpha^2 \overline{\psi}_{1,xx} + 2\overline{\psi}_{1,xxyy} + \overline{\psi}_{1,yyyy} - u_0 \overline{\psi}_{1,xyy}$$

$$- \lambda_1^2 \overline{\psi}_{1,yy} + u_0'' \overline{\psi}_{1,x}$$

$$= G_r \theta_{1,y} + k_1 [u_0 \overline{\psi}_{1,xxxxx} + 2u_0 \overline{\psi}_{1,xxxyy} + u_0 \overline{\psi}_{1,xyyyy} - u_0^{iv} \overline{\psi}_{1,x}]. \tag{3.14}$$

To solve (3.11) and (3.14), we take

$$\overline{\psi}_1(x, y) = e^{i\lambda x} \psi(y) \text{ and } \theta_1(x, y) = e^{i\lambda x} \phi(y). \tag{3.15}$$

Applying this in (3.11) and (3.14), we obtain

$$\psi^{iv} - \psi''(2\lambda^2 + i\lambda u_0 + \lambda_1^2) + \psi(i\lambda u_0'' + i\lambda^3 u_0 + \lambda^4 + \alpha^2 \lambda^2)$$

$$= G_r \phi' + ik_1 [\lambda u_0 \psi^{iv} - 2u_0 \lambda^3 \psi'' - \psi(\lambda u_0^{iv} - u_0 \lambda^5)], \tag{3.16}$$

$$\phi'' - \phi(\lambda^2 + iP_r \lambda u_0) = iP_r \lambda \theta_0' \psi. \tag{3.17}$$

Thus, the corresponding boundary conditions are:

$$y = 0;\ \psi' = \lambda_1(C_2 - C_1) + \frac{G_r}{\lambda_1^2}(m-1),\ \psi = 0,\ \phi = 1 - m,$$

$$y = 1;\ \psi' = \alpha_1\left\{\lambda_1(C_2 e^{\lambda_1} - C_1 e^{-\lambda_1}) + \frac{G_r}{\lambda_1^2}(m-1)\right\},\ \psi = 0,\ \phi = \alpha_1(1-m). \quad (3.18)$$

If we consider only small values of λ, let

$$\psi(\lambda,\ y) = \sum_i \lambda^i \psi_i \ \text{ and } \ \phi(\lambda,\ y) = \sum_i \lambda^i \phi_i, \quad (3.19)$$

and substituting it in (3.16) and (3.17) and equating the like powers of λ, we obtain

$$\psi_0^{iv} - \lambda_1^2 \psi_0'' = G_r \theta_0', \quad (3.20)$$

$$\phi_0'' = 0, \quad (3.21)$$

$$\psi_1^{iv} - \lambda_1^2 \psi_1'' + iu_0''\psi_0 - iu_0\psi_0'' = G_r\phi_1' + ik_1[u_0\psi_0^{iv} - \psi_0 u_0^{iv}], \quad (3.22)$$

$$\phi_1'' = iP_r(u_0\phi_0 + \theta_0'\psi_0). \quad (3.23)$$

The corresponding boundary conditions are:

$$y = 0;\ \psi_0' = \lambda_1(C_2 - C_1) + \frac{G_r}{\lambda_1^2}(m-1),\ \psi_0 = 0,\ \phi_0 = 1 - m,$$

$$y = 1;\ \psi_0' = \alpha_1\left\{\lambda_1(C_2 e^{\lambda_1} - C_1 e^{-\lambda_1}) + \frac{G_r}{\lambda_1^2}(m-1)\right\},\ \psi_0 = 0,\ \phi_0 = \alpha_1(1-m),$$

$$\psi_i' = \psi_i = \phi_i = 0,\ i \geq 1 \ \text{ and } \ y = 0 \ \text{ and } \ y = 1. \quad (3.24)$$

Equations (3.20) to (3.23) are ordinary differential equations and can be solved with the help of the boundary conditions (3.24).

The solutions of equations (3.20) to (3.23) are as follows:

$$\phi_0 = (1 - m)[1 + (\alpha_1 - 1)]y, \quad (3.25)$$

$$\psi_0 = A_3 + A_4 y + C_3 e^{\lambda_1 y} + C_4 e^{-\lambda_1 y} + \frac{M_1}{2\lambda_1^2} y^2, \quad (3.26)$$

$$\phi_1 = D_1 + D_2 y + iP_r(1-m)B_1 \frac{y^2}{2} + iP_r(1-m)B_2 \frac{y^3}{6} + iP_r(1-m)B_3 \frac{y^4}{12}$$
$$+ iP_r(1-m)\left[\frac{B_4}{\lambda_1^2} - \frac{2}{\lambda_1^3} B_6\right] e^{\lambda_1 y} + iP_r(1-m)\left[\frac{B_5}{\lambda_1^2} + \frac{2}{\lambda_1^3} B_7\right] e^{-\lambda_1 y}$$
$$+ iP_r(1-m)\frac{B_6}{\lambda_1^2} ye^{\lambda_1 y} + iP_r(1-m)\frac{B_7}{\lambda_1^2} ye^{-\lambda_1 y}, \tag{3.27}$$

$$\psi_1 = E_1 + E_2 y + L_1 y^2 + L_2 y^3 + L_3 y^4 + E_3 e^{\lambda_1 y} + E_4 e^{-\lambda_1 y}$$
$$+ L_5 ye^{\lambda_1 y} + L_6 ye^{-\lambda_1 y} + L_7 y^2 e^{\lambda_1 y}$$
$$+ L_8 y^2 e^{-\lambda_1 y} + L_9 y^3 e^{\lambda_1 y} + L_{10} y^3 e^{-\lambda_1 y}, \tag{3.28}$$

where the constants are obtained but not presented here, for the sake of brevity.

4. Results and Discussion

The dimensionless shearing stress σ_{nx} acting on the wall in vertical upward direction is given by

$$\sigma_{nx} = l\sigma_{xx} + m\sigma_{yx} + n\sigma_{zx},$$

where l, m and n are the directions cosines of the normal to the wall $y = \varepsilon \cos \lambda x$ given by

$$l = \hat{n} \cdot \hat{i},\ m = \hat{n} \cdot \hat{j},\ n = \hat{n} \cdot \hat{k},$$

where $\hat{i}$, $\hat{j}$, $\hat{k}$ are the unit vectors in x, y and z directions respectively, and $\hat{n}$ is the outward drawn unit normal vector at the surface $y = \varepsilon \cos \lambda x$.

$$\sigma_{nx} = \left[\frac{du_0}{dy} - \varepsilon(\psi_0'' + \lambda\psi_1'')e^{i\lambda x} - \varepsilon\lambda^2(\psi_0 + \lambda\psi_1)e^{i\lambda x}\right]$$
$$- k_1[-2\varepsilon\lambda \sin \lambda x u_0'^2 - i\varepsilon u_0 \lambda(\psi_0'' + \lambda\psi_1'')e^{i\lambda x}$$
$$- iu_0\lambda^3 e^{i\lambda x}(\psi_0 + \lambda\psi_1) - 2i\varepsilon\lambda^3 e^{2i\lambda x}(\psi_0 + \lambda\psi_1)(\psi_0' + \lambda\psi_1')$$
$$- i\varepsilon\lambda e^{2i\lambda x}(\psi_0 + \psi_1)(\psi_0''' + \lambda\psi_1''') - 6i\varepsilon u_0'\lambda e^{i\lambda x}(\psi_0' + \lambda\psi_1')] - p_0.$$

Similarly, the dimensionless shearing stress σ'_{nx} acting on the wall $y = 1 + \alpha_1 \varepsilon \cos \lambda x$ in vertical upward direction is given by

$$\sigma'_{nx} = \left[\frac{du_0}{dy} - \varepsilon(\psi''_0 + \lambda\psi''_1)e^{i\lambda x} - \varepsilon\lambda^2(\psi_0 + \lambda\psi_1)e^{i\lambda x}\right]$$

$$- k_1[-2\varepsilon\lambda \sin \lambda x \alpha_1 u'^2_0 - i\varepsilon u_0 \lambda(\psi''_0 + \lambda\psi''_1)e^{i\lambda x}$$

$$- iu_0\lambda^3 e^{i\lambda x}(\psi_0 + \lambda\psi_1) - 2i\varepsilon\lambda^3 e^{2i\lambda x}(\psi_0 + \lambda\psi_1)(\psi'_0 + \lambda\psi'_1)$$

$$- i\varepsilon\lambda e^{2i\lambda x}(\psi_0 + \psi_1)(\psi'''_0 + \lambda\psi'''_1) - 6i\varepsilon u'_0 \lambda e^{i\lambda x}(\psi'_0 + \lambda\psi'_1)] - p_0.$$

The heat transfer co-efficient N_u defined as:

$$N_u = \frac{d\theta_0}{dy} + \varepsilon R_e\left(e^{i\lambda x}\frac{d\phi_0}{dy} + \lambda e^{i\lambda x}\frac{d\phi_1}{dy}\right).$$

Hence the Nusselt number at the walls, $y = \varepsilon \cos \lambda x$ and $y = 1 + \alpha_1 \varepsilon \cos \lambda x$ are given by

$$N_{u_w} = N^0_{u_0} + \varepsilon R_e[e^{i\lambda x}\phi'_0(0) + \lambda e^{i\lambda x}\phi'_1(0)],$$

and

$$N_{u_w} = N^0_{u_1} + \varepsilon R_e[e^{i\lambda x}\phi'_0(1) + \lambda e^{i\lambda x}\phi'_1(1)],$$

where

$$N^0_{u_0} = \theta'_0(0) = m - 1 \text{ and } N^0_{u_1} = \theta'_0(1) = m - 1.$$

The purpose of this study is to bring out the effects of non-Newtonian parameter on the MHD flow and heat transfer characteristics as the effects of other parameters have been discussed by Ahmed et al. [1]. The non-Newtonian effect is exhibited through the non-dimensional parameter k_1. The corresponding results for Newtonian fluid is obtained by setting $k_1 = 0$ and it is worth mentioning that these results coincide with Ahmed et al. [1]. The velocity components u and v are not significantly affected by non-Newtonian fluid for both the walls $y = \varepsilon \cos \lambda x$ and $y = 1 + \alpha_1 \varepsilon \cos \lambda x$.

The dimensionless shearing stress components σ_{nx} at the wall $y = \varepsilon \cos \lambda x$ and σ'_{nx} at the wall $y = 1 + \alpha_1 \varepsilon \cos \lambda x$ against the Hartmann number M have been presented in Figures 1 to 4 for various combinations of parameters involved in the solution. It is observed that the shearing stress components σ_{nx} and σ'_{nx} decrease with the increase of the value of the Hartmann number M in both Newtonian and non-Newtonian cases, for $\alpha = 1$, $\alpha_1 = 2$, $m = 1$ $G_r = 5$ and $P_r = 0.7$ for both $x = 25$ and $x = 75$. Also, the components σ_{nx} and σ'_{nx} increase with the increasing value of the non-Newtonian parameter k_1(=0, 0.2, 0.4), for $x = 25$ and reverse behaviour of co-efficient of skin-friction is seen at both the walls for $x = 75$.

It has also been observed from the expression of θ that the temperature field is not significantly affected by the non-Newtonian parameter k_1.

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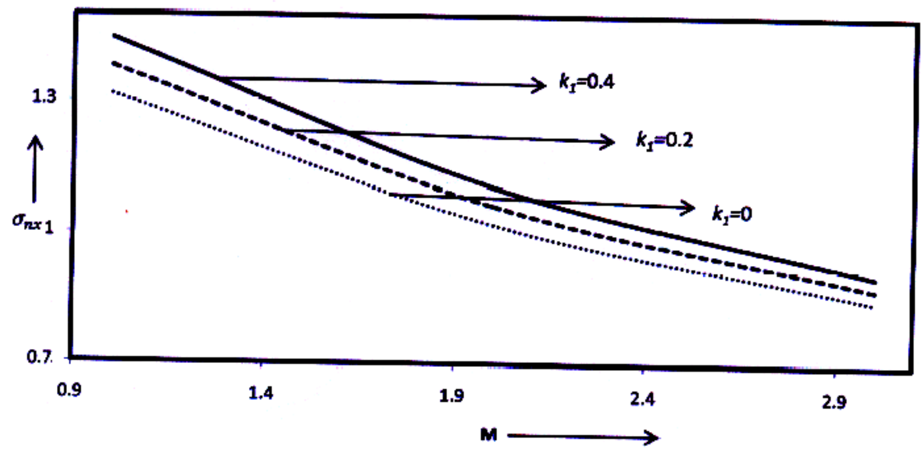

Figure 1. The shearing stress σ_{nx} acting on the wall $y = \varepsilon \cos \lambda x$ against the Hartmann number M for $\alpha = 1$, $\alpha_1 = 2$, $m = 1$, $G_r = 5$ and $P_r = 0.7$ and $x = 25$.

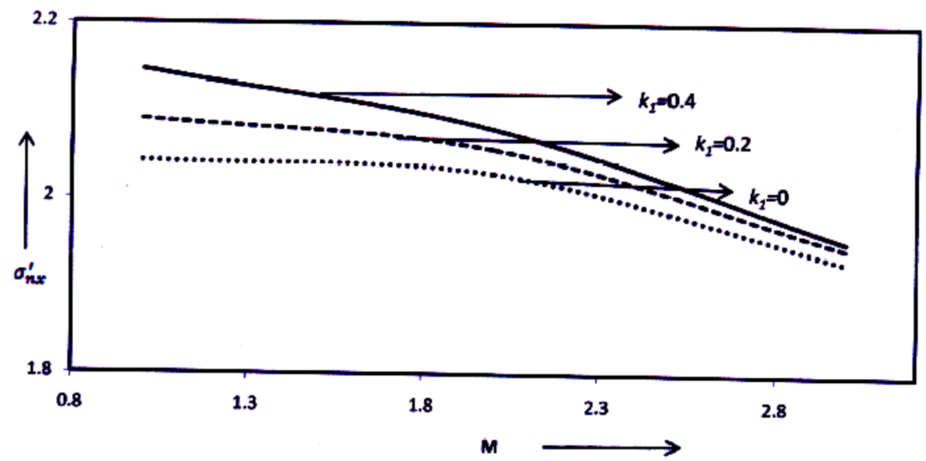

Figure 2. The shearing stress σ'_{nx} acting on the wall $y = 1 + \alpha_1 \varepsilon \cos \lambda x$ against the Hartmann number M for $\alpha = 1$, $\alpha_1 = 2$, $m = 1$, $G_r = 5$ and $P_r = 0.7$ and $x = 25$.

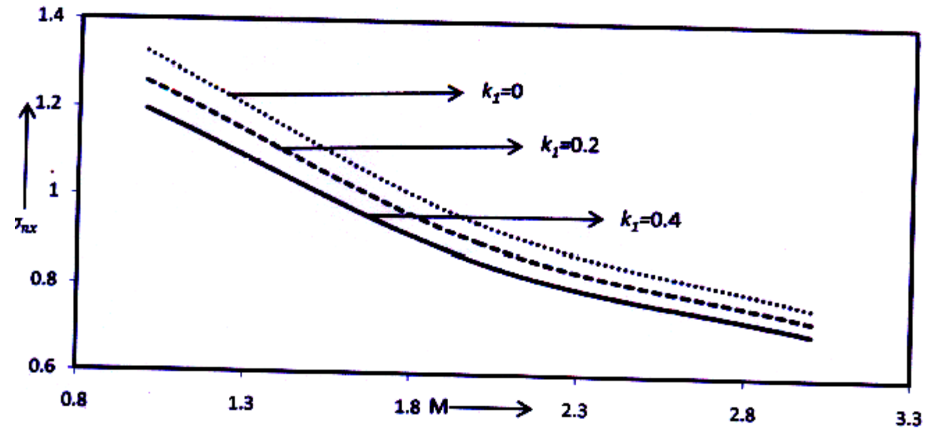

Figure 3. The shearing stress σ_{nx} acting on the wall $y = \varepsilon \cos \lambda x$ against the Hartmann number M for $\alpha = 1$, $\alpha_1 = 2$, $m = 1$, $G_r = 5$ and $P_r = 0.7$ and $x = 75$.

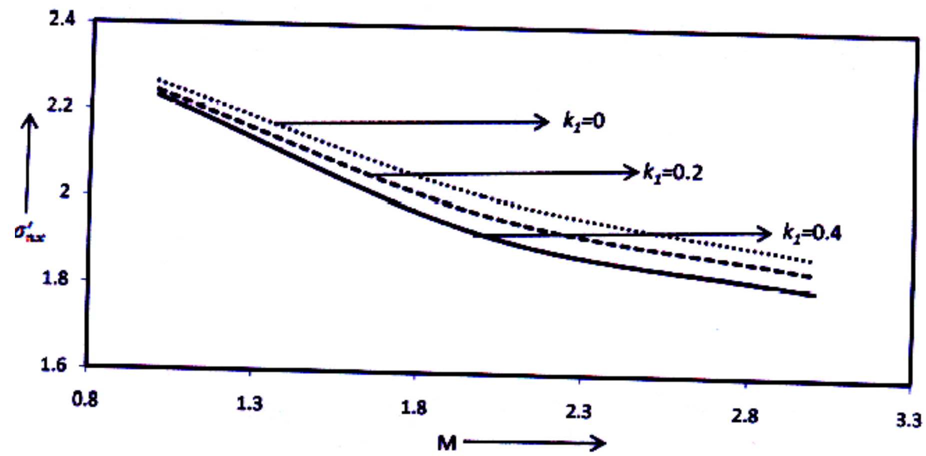

Figure 4. The shearing stress σ'_{nx} acting on the wall $y = 1 + \alpha_1 \varepsilon \cos \lambda x$ against the Hartmann number M for $\alpha = 1$, $\alpha_1 = 2$, $m = 1$, $G_r = 5$ and $P_r = 0.7$ and $x = 75$.

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