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ANOTHER DECOMPOSITION OF IRRESOLUTENESS

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Abstract

The aim of this paper is to study properties of λ-semi-closed sets and to provide other decompositions of semi-continuity and irresoluteness. We prove that a function $f : (X, \tau) \to (Y, \sigma)$ is semi-continuous (resp. irresolute) if and only if f is gs-continuous and λ-semi-continuous (resp. pre gs-continuous and strongly λ-semi-continuous).

1. Introduction and Preliminaries

As the decomposition of continuity is one of the many problems in general topology, many authors [6, 13-16, 32, 33] used generalized

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concepts of closed (or, open) set to solve the problem. Recently, Balachandran et al. [3] used 'generalized' closed (for example, g-closed, sg-closed, semi-closed) sets to get generalized concepts of locally closed set due to Bourbaki [7] and studied the relationship among those classes and some of their topological properties. Dontchev and Ganster [12] showed that the concept of semi-locally closed sets coincides with that of simply-open sets and gave a decomposition of irresoluteness by the help of pre sg-continuity. Dontchev and Maki [11] also solved Bhattacharyya and Lahiri's open problem (i.e., whether the intersection of sg-closed sets is sg-closed) and introduced the concept of semi-λ-closed sets, which contains the concept of semi-locally closed sets and define semi-λ-continuous function to give a decomposition of semi-continuity.

In this paper, we first introduce the concept of a λ-semi-closed set which is strictly placed between the notions of λ-closed and semi-λ-closed sets, and study its properties related to those of locally semi-closed sets. Finally, using these concepts, we define λ-semi-continuous and semi-λ-continuous functions and provide other decompositions of semi-continuity and irresoluteness.

Let (X, τ) be a topological space and $A \subset X$. The closure of A and the interior of A with respect to τ are denoted by $cl(A)$ and $int(A)$, respectively. The kernel [22] of A is the intersection of all open supersets of A and is denoted by $\ker(A)$. A subset A is said to be *semi-open* (resp. *semi-closed*) [21] if $A \subset cl(int(A))$ (resp. $int(cl(A)) \subset A$). The intersection of all semi-closed sets containing A is called the *semi-closure* [8] of A and is denoted by $scl(A)$. Dually, the semi-interior of A, denoted by $sint(A)$, is the union of all semi-open sets contained by A.

2. gs-closed Sets and λ-semi-closed Sets

Definition 2.1. A subset A of a space (X, τ) is called

(a) *sg-closed* [4] if $scl(A) \subset G$ whenever $A \subset G$ and G is semi-open,

(b) *gs-closed* [2] if $scl(A) \subset G$ whenever $A \subset G$ and G is open,

(c) *$\mathcal{g}s$-open* [2] if $F \subset sint(A)$ whenever $F \subset A$ and F is closed,

(d) *locally semi-closed* [30] if $A = G \cap F$ where G is open and F is semi-closed,

(e) *semi-locally closed* [30] if $A = G \cap F$ where G is semi-open and F is semi-closed,

(f) *simply-open* [27] if $A = U \cup N$ where U is open and N is nowhere dense.

In [18], Ganster et al. showed that the notions of semi-locally closed and simply-open sets are same. Arya and Nour [2] pointed out that the union (resp. intersection) of $\mathcal{g}s$-open (resp. $\mathcal{g}s$-closed) sets is not, in general, $\mathcal{g}s$-open (resp. $\mathcal{g}s$-closed). But we have

Theorem 2.2. (a) *If A and B are separated* (*i.e.*, $A \cap cl(B) = cl(A) \cap B = \varnothing$) *$\mathcal{g}s$-open sets, then $A \cup B$ is $\mathcal{g}s$-open.*

(b) *If A and B are $\mathcal{g}s$-closed sets such that their complements are separated, then $A \cap B$ is $\mathcal{g}s$-closed.*

Proof. (a) Let F be closed and $F \subset A \cup B$. Then $F \cap cl(A) \subset A$ and hence $F \cap cl(A) \subset sint(A)$. Similarly, $F \cap cl(B) \subset sint(B)$. Now, we have

$$\begin{aligned} F = F \cap (A \cup B) &\subset (F \cap cl(A)) \cup (F \cap cl(B)) \\ &\subset sint(A) \cup sint(B) \\ &\subset sint(A \cup B). \end{aligned}$$

Hence $A \cup B$ is $\mathcal{g}s$-open.

(b) It follows from (a) by taking complements.

Theorem 2.3. *Let (X, τ) be a space. Then a subset A of X is $\mathcal{g}s$-closed if and only if $scl(A) \subset \ker(A)$.*

Proof. Let G be any open set with $A \subset G$. Since A is $\mathcal{g}s$-closed, $scl(A) \subset G$ and hence $scl(A) \subset \ker(A)$. Conversely, let G be any open set such that $A \subset G$. By hypothesis, $scl(A) \subset \ker(A) \subset G$ and hence A is gs-closed.

Definition 2.4. A subset A of (X, τ) is said to be

(a) Λ-*set* [22] if A is intersection of open sets,

(b) *semi*-Λ-*set* [11] if A is intersection of semi-open sets,

(c) λ-*closed* [1] if $A = G \cap F$ where G is a Λ-set and F is closed,

(d) *semi*-λ-*closed* [11] if $A = G \cap F$ where G is a semi-Λ-set and F is semi-closed,

(e) λ-*semi-closed* if $A = G \cap F$ where G is a Λ-set and F is semi-closed.

Remark 2.5. (a) Every locally semi-closed set is λ-semi-closed (see Example 2.6 (a)). Every λ-closed set is λ-semi-closed and every λ-semi-closed set is semi-λ-closed (see Example 2.6(b)).

(b) In [11], Dontchev and Maki pointed out that the set $SLC(X)$ of all semi-locally closed sets of space (X, τ) is always a topology on X. However, the set $LSC(X)$ of all locally semi-closed sets is not, in general, a topology (see Example 2.6). If (X, τ) is a T_1 space, then the set $LSC(X)$ is the discrete topology on X. Moreover, if X is finite, then $LSC(X)$ is a base for a partition topology (i.e., open sets are closed) on X.

Example 2.6. (a) Let $\mathbb{N}$ be the set of all positive integers with the cofinite topology. Then the set of all even integers is λ-semi-closed but not locally semi-closed.

(b) Let $X = \{a, b, c\}$ with topology $\tau = \{X, \varnothing, \{a\}\}$. Then $\{c\}$ is λ-semi-closed but not λ-closed. Also, $\{a, c\}$ is semi-λ-closed but not λ-semi-closed.

Theorem 2.7. *For a subset A of a space (X, τ), the following are equivalent*:

(a) *A is λ-semi-closed.*

(b) $A = L \cap scl(A)$, *where L is a* Λ*-set*.

(c) $A = \ker(A) \cap scl(A)$.

(d) *A is intersection of locally semi-closed sets*.

Proof. The proofs are easy and hence omitted.

Theorem 2.8. *For a subset A of* (X, τ), *the following are equivalent*:

(a) *A is semi-closed*.

(b) *A is* g*s-closed and locally semi-closed*.

(c) *A is* g*s-closed and* λ*-semi-closed*.

Proof. (a) $\Rightarrow$ (b) $\Rightarrow$ (c) are clear from the facts that every semi-closed set is both gs-closed and locally semi-closed, and every locally semi-closed set is λ-semi-closed.

(c) $\Rightarrow$ (a) Since A is gs-closed, $scl(A) \subset \ker(A)$. On the other hand, since A is λ-semi-closed, by Theorem 2.7, $A = \ker(A) \cap scl(A)$. Thus, we have $scl(A) \subset \ker(A) \cap scl(A) = A$. This shows that A coincides with its semi-closure, i.e., A is semi-closed.

Definition 2.9. A space (X, τ) is *SG-space* [3] (resp. *SC-space*) if the intersection of a semi-closed set with a g-closed (resp. closed) set is g-closed (resp. closed).

Every *SC*-space is an *SG*-space but the converse is not true.

Example 2.10. Let $X = \{a, b, c\}$ with topology $\tau = \{X, \varnothing, \{a\}, \{a, b\}, \{a, c\}\}$. Since $\{b, c\}$ is semi-closed but not g-closed, (X, τ) is an *SC*-space which is not an *SG*-space.

Theorem 2.11. *For a subset A of an SC-space* (X, τ), *the following are equivalent*:

(a) *A is* g*s-closed*,

(b) $cl\{x\} \cap A \neq \varnothing$ *for each* $x \in scl(A)$,

(c) $scl(A) \setminus A$ *contains no nonempty closed set*.

Proof. (a) $\Rightarrow$ (b) Let $x \in scl(A)$. If $cl\{x\} \cap A = \varnothing$, then $A \subset (X \setminus cl\{x\})$ and so $scl(A) \subset (X \setminus cl\{x\})$, contradicting $x \in scl(A)$.

(b) $\Rightarrow$ (c) Let F be a closed set such that $F \subset scl(A) \setminus A$. If there exists an $x \in F$, then by (b), $\varnothing \neq cl\{x\} \cap A \subset F \cap A \subset (scl(A) \setminus A) \cap A$, a contradiction. Hence, $F = \varnothing$.

(c) $\Rightarrow$ (a) Let $A \subset G$ and G be open in X. If $scl(A) \not\subset G$, then $scl(A) \cap (X \setminus G)$ is nonempty semi-closed. Since the space is an SC-space, $scl(A) \cap (X \setminus G)$ is a nonempty closed subset of $scl(A) \setminus A$, a contradiction. Hence, $scl(A) \subset G$. This shows that A is gs-closed.

Corollary 2.12. *Let A be a gs-closed set of an SC-space (X, τ). Then A is semi-closed if and only if $scl(A) \setminus A$ is closed.*

Proof. Since A is semi-closed, $scl(A) \setminus A = \varnothing$ is closed. Conversely, suppose $scl(A) \setminus A$ is closed. Since A is gs-closed and $scl(A) \setminus A$ is closed subset of itself, by Theorem 2.11, $scl(A) \setminus A = \varnothing$. Hence, $scl(A) = A$.

Corollary 2.13. *Let (X, τ) be an SC-space.*

(a) *If $A \subset B \subset scl(A)$ and A is gs-closed, then B is gs-closed.*

(b) *If $sint(A) \subset B \subset A$ and A is gs-open, then B is gs-open.*

Proof. (a) Since $scl(B) \setminus B \subset scl(A) \setminus A$ and $scl(A) \setminus A$ has no nonempty closed subsets, neither does $scl(B) \setminus B$. Hence, B is gs-closed.

(b) It follows from (a) by taking complements.

Theorem 2.14. *For a subset A of an SC-space (X, τ), the following are equivalent:*

(a) *A is locally semi-closed.*

(b) *$scl(A) \setminus A$ is closed.*

(c) *$A \cup (X \setminus scl(A))$ is open.*

Proof. (a) $\Rightarrow$ (b) Since A is locally semi-closed, using Proposition 4.11 in [3], $A = G \cap scl(A)$ where G is open. Now $scl(A) \setminus A = scl(A) \setminus G$ $= scl(A) \cap (X \setminus G)$ where $scl(A)$ is semi-closed and $X \setminus G$ is closed. Since X is an SC-space, $scl(A) \cap (X \setminus G)$ is closed, i.e., $scl(A) \setminus A$ is closed.

(b) $\Rightarrow$ (c) Since $scl(A) \setminus A$ is closed, $A \cup (X \setminus scl(A)) = X \setminus (scl(A) \setminus A)$ is open.

(c) $\Rightarrow$ (a) Since $A = (X \setminus (scl(A) \setminus A)) \cap scl(A)$, by (c) $X \setminus (scl(A) \setminus A)$ is open. Hence A is locally semi-closed.

Definition 2.15. A subset A of (X, τ) is called *semi-dense* [3] if $scl(A) = X$.

Definition 2.16. A space (X, τ) is *sg-submaximal* [3] (resp. *submaximal* [7]) if every semi-dense (resp. dense) subset is g-open (resp. open) in (X, τ).

Every submaximal space is sg-submaximal but the converse is not true [3].

Theorem 2.17. *An SC-space* (X, τ) *is submaximal if and only if every subset of X is locally semi-closed.*

Proof. Let (X, τ) be submaximal and A be any subset of X. Put $U = A \cup (X \setminus scl(A))$. Then $scl(U) = X$, i.e., U is semi-dense in (X, τ). By hypothesis, U is open and hence, by Theorem 2.14, A is locally semi-closed.

Conversely, let A be dense in (X, τ) and suppose that every subset is locally semi-closed. Since A is locally semi-closed and $A = A \cup (X \setminus scl(A))$, by Theorem 2.14, A is open and hence X is submaximal.

3. Decompositions of Semi-continuity and Irresoluteness

Definition 3.1. A function $f : (X, \tau) \to (Y, \sigma)$ is called

(a) *semi-continuous* [21] (resp. *irresolute* [9]) if $f^{-1}(V)$ is semi-open in X for each open (resp. semi-open) set V of Y,

(b) *sg-continuous* [31] (resp. *pre-sg-irresolute* [24]) if $f^{-1}(V)$ is sg-closed in X for each closed (resp. semi-closed) set V of Y,

(c) *gs-continuous* [10] (resp. *pre-gs-continuous* [28]) if $f^{-1}(V)$ is gs-closed in X for each closed (resp. semi-closed) set V of Y.

Definition 3.2. A function $f : X \to Y$ be a mapping is called

(a) *simply-continuous* [27], or *SLC-continuous* [3] (resp. *strongly simply-continuous* [12]) if $f^{-1}(V)$ is simply-open in X for each closed (resp. semi-closed) set V of Y,

(b) *LSC-continuous* [3] (resp. *strongly LSC-continuous*) if $f^{-1}(V)$ is locally semi-closed in X for each closed (resp. semi-closed) set V of Y,

(c) *semi-λ-continuous* [11] (resp. *strongly semi-λ-continuous*) if $f^{-1}(V)$ is semi-λ-closed in X for each closed (resp. semi-closed) set V of Y,

(d) *λ-semi-continuous* (resp. *strongly λ-semi-continuous*) if $f^{-1}(V)$ is λ-semi-closed in X for each closed (resp. semi-closed) set V of Y.

Theorem 3.3. (a) *If $f : (X, \tau) \to (Y, \sigma)$ is semi-λ-continuous (resp. strongly semi-λ-continuous) and A is preopen in (X, τ), then $f|_A : (A, \tau_A) \to (Y, \sigma)$, the restriction of f to A, is also semi-λ-continuous (resp. strongly semi-λ-continuous).*

(b) *If $f : (X, \tau) \to (Y, \sigma)$ is λ-semi-continuous (resp. strongly λ-semi-continuous) and A is preopen in (X, τ), then $f|_A : (A, \tau_A) \to (Y, \sigma)$, the restriction of f to A, is also λ-semi-continuous (resp. strongly λ-semi-continuous).*

Proof. (a) We prove only in case f is semi-λ-continuous. Let V be open in Y. Since $f^{-1}(V)$ is semi-λ-closed, there exist a semi-Λ-set G and a semi-closed set F such that $(f|_A)^{-1}(V) = (G \cap A) \cap (F \cap A)$. By Lemma

2.2 in [26], $G \cap A$ is semi-Λ-set in (A, τ_A) and $F \cap A$ is semi-closed in (A, τ_A) since A is preopen. Hence, $(f|_A)^{-1}(V)$ is semi-λ-closed in (A, τ_A). This implies that $f|_A$ is semi-λ-continuous.

(b) The proof is similar to (a) using Lemma 2.3 in [25].

Lemma 3.4. *Suppose that a family of all semi-λ-closed (resp. λ-semi-closed) sets in (X, τ) is closed under finite union. Let $\{G_i \mid G_i$ is semi-λ-closed (resp. λ-semi-closed), $i \in \Gamma\}$ be a cover of X, where Γ is finite. If $A \cap G_i$ is semi-λ-closed (resp. λ-semi-closed) in (A, τ_A) for each $i \in \Gamma$, then A is semi-λ-closed (resp. λ-semi-closed).*

Proof. We prove in case of λ-semi-closed sets. Let $i \in \Gamma$. Since $A \cap G_i$ is λ-semi-closed in (A, τ_A), $A \cap G_i = H_i \cap K_i$ for some Λ-set H_i and semi-closed set K_i in (A, τ_A). Then there exist a Λ-set U_i and a semi-closed set V_i [25, Lemma 2.1] in (X, τ) such that $A \cap G_i = (U_i \cap G_i) \cap (V_i \cap G_i)$. Since G_i is semi-closed in (X, τ), $A \cap G_i = U_i \cap (G_i \cap V_i)$ is λ-semi-closed. Using assumption we have $A = \bigcup\{A \cap G_i \mid i \in \Gamma\}$ to be λ-semi-closed.

Theorem 3.5. *Suppose that a family of all semi-λ-closed (resp. λ-semi-closed) sets in (X, τ) is closed under finite unions. Let $X = G_1 \cup G_2$ where G_1, G_2 are semi-closed in (X, τ) and $f : (G_1, \tau_{G_1}) \to (Y, \sigma)$ and $g : (G_2, \tau_{G_2}) \to (Y, \sigma)$ be compatible functions.*

(a) *If f and g are semi-λ-continuous (resp. strongly semi-λ-continuous), then $f \nabla g : (X, \tau) \to (Y, \sigma)$ is also semi-λ-continuous (resp. strongly semi-λ-continuous).*

(b) *If f and g are λ-semi-continuous (resp. strongly λ-semi-continuous), then $f \nabla g : (X, \tau) \to (Y, \sigma)$ is also λ-semi-continuous (resp. strongly λ-semi-continuous).*

Proof. (a) We prove only the case of semi-λ-continuous. Let V be open in (Y, σ). Then for each $i \in \{1, 2\}$, $(f \nabla g)^{-1}(V) \cap G_i = f^{-1}(V)$ is semi-λ-

closed in (G_i, τ_{G_i}). Using Lemma 3.4, we have $(f\nabla g)^{-1}(V)$ to be a semi-λ-closed in (X, τ). Hence, $f\nabla g$ is semi-λ-continuous.

(b) The proof is similar to (a) using Lemma 2.3 in [25].

The proofs of the following results are immediate.

Theorem 3.6. *Let* $f : (X, \tau) \to (Y, \sigma)$ *and* $g : (Y, \sigma) \to (Z, \omega)$ *be two functions.*

(a) *If f is semi-λ-continuous (resp. strongly semi-λ-continuous) and* g *is continuous (resp. semi-continuous), then* $g \circ f$ *is semi-λ-continuous (resp. strongly semi-λ-continuous).*

(b) *If f is λ-semi-continuous (resp. strongly λ-semi-continuous) and* g *is continuous (resp. semi-continuous), then* $g \circ f$ *is λ-semi-continuous (resp. strongly λ-semi-continuous).*

Remark 3.7. (a) Every *LSC*-continuous (resp. strongly *LSC*-continuous) function is simply-continuous (resp. strongly simply-continuous) and every λ-semi-continuous (resp. strongly λ-semi-continuous) function is semi-λ-continuous (resp. strongly semi-λ-continuous) but the converses are not true.

(b) Every *LSC*-continuous (resp. strongly *LSC*-continuous, simply-continuous, strongly simply-continuous) function is λ-semi-continuous (resp. strongly λ-semi-continuous, semi-λ-continuous, strongly semi-λ-continuous) but the converses are not true.

(c) Suppose that (X, τ) is globally disconnected [14] (i.e., every set which can be placed between an open set and its closure is open). Then $f : (X, \tau) \to (Y, \sigma)$ is semi-λ-continuous (resp. strongly semi-λ-continuous, sg-continuous, pre-sg-continuous) if and only if f is λ-semi-continuous (resp. strongly λ-semi-continuous, gs-continuous, pre-gs-continuous).

Example 3.8. (a) Let $X = \{a, b, c\}$, $\tau = \{X, \varnothing, \{a\}, \{b, c\}\}$ and $\sigma = \{X, \varnothing, \{a\}\}$. Let $f : (X, \tau) \to (X, \sigma)$ be the identity function. Then f

is *LSC*-continuous but neither strongly *LSC*-continuous nor strongly simply-continuous.

(b) Let $X = \{a, b, c\}$, $\tau = \{X, \varnothing, \{a\}, \{a, b\}\}$ and $\sigma = \{X, \varnothing, \{a\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be a function defined by $f(a) = b$, $f(b) = a$, $f(c) = c$. Then f is strongly simply-continuous but neither *LSC*-continuous nor strongly *LSC*-continuous.

(c) Let $X = \{a, b, c\}$, $\tau = \{X, \varnothing, \{a\}\}$ and $\sigma = \{X, \varnothing, \{b\}\}$. Let $f : (X, \tau) \to (X, \sigma)$ be the identity function. Then f is simply-continuous (and hence semi-λ-continuous) but neither λ-semi-continuous nor *LSC*-continuous.

(d) Let N be the set of all positive integers with the cofinite topology τ_f and $X = \{a, b\}$ with topology $\{X, \varnothing, \{a\}\}$. Let $(N, \tau_f) \to (X, \tau)$ be a function defined by $f(n) = a$ if n is odd, $f(n) = b$ if n is even. Then f is strongly λ-semi-continuous but neither simply-continuous nor *LSC*-continuous.

Borsik and Dobos [6] gave decomposition of quasi-continuity: A function $f : (X, \tau) \to (Y, \sigma)$ is quasi-continuous if and only if f is almost quasi-continuous and simply-continuous. Recently, Dontchev and Maki [11] and Dontchev and Ganster [12] gave decompositions of quasi-continuity and irresoluteness as follows:

Theorem 3.9. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be a function. Then*

(a) f *is quasi-continuous if and only if* f *is* sg*-continuous and semi-*λ*-continuous.*

(b) f *is irresolute if and only if* f *is strongly simply-continuous and pre-*sg*-continuous.*

Note that quasi-continuous functions are usually called *semi-continuous*. From Theorem 2.8, we have other decompositions of semi-continuity and irresoluteness.

Theorem 3.10. *For a function* $f : (X, \tau) \to (Y, \sigma)$, *the following are equivalent*:

(a) *f is semi-continuous*.

(b) *f is $\mathcal{g}$s-continuous and LSC-continuous*.

(c) *f is $\mathcal{g}$s-continuous and λ-semi-continuous*.

Theorem 3.11. *For a function* $f : (X, \tau) \to (Y, \sigma)$, *the following are equivalent*:

(a) *f is irresolute*.

(b) *f is pre-$\mathcal{g}$s-continuous and strongly LSC-continuous*.

(c) *f is pre-$\mathcal{g}$s-continuous and strongly λ-semi-continuous*.

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