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RADIATION EFFECTS ON FLOW PAST AN IMPULSIVELY STARTED INFINITE VERTICAL PLATE IN A ROTATING FLUID

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Abstract

The effect of thermal radiation on the natural convection flow past an impulsively started infinite vertical plate in a rotating fluid is investigated in the form of exact solution by applying Laplace transformation technique. The optically thin radiation limit is considered and the working fluid is taken to have Prandtl numbers $\Pr = 0.71,\ 7.0$ and 100.0 and the effects of varying the radiation parameter F, the variation of Grashof number G, rotation parameter Ω and time are discussed.

1. Introduction

The first exact solution of the Navier-Stokes equation was given by Stokes [21], which explains the motion of a viscous incompressible fluid past an impulsively started infinite horizontal plate in its own plane. This is known as Stokes's first problem in the literature. If the plate is in a vertical direction and given an impulsive motion in its own plane in a stationary fluid, then the resulting effect of buoyancy forces was first

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studied by Soundalgekar [18] by the Laplace transformation technique and the effects of heating or cooling of the plate by free convection currents were discussed. Now if the stationary mass of fluid is made to rotate about this vertical plate, how buoyancy forces, centrifugal forces and Coriolis forces affect the flow past this moving plate. This has not been studied in the literature. From the physical point of view, this is very complex situation, and then an exact solution is not possible. However, if the system is rotating very slowly, the square and higher order terms in the centrifugal forces can be neglected and then the system is acted only by the thermal buoyancy force and Coriolis force, respectively, given by $-\rho_0 \beta \overline{g}(T - T_\infty)$ and $-2\rho_0 \overline{\Omega} \times \overline{V}$, where $\overline{g}$, $\overline{\Omega}$ and $\overline{V}$ are the gravitational vector force, rotation and velocity vectors respectively. A more general case where all these forces are taken into account is studied by Ker and Lin [13], while studying the combined convection in a rotating cubic cavity. The interaction of natural convection with thermal radiation has increased greatly during the last decade due to its importance in many practical applications. Radiation effects on the free convection flow are important in context of space technology and processes involving high temperature and very little is known about the effects of radiation on the boundary layer flow of a radiating fluid past a body.

Several investigations have been carried out on problem of heat transfer by radiation as an important application of space and temperature related problems. Greif, Habib and Lin [8] obtained an exact solution for the problem of laminar convective flow in a vertical heated channel in the optically thin limit. In the optically thin limit, the fluid does not absorb its own emitted radiation which means that there is no self absorption but the fluid does absorb radiation emitted by the boundaries. Viskanta [23] investigated the forced convective flow in a horizontal channel permeated by uniform vertical magnetic fluid taking radiation into account. He studied the effects of magnetic field and radiation on the temperature distribution and the rate of heat transfer in the flow. Later Gupta and Gupta [9] studied the effect of radiation on the combined free and forced convection of an electrically conducting fluid flowing inside an open-ended vertical channel in the presence of a uniform transverse magnetic field for the case of optically thin limit. They found that radiation tends to increase the rate of heat transfer to the fluid there by reducing the effect of natural convection. Soundalgekar and Takhar [19] first, studied the effect of radiation on the natural convection flow of a gas past a semi-infinite plate using the Cogly-Vincentine-Gilles equilibrium model (Cogly et al. [3]). Later, Hossain and Takhar [11] analyzed the effect of radiation using the Rosseland diffusion approximation which leads to non-similar solution for the forced and free convection of an optically dense viscous incompressible fluid past a heated vertical plate with uniform free stream and uniform surface temperature, while Hossain et al. [10] studies the effect of radiation on free convection from a porous vertical plates. Muthucumaraswamy and Kumar [15] studied the thermal radiation effects on moving infinite vertical plate in the presence of variable temperature and mass diffusion. Further studies on radiation effects with different physical have been done by different authors. Few of them are Takhar et al. [22], Raptis and Perdikas [17], Das et al. [4], Pop et al. [16], Kumari and Nath [14], Ibrahim et al. [12], Abo-Eldahab and Gendy [1], El Arabawy [5], Elbashbeshy and Dimian [6], Ganesan and Loganathan [7].

The objective of the present study is to investigate free convection flow of an optically thin viscous incompressible fluid from an impulsively started vertical isothermal flat plate. The plate being rotating in unison with the stationary fluid when radiation effect is included involving the Cogly-Vincentine-Giles equilibrium model. Solutions are presented in graphical as well as tabular form. Various values of the radiation parameter, rotation parameter, Grashof number are considered for fluid having Prandtl number $\mathrm{Pr} = 0.71,\ 7.0$ and 100.0.

2. Mathematical Analysis

Consider an infinite plate maintained placed vertically in an infinite expanse of stationary radiating fluid at constant temperature T'_∞ coinciding with the plane $z' = 0$. The x'-axis is taken along the plate in the vertically upward direction and the z'-axis is taken normal to the plate while the y'-axis is assumed to be in the plane of the plate and normal to both x' and z'-axes. Initially, the fluid and the plate rotate in unison with a uniform angular velocity Ω' about z'-axis. Relative to the rotating fluid, the plate is given an impulsive motion, so that it moves with a velocity U_0 in its own plane along the x'-axis and the plate temperature is raised or lowered to T'_w. The plate being of infinite length, all the physical variables are functions of z' and t' only. The following assumptions are made in this investigation:

(a) The fluid physical properties are assumed constant and the density variation in the body force term in momentum equation is assumed where the Bussinesq approximation is involved.

(b) The fluid is assumed to be gray, emitting and absorbing but non-scattering medium.

(c) The optically thin radiation limit is considered where the radiative heat flux term appearing in the energy equation can be simplified by using the Cogly-Vincentine-Giles equilibrium model.

Under these assumptions, the governing equations of the laminar boundary layer flow problem under consideration can be written as

$$\frac{\partial U'}{\partial t'} - 2\Omega' V' = \nu \frac{\partial^2 U'}{\partial z'^2} + g\beta(T' - T'_\infty)\cos \Omega' t' \tag{1}$$

$$\frac{\partial V'}{\partial t'} + 2\Omega' U' = \nu \frac{\partial^2 V'}{\partial z'^2} - g\beta(T' - T'_\infty)\sin \Omega' t' \tag{2}$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial z'^2} - \frac{\partial q_r}{\partial z'}. \tag{3}$$

In the optically thin limit, the fluid does not absorb its own emitted radiation that is there is no self absorption, but it does absorb radiation emitted by the boundaries. Under this assumption Sparrow and Cess [20] showed that the following relation holds

$$\frac{\partial q_r}{\partial z'} = 4K_p \sigma T'^4 - 4K_M \sigma T'^4_w,$$

where K_p and K_M are the Planck mean and the modified Planck mean absorption coefficient respectively and σ being the Stefan-Boltzmann constant. Cogley et al. [3], have shown that in the optically thin limit for a non-gray gas near equilibrium, that

$$\frac{\partial q_r}{\partial z'} = 4(T' - T'_w)\int_0^\infty K_{\lambda w}\left(\frac{de_{b\lambda}}{dT'}\right)_w d\lambda, \tag{4}$$

where $K_{\lambda w}$ is the absorption coefficient, $e_{b\lambda}$ is the Planck function and the subscript w refers to values at the wall. Further simplifications can be made concerning the spectral properties of radiating gases, but are not necessary for our investigation. Substitution of equation (4) in (3) yields

$$\frac{\partial T'}{\partial t} = \alpha \frac{\partial^2 T'}{\partial z'^2} - CT',$$

where

$$C = \frac{4}{\rho C_p}\int_0^\infty K_{\lambda w}\left(\frac{de_{b\lambda}}{dT'}\right)_w d\lambda.$$

Here the subscript w indicates that all quantities have been evaluated at the reference temperature T_w (plate temperature at $z' = 0$).

The initial and boundary conditions are

$$\left.\begin{array}{l} U' = V' = 0,\ T' = T'_\infty, \text{ for all } z', t' \le 0 \\ \left.\begin{array}{l} U' = U_0,\ V' = 0,\ T' = T'_w \text{ at } z' = 0 \\ U' = 0,\ V' = 0,\ T' \to T'_\infty \text{ as } z' \to \infty \end{array}\right\} t' > 0 \end{array}\right\}. \tag{5}$$

All the physical variables are defined in the Nomenclature. On introducing the following non-dimensional quantities,

$$\left.\begin{array}{l} (U, V) = \dfrac{(U', V')}{U_0},\ z = \dfrac{z'U_0}{\nu},\ t = \dfrac{t'U_0^2}{\nu},\ \Omega = \dfrac{\Omega'\nu}{U_0^2},\ \Pr = \dfrac{\mu C_p}{k} \\ \theta = \dfrac{T' - T'_\infty}{T'_w - T'_\infty},\ G = \dfrac{\nu g\beta(T'_w - T'_\infty)}{U_0^3},\ F = \dfrac{C \Pr \nu}{U_0^2} \end{array}\right\}, \tag{6}$$

then the equations (1) to (6), we have

$$\frac{\partial q}{\partial t} + 2i\Omega q = \frac{\partial^2 q}{\partial z^2} + G\theta e^{-i\Omega t} \tag{7}$$

$$\Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial z^2} - F\theta \tag{8}$$

with following initial and boundary conditions:

$$\left.\begin{array}{l} q = \theta = 0, \text{ for all } z, t \le 0 \\ \left.\begin{array}{l} q = 1, \theta = 1, \text{ at } z = 0 \\ q = 0, \theta = 0, \text{ as } z \to \infty \end{array}\right\} t > 0 \end{array}\right\}. \tag{9}$$

Here $q = U + iV$.

We now solve equations (7) and (8) subject to the initial and boundary conditions (9) by the usual Laplace-transform technique. The solutions are given by

$$\theta = \frac{1}{2}\left[e^{-z\sqrt{F}} erfc\left(\frac{z\sqrt{\Pr}}{2\sqrt{t}} - \sqrt{\frac{F}{\Pr}t}\right) + e^{z\sqrt{F}} erfc\left(\frac{z\sqrt{\Pr}}{2\sqrt{t}} + \sqrt{\frac{F}{\Pr}t}\right)\right] \tag{10}$$

$$\begin{aligned}
q = {} & \frac{1}{2}\left[e^{-z\sqrt{2i\Omega}} erfc\left(\frac{z}{2\sqrt{t}} - \sqrt{2i\Omega t}\right) + e^{z\sqrt{2i\Omega}} erfc\left(\frac{z}{2\sqrt{t}} + \sqrt{2i\Omega t}\right)\right] \\
& - \frac{G}{2(i\Omega - F)} e^{-i\Omega t}\left[e^{-z\sqrt{i\Omega}} erfc\left(\frac{z}{2\sqrt{t}} - \sqrt{i\Omega t}\right) + e^{z\sqrt{i\Omega}} erfc\left(\frac{z}{2\sqrt{t}} + \sqrt{i\Omega t}\right)\right] \\
& + \frac{G}{2(i\Omega - F)} e^{\left(\frac{i\Omega(2-\Pr)-F}{\Pr-1}\right)t}\left[e^{-z\sqrt{\frac{\Pr i\Omega - F}{\Pr - 1}}} erfc\left(\frac{z}{2\sqrt{t}} - \sqrt{\left\{\frac{\Pr i\Omega - F}{\Pr - 1}\right\}t}\right)\right. \\
& \qquad \left. + e^{z\sqrt{\frac{\Pr i\Omega - F}{\Pr - 1}}} erfc\left(\frac{z}{2\sqrt{t}} + \sqrt{\left\{\frac{\Pr i\Omega - F}{\Pr - 1}\right\}t}\right)\right] \\
& - \frac{G}{2(i\Omega - F)} e^{\left(\frac{i\Omega(2-\Pr)-F}{\Pr-1}\right)t}\left[e^{-z\sqrt{\frac{\Pr i\Omega - F}{\Pr - 1}}} erfc\left(\frac{z\sqrt{\Pr}}{2\sqrt{t}} - \sqrt{\left\{\frac{\Pr i\Omega - F}{\Pr(\Pr - 1)}\right\}t}\right)\right. \\
& \qquad \left. + e^{z\sqrt{\frac{\Pr i\Omega - F}{\Pr - 1}}} erfc\left(\frac{z\sqrt{\Pr}}{2\sqrt{t}} + \sqrt{\left\{\frac{\Pr i\Omega - F}{\Pr(\Pr - 1)}\right\}t}\right)\right] \\
& + \frac{G}{2(i\Omega - F)} e^{-i\Omega t}\left[e^{-z\sqrt{F}} erfc\left(\frac{z\sqrt{\Pr}}{2\sqrt{t}} - \sqrt{\frac{F}{\Pr}t}\right)\right.
\end{aligned}$$

$$+ e^{z\sqrt{F}} erfc\left(\frac{z\sqrt{\Pr}}{2\sqrt{t}} + \sqrt{\frac{F}{\Pr} t}\right)\Bigg]. \tag{11}$$

We have separated $q = U + iV$ into real and imaginary parts and the numerical values of U and V are computed. During computation, it is observed that the arguments of the error function involved in (11) are complex, so we use the following well known formula (Abramowitz and Stegun [2]) to separate into real and imaginary parts.

$$erf(X + iY) = erf(X) + \frac{e^{-X^2}}{2\pi X}\{1 - \cos(2XY) + i\sin(2XY)\}$$

$$+ \frac{2}{\pi} e^{-X^2} \sum_{n=1}^{\infty} \frac{e^{-n^2/4}}{n^2 + 4X^2}\{f_n(X, Y) + ig_n(X, Y)\} + \varepsilon(X, Y),$$

where

$$f_n(X, Y) = 2X - 2X\cosh(nY) + n\sinh(nY)\sin(2XY)$$

$$g_n(X, Y) = 2X\cosh(nY)\sinh(2XY) + n\sinh(nY)\cos(2XY)$$

$$|\varepsilon(X, Y)| \approx 10^{-16}|erf(X + iY)|.$$

3. Results and Discussions

The results incorporating the effects of radiation on the velocity and temperature profiles are presented in Figures 1 and 2. Figure 1 shows the variation of U (the component of velocity in the direction of motion of the plate) with Z for different values of the radiation parameter F, at time $T = 0.2$. It is observed that the axial velocity decreases for increasing the radiation parameter. Likewise the transverse velocity profiles are presented in Figure 2, which demonstrates the decreasing (increasing) behaviors. The negative sign for V in this figure indicates that this component is transverse to the main flow direction x in the clockwise sense. To see the effect of the Grashof number G, the axial and transverse velocity profiles are presented in Figures 3 and 4 respectively keeping the other parameters fixed. It is showed that increasing the values of G, both axial and transverse velocity increases.

The results for the non-radiating problem in the presence of rotation are presented in Figures 5 and 6 and show the effect of rotation on the axial and transverse velocity profiles. Figure 5 shows that the axial velocity at any given instant and at a given height from the plate increases with increasing rotation. On the other hand Figure 6 shows that at a given instant the transverse velocity component V increases with increasing in rotation. We observe from the Figures 7 and 8, that the axial and transverse velocity decreases with increasing Prandtl number Pr.

4. Skin-friction

From the velocity field, we now study the effects of these parameters on the skin-friction. It is given by

$$\tau' = -\mu \left.\frac{\partial q'}{\partial y'}\right|_{z'=0}. \tag{12}$$

And in non-dimensional form, it becomes

$$\tau_x = \frac{\tau'_x}{\rho U_0^2} = -\left.\frac{dU}{dz}\right|_{z=0} \quad \text{and} \quad \tau_y = \frac{\tau'_y}{\rho U_0^2} = -\left.\frac{dV}{dz}\right|_{z=0}. \tag{13}$$

We have computed numerical values of τ_x and τ_y and these are listed in Table I.

Table I. Values of skin-friction

t	F	Pr	Ω	G	τ_x	τ_y
0.2	0.0	0.71	0.4	0.4	1.0746	0.2715
0.2	0.5	0.71	0.4	0.4	1.2712	0.2965
0.2	0.5	0.71	0.4	0.6	1.2732	0.3438
0.2	0.5	0.71	0.4	0.8	1.2752	0.3911
0.2	0.5	0.71	0.6	0.4	1.2612	0.4095
0.2	0.5	0.71	0.8	0.4	1.2556	0.5124
0.2	0.7	0.71	0.4	0.4	1.2611	0.2475
0.2	1.0	0.71	0.4	0.4	1.2366	0.2080
0.2	0.5	7.0	0.4	0.4	1.4727	0.1988
0.2	0.5	100.0	0.4	0.4	1.8082	0.0568

We observe from the table that as compared to non-radiating case $F = 0$, the axial skin-friction τ_x is always greater in the presence of optically thin limit and decreases with increasing the radiation parameter F. But transverse skin-friction τ_y is always less for optically thin limit and decreases with increasing F. Also τ_x decreases with rotation parameter Ω and increases with increase in the Prandtl number $\Pr$. The transverse skin-friction τ_y is found to increase with Ω and decreases as $\Pr$ increases. From the table we conclude that for greater values of G, both τ_x and τ_y increases.

5. Conclusions

i. Axial and transverse velocity decreases for increasing the radiation parameter.

ii. Increasing the values of G and Ω, both axial and transverse velocity increases.

iii. Axial and transverse velocity decreases with increasing Prandtl number.

iv. The axial skin-friction τ_x is always greater in the presence of optically thin limit and decreases with increasing the radiation parameter F. But transverse skin-friction τ_y is always less for optically thin limit and decreases with increasing F.

Nomenclature

C_p	specific heat at constant pressure
erfc	complementary error function
g	gravitational acceleration
G	Grashof number
K	thermal conductivity
Pr	Prandtl number
t'	time

t	dimensionless time
T'	fluid temperature
T'_w	temperature of the plate
T'_∞	temperature of the fluid away from the plate
U_0	reference velocity of the plate
U'	fluid velocity in x'-direction
U	non-dimensional fluid velocity in x-direction
V'	fluid velocity in y'-direction
V	non-dimensional fluid velocity in y-direction
q_r	radiative flux
F	radiation parameter
x'	co-ordinate along the plate
y'	co-ordinate normal to $x'z'$-plane
z'	co-ordinate normal to the plate

Symbols

β	thermal expansion coefficient
ρ_0	reference fluid-density
μ	fluid viscosity
τ'	skin-friction
Ω	non-dimensional rotation parameter
θ	dimensionless temperature
Ω'	rotation parameter
ν	kinematic viscosity
τ	dimensionless skin-friction
ρ	fluid density

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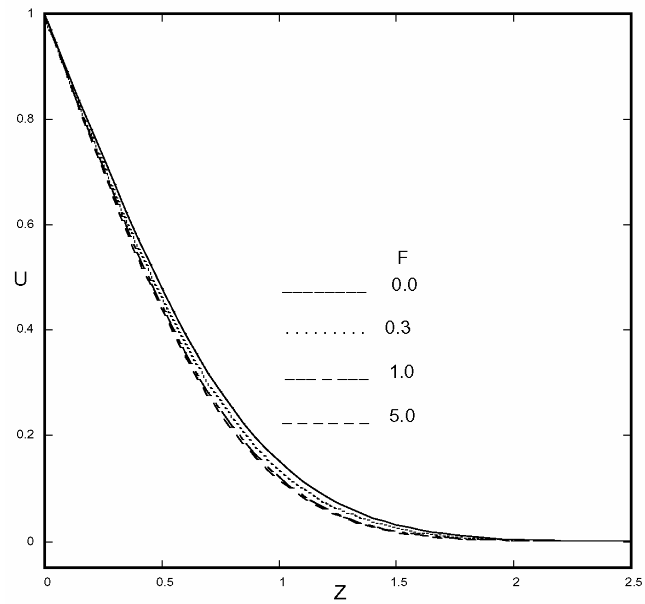

Figure 1. Axial velocity profiles for $t = 0.2$, $\mathrm{Pr} = 0.71$, $G = 0.4$, $\Omega = 0.4$.

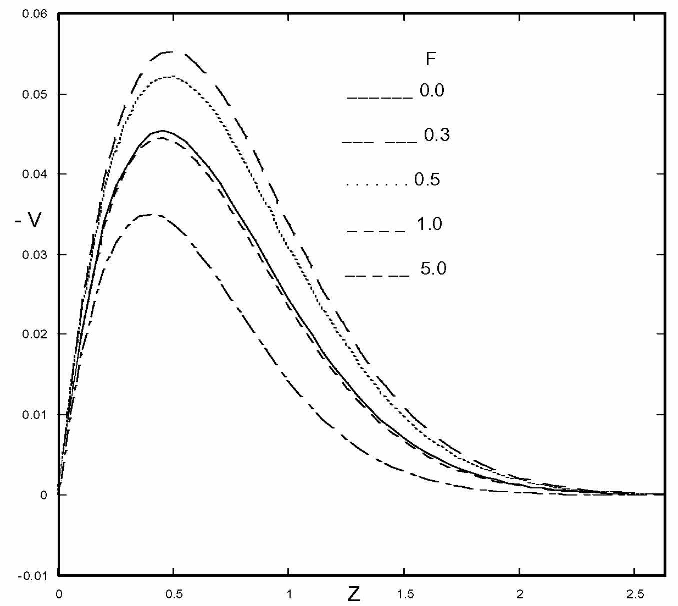

Figure 2. Transverse velocity profiles for $t = 0.2$, $\mathrm{Pr} = 0.71$, $G = 0.4$, $\Omega = 0.4$.

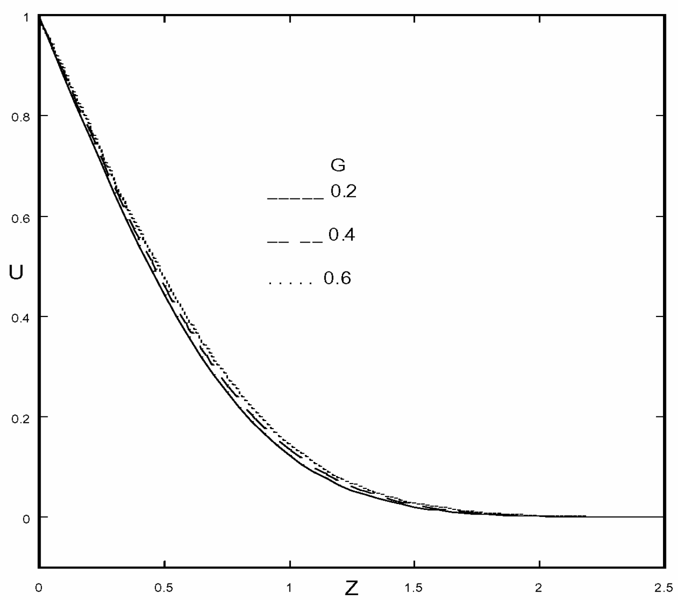

Figure 3. Axial velocity profiles for $t = 0.2$, $\Pr = 0.71$, $F = 0.3$, $\Omega = 0.4$.

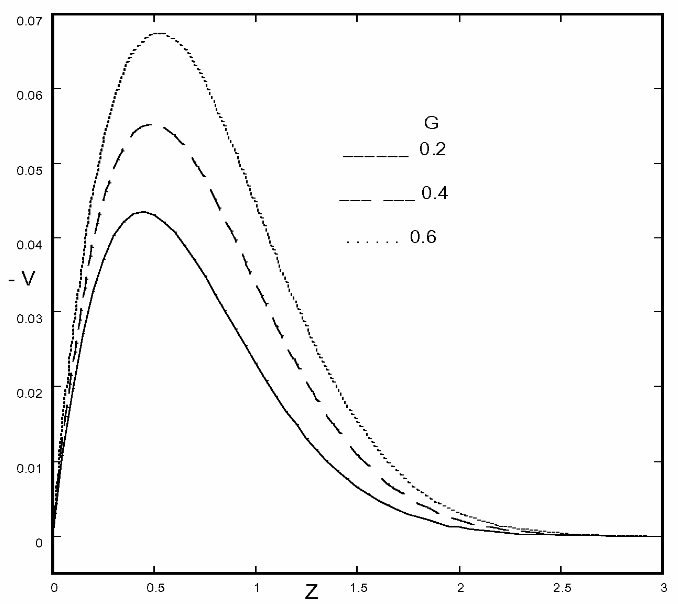

Figure 4. Transverse velocity profiles for $t = 0.2$, $\Pr = 0.71$, $F = 0.3$, $\Omega = 0.4$.

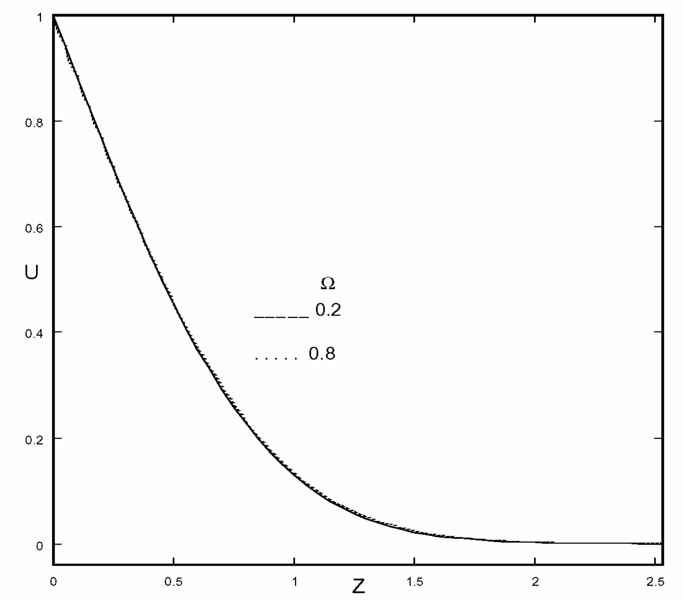

Figure 5. Axial velocity profiles for $t = 0.2$, Pr $= 0.71$, $F = 0.3$, $G = 0.4$.

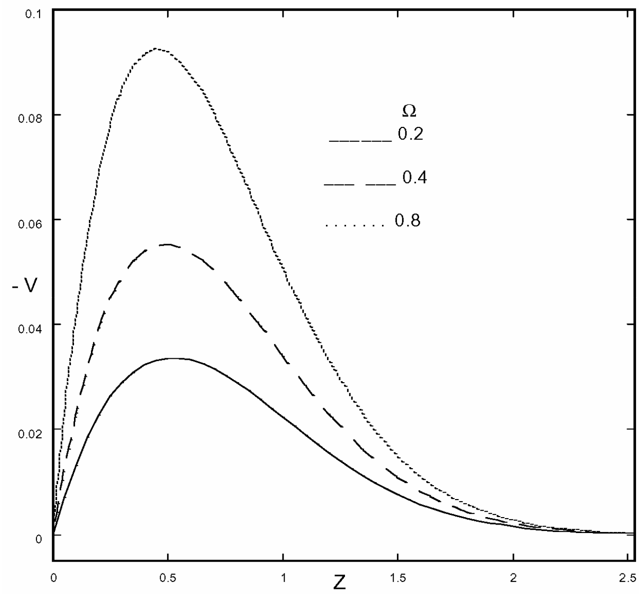

Figure 6. Transverse velocity profiles for $t = 0.2$, Pr $= 0.71$, $F = 0.3$, $G = 0.4$.

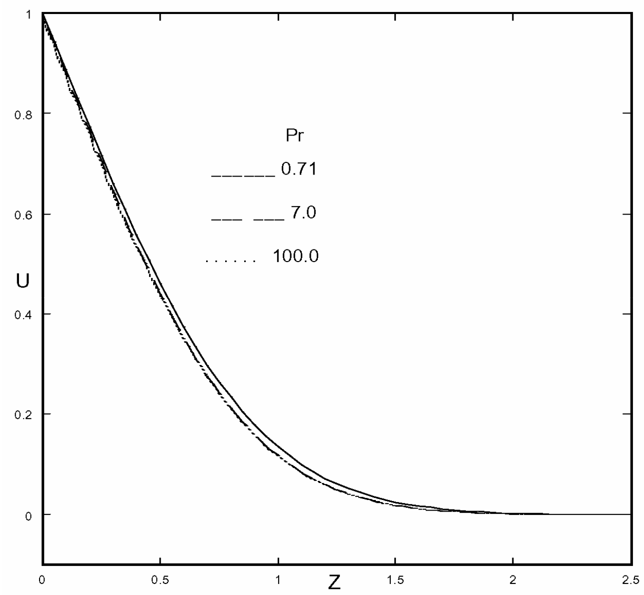

Figure 7. Axial velocity profiles for $t = 0.2$, $G = 0.4$, $F = 0.3$, $G = 0.4$.

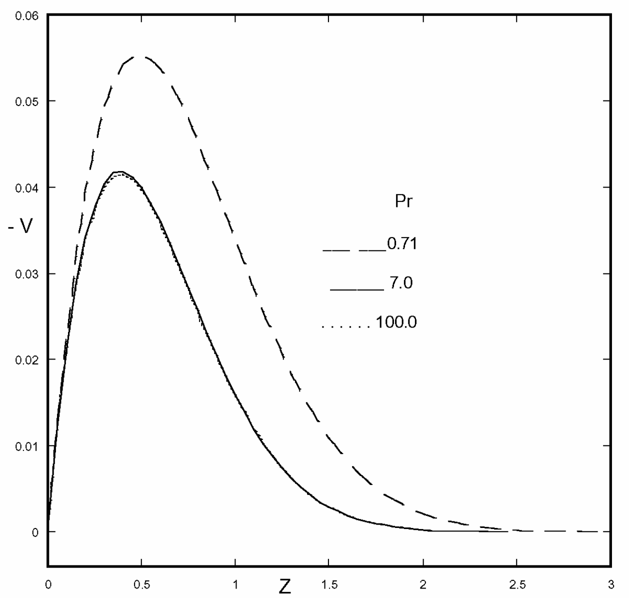

Figure 8. Transverse velocity profiles for $t = 0.2$, $G = 0.4$, $F = 0.3$, $\Omega = 0.4$.