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NATURAL CONVECTION FLOW PAST A VERTICAL POROUS PLATE WITH INTERNAL HEAT GENERATION AND CONSTANT HEAT FLUX

M. FERDOWS, KOJI KAINO and J. C. CREPEAU

Department of Mathematics
University of Dhaka
Dhaka-1000, Bangladesh

Department of Advanced Science and Technology
Toyota Technological Institute
2-12-1, Hisakata, Tempaku-ku
Nagoya 468-8511, Japan

Department of Mechanical Engineering
University of Idaho
1776 Science Center Drive, U. S. A.

Abstract

Using similarity solution technique natural convection from a semi-infinite vertical porous plate in the presence of internal heat generation (IHG) is studied while the heat flux at the plate is constant. The effects of Prandtl number on the velocity and temperature distributions are examined. It is found that at higher Prandtl number, the flow field deviates greatly.

1. Introduction

Steady two-dimensional boundary layer solutions are studied in order

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to reveal the combined convection from a vertical semi-infinite plate to a micropolar fluid considering uniform surface heat flux [2]. They have pointed out that the flow and heat transfer characteristics and asymptotic solutions are presented for the region close and far away from the leading edge. Hossain et al. [3] have presented finite difference solutions to study the flow and heat transfer of a viscous incompressible fluid with temperature dependent viscosity and thermal conducting past a wedge with a uniform heat flux. They examined different parameters on the flow field. Free convection flow through a porous medium bounded by vertical porous plate when the heat flux at the plate is constant was studied by Raptis et al. [4]. They have discussed the permeability parameter and Grashof number effects on the velocity profile as well as rate of heat transfer. A similarity solution for a fluid with an exponentially decaying heat generation term and a constant temperature vertical plate was developed by Crepeau and Clarksean [1]. They found that the presence of internal energy generation increases the flow velocity by enhancing the buoyancy of the fluid.

In this paper, we propose to analyze the parameter effects on natural convection with IHG considering constant heat flux at the plate.

2. Flow Analysis and Discussions

A two-dimensional natural convection flow past a vertical porous plate with constant heat flux considering IHG is assumed. The flow is assumed to be in the x-direction which is taken along the plate in the upward direction and the y-axis is taken normal to it. The governing equations within boundary layer relevant to the problem are

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{1}$$

Momentum equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \upsilon \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty). \tag{2}$$

Energy equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = k\frac{\partial^2 T}{\partial y^2} + q'''. \tag{3}$$

The boundary conditions for the model are

$$\left.\begin{array}{l} u = 0, \quad v = 0, \quad \dfrac{\partial T}{\partial y} = -\dfrac{q}{k} \text{ at } y = 0 \\ u = 0, \quad T = T_\infty \quad \text{as } y \to \infty \end{array}\right\}. \tag{4}$$

We now introduce the following dimensionless variables:

$$\eta(x, y) = \frac{y}{x}\left(\frac{\text{Gr}}{5}\right)^{1/5}, \quad u = \frac{1}{x}5\upsilon f'(\eta)\left(\frac{\text{Gr}}{5}\right)^{2/5},$$

$$T = T_\infty + \left(\frac{\text{Gr}}{5}\right)^{-1/5}\frac{qx}{k}\theta(\eta), \quad \text{Gr} = \frac{g\beta q x^4}{k\upsilon^2}. \tag{5}$$

We then consider the internal heat generation (IHG) of the form

$$q''' = \frac{q\upsilon}{kx}\left(\frac{\text{Gr}}{5}\right)^{1/5}e^{-\eta}. \tag{6}$$

Introducing equations (5) and (6) into equations (2)-(3) we, respectively, have

$$f''' + 4ff'' - 3f'^2 + \theta(\eta) = 0, \tag{7}$$

$$\theta'' + \text{Pr}(4f\theta' - f'\theta) + \text{Pr}\, e^{-\eta} = 0, \tag{8}$$

where $\text{Pr} = \frac{\upsilon}{k}$ is the Prandtl number.

The corresponding boundary conditions are

$$\left.\begin{array}{l} f = 0, \quad f' = 0, \quad \theta' = -1 \quad \text{at } \eta = 0 \\ f' = 0, \quad \theta = 0 \quad \text{as } \eta \to \infty \end{array}\right\}. \tag{9}$$

Equations (7) and (8) with boundary conditions (9) are solved numericaly for non-dimensional velocity and temperature distributions and the results are shown in Figures 1-2 and in Table 1. For this, we consider the Prandtl numbers $\text{Pr} = 0.71, 0.9, 1.06$ and 7.0 which are equivalent for the air, ammonia, steam and water, respectively.

The Prandtl numbers effects on the velocity distribution are shown in Figure 1. It is seen that with the increase of Prandtl number, the velocity decreases and this influence greatly affects the velocity field at $\Pr = 7.0$.

The Prandtl numbers effects on temperature profiles are shown in Figure 2 for different Pr. It is observed that the dimensionless temperature decreases with the increase of Prandtl number.

Finally the effects of the Skin-friction are shown in Table 1 for different Pr. It is seen that the local Skin-friction decreases as Pr increases.

Table 1. Values of Skin-friction for different Pr

Parameter	$f''(0)$
$\Pr = 0.71$	1.049
$\Pr = 0.9$	1.0086
$\Pr = 1.06$	0.9851
$\Pr = 7.0$	0.9082

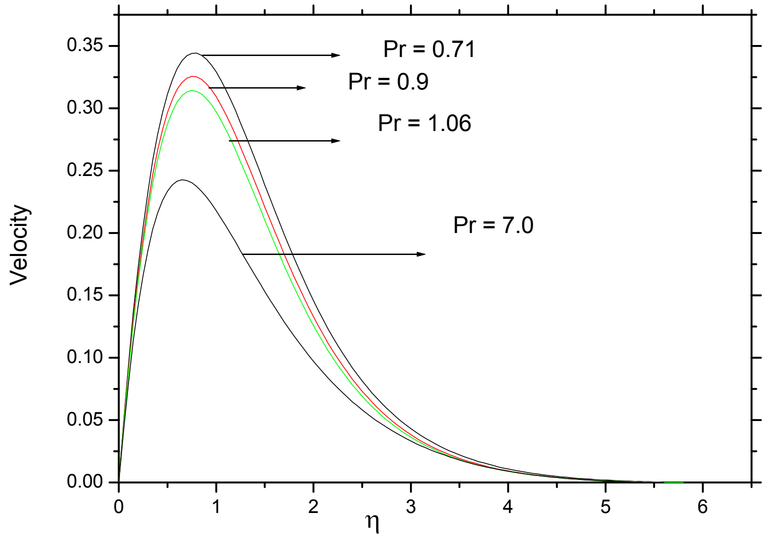

Figure 1. Velocity distribution for different Pr.

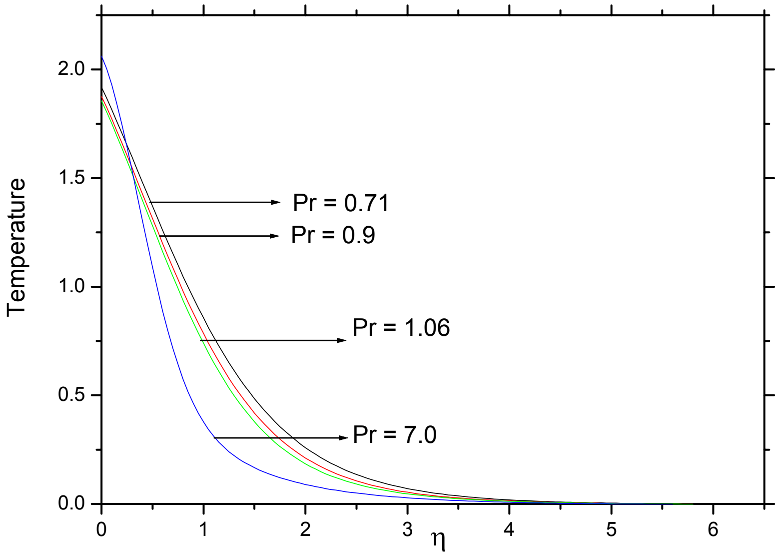

Figure 2. Temperature distribution for different Pr.

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