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ON MODELS AND METHODS OF DYNAMIC OPTIMAL MANAGEMENT. MODEL 17

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Abstract

The model built allows to calculate optimal orders of a mechanical system in movement for variable regimes and succession of its moment of commutation. For example, in the case of management of the train, the used method puts in an obvious way the dependence between energy consumed E, the mass M, the slope Q, and the power of the engine u.

1. Introduction

In this section, we introduce the optimal control problem. In the construction of an optimal functioning plan of a dynamic object, the placement in an obvious way influence the necessary perturbations. These different perturbations change real situations of the object. They can be put back to have decisions. This is why, it is necessary to have methods allowing to introduce in calculations, random factors and to anticipate possible changes in the plan.

In Section 2, we present the model 17 of the consumption of energy of a dynamic object.

In Section 3, we present results of optimal management of the train. Conclusions and perspectives of works are presented in Section 4.

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2. Consumption Energy of a Dynamic Object: Model 17

Provoking the principle of maximum of Pontriaguine [1], we build the model 17 of consumption of energy of a dynamic object under the influence of which perturbations with characteristics are approximated by theoretical distributions:

$$\ddot{x}(t) + g(\dot{x}(t)) + f(x(t)) = u(t) \tag{17}$$

$$J[u] = E_C + E_P + \int_{x_0}^{x_f} g(y(x))dx, \tag{18}$$

where the vector $u = (u_1, u_2, ..., u_n)$ describes an influence controlled by this dynamic object;

$u \in C^1$ (class of admissible functions);

$x = (x_1, x_2, ..., x_n)$ describes a state of the object with:

$$\dot{x} = y; \quad x_i(0) = x_i^0; \quad x_i(T) = x_i^f; \quad i = 1, ..., n.$$

Conditions above justify the dirigibility of the object x^0 as compared to x^f. We denote by

$U(x^f)$: the totality of dirigibility;

E_C : kinetic energy and E_P : potential energy;

$E_g = \int_{x_0}^{x_f} g(y(x))dx$: a functional objective.

During optimal management, we pose the problem to minimize the functional economic $J[u^*]$ with

$$J[u^*] = \min J[u] = \min \int_{x_0}^{x_f} g(y(x))dx. \tag{19}$$

We show that if $u^*(t)$ is an optimal order, then

$$J[u^*] \le J[u], \tag{20}$$

where $u \in C_1$: under totality of admissible orders.

Indeed, $g(y)$ is the objective function verifying conditions

$$g(y(x)) > 0; \quad g'(y(x)) > 0; \quad g''(y(x)) \geq 0. \tag{21}$$

Is $+1$ or -1 the commutation allowing to increase or decrease the consumption of energy (that is, the entire first of the power consumed) in function of conditions of the movement?

We suppose that an area where all have admissible paths, we have

$$\begin{cases} +1 - f(x) - g(y) > 0 \\ -1 - f(x) - g(y) < 0. \end{cases} \tag{22}$$

The conditions above will be sufficient for optimality of the path.

By making the development limited to the vicinity of y_0, we obtain

$$g(y) = g(y_0) + (y - y_0)g'(y_0) + \frac{(y - y_0)^2}{2} g''(c),$$

where $c \in]\, y_0, y_1 \,[$, c being fixed.

On integration of the relationship above, we obtain

$$\int_{x_0}^{x_f} g(y(x))dx \geq \int_{x_0}^{x_f} g(y_0(x))dx + \int_{x_0}^{x_f} (y(x) - y_0(x)) \cdot g'(y_0)dx.$$

It is necessary to justify the condition of optimality

$$\int_{x_0}^{x_f} (g(y(x)) - g(y_0(x)))dx \geq 0.$$

According to relationship (20), we have $g(y(x)) \cdot g'(y(x)) > 0$. Two possibilities:

(a) $y(x) \leq y_0(x) = K_1 \Rightarrow y_0(x) \cdot y(x) \leq K_1^2$.

(b) $y(x) > y_0(x) = K_1 \Rightarrow y_0(x) \cdot y(x) \geq K_1^2$.

Now, we consider the first possibility:

If $x \in \left]x_2, x_f\right[$, then

$$y(x) \leq y_0(x) = K_1 \Rightarrow y_0(x) \cdot y(x) \leq K_1^2$$

and we obtain

$$\int_{x_2}^{x_f}(g(y(x)) - g(y_0(x)))dx \geq \int_{x_2}^{x_f}(y(x) - y_0(x))g'(y_0(x))dx$$

$$= \int_{x_2}^{x_f} g'(y_0(x)) \cdot y_0(x)y(x)\left[\frac{1}{y(x)} - \frac{1}{y_0(x)}\right]dx$$

$$\geq \int_{x_2}^{x_f} g'(K_1)K_1^2\left[\frac{1}{y(x)} - \frac{1}{y_0(x)}\right]dx = 0$$

$$\Rightarrow \int_{x_2}^{x_f}(g(y(x)) - g(y_0(x)))dx \geq 0.$$

Using the same reasoning on $]x_0, x_1[$, we obtain the following result:

$$\int_{x_0}^{x_f}(g(y(x)) - g(y_0(x)))dx \geq 0.$$

Next, we consider the second possibility:

If $]x_1^*, x_2^*[\subset]x_1, x_2[; \ \forall x \in]x_1^*, x_2^*[$, then we have

$$y(x) > y_0(x) = K_1 \Rightarrow y_0(x) \cdot y(x) \geq K_1^2$$

and, moreover, $\frac{1}{y_0(x)} - \frac{1}{y(x)} > 0$, and hence

$$\int_{x_1^*}^{x_2^*}(g(y(x)) - g(y_0(x)))dx \geq \int_{x_1^*}^{x_2^*}(y(x) - y_0(x))g'(y_0(x))dx$$

$$= \int_{x_1^*}^{x_2^*} g'(y_0(x)) \cdot y_0(x)y(x)\left[\frac{1}{y_0(x)} - \frac{1}{y(x)}\right]dx$$

$$\geq \int_{x_1^*}^{x_2^*} g'(K_1)K_1^2\left[\frac{1}{y_0(x)} - \frac{1}{y(x)}\right]dx$$

$$= g'(K_1)K_1^2\left[\int_{x_1}^{x_2}\frac{dx}{y_0} - \int_{x_1}^{x_2}\frac{dx}{y}\right] = 0.$$

Consequently,

$$\int_{x_1}^{x_2}(g(y(x)) - g(y_0(x)))dx \geq 0.$$

Therefore, we conclude

$$\int_{x_0}^{x_f}(g(y(x)) - g(y_0(x)))dx \geq \int_{x_0}^{x_f} g'(K_1)K_1^2\left[\frac{1}{y_0(x)} - \frac{1}{y(x)}\right]dx$$

$$= g'(K_1)K_1^2\left[\int_{x_1}^{x_2}\frac{dx}{y_0} - \int_{x_1}^{x_2}\frac{dx}{y}\right] = 0.$$

$$\Rightarrow \int_{x_0}^{x_f} (g(y(x)) - g(y_0(x)))dx \geq 0.$$

We thus obtain the necessary and sufficient condition of optimality.

If $u^*(t)$ is an optimal order, for all $u \in C_1$ (under totality of admissible orders), then

$$J[u^*] \leq J[u]. \tag{20}$$

The behaviour of this object with variable parameters and under influence of perturbations is a consequence of variation of the consumption of energy and the constant of optimality of the optimal control u^*.

Indeed, consider an admissible control u_1 defined by:

$$u_1(t) = u^*(t) + \partial u(t) \text{ on } 0 \leq t \leq T.$$

According to the principle of the maximum, there exists x_1 corresponding to u_1 such that

$$x_1(t) = x^*(t) + \partial x(t)$$

$$\begin{cases} \dot{x}_i = f_i(x, u) \\ i = 0, n; \quad x(0) = x^*; \quad f_i(x, u) \in C^2[\Omega], \end{cases} \tag{23}$$

$$\dot{x}_i^*(t) = f_i(x^*(t), u^*(t)),$$

$$\frac{d}{dt}(x_i^*(t) + \partial x(t)) = f_i(x_i^*(t) + \partial x(t), u^*(t) + \partial u(t))$$

$$\dot{w}_l^*(t) \cdot \dot{x}_i^*(t) = \dot{w}_i^*(t) \cdot f_i(x^*(t), u^*(t)) \tag{24}$$

and

$$(w_i^*(t) + \partial w(t))(x_i^*(t) + \partial x(t)) = [f_i(x_i^*(t) + \partial x(t), u_i^*(t) + \partial u(t))][w_i^*(t) + \partial w(t)]. \quad (25)$$

By (25)-(24), we obtain

$$\frac{d(w_i^*(t) + \partial w(t))}{dt}(x_i^*(t) + \partial x(t), u^*(t) + \partial u(t)) - \dot{w}_i^*(t) \cdot \dot{x}_i^*(t) = (w_i^*(t) + \partial w(t)),$$

$$f_i(x^*(t) + \partial x(t), u^*(t) + \partial u(t)) - \dot{w}_i^*(t)\dot{x}_i(t)$$

$$= \sum_i (w_i^*(t) + \partial w(t))(\dot{x}_i^*(t) + \partial \dot{x}(t)) - \dot{w}_i^*(t)\dot{x}_i^*(t),$$

$$\int_0^T \sum_i w_i^*(t)\partial \dot{x}_i(t)dt + \int_0^T \sum_i \partial w(t)\dot{x}_i^*(t)dt + \int_0^T \sum_i \partial w_i(t)\partial \dot{x}_i(t)dt$$

$$= H(x^* + \partial x, w^* + \partial w, u^* + \partial u) - H(x^*, w^*, u^*).$$

Integrating by parts of the first term, we have

$$I_1 = \left[\sum_i w_i^*(t) \cdot \partial x_i(t)\right]_0^T - \int_0^T \sum_i \dot{w}_i^*(t)\partial x_i(t)dt$$

$$= -\sum_i \frac{\partial Q(x(t))}{\partial x_i}\partial x_i(t) + \int_0^T \sum_i \left(\frac{H(x^*, w^*, u^*)}{\partial x_i}\partial x_i\right)dt$$

with initial conditions $w_i^*(t) = -\dfrac{\partial Q(x(T))}{\partial x_i}$, $i = 1, n.$ According to Hamilton,

$$\dot{w}_i = -\frac{\partial H}{\partial x_i} = -\sum_i w_k(t)\frac{\partial f_k(x, u)}{\partial x_i},$$

$$I_2 = \int_0^T \sum_i \partial w_i(t)\dot{x}_i^*(t)dt = \int_0^T \sum_i \left(\frac{\partial H(x^*, w^*, u^*)}{\partial w_i}\partial w_i\right)dt,$$

where

$$\dot{x}_i = \frac{\partial H}{\partial w_i} = f_i(x, u),$$

$$I_3 = \int_0^T \sum_i \partial w_i(t) \dot{x}_i^*(t) dt = \frac{1}{2} \int_0^T \sum_i \left(\frac{\partial H(x^*, w^*, u^*)}{\partial w_i} \partial w_i \right) dt + \frac{1}{2} I_4,$$

$$I_4 = \left[\sum_i \partial w_i(t) \cdot \partial x_i(t) \right]_0^T - \int_0^T \sum_i \partial \dot{w}_i^*(t) \partial x_i(t) dt,$$

where $\dot{x}_i = \dfrac{\partial H}{\partial w_i}$ and $\dot{w}_i = -\dfrac{\partial H}{\partial x_i}$.

This gives

$$\begin{aligned} & I_1 + I_2 + I_3 \\ &= \int_0^T \partial H dt = -\sum_i \frac{\partial Q(x(T))}{\partial x_i} \partial x_i(t) + \int_0^T \sum_i \left(\frac{H(x^*, w^*, u^*)}{\partial x_i} \partial x_i \right) dt \\ &+ \int_0^T \left(\sum_i \frac{\partial H^*}{\partial w_i} \partial w_i(t) \right) dt + \frac{1}{2} \int_0^T \sum_i \left(\partial \left(\frac{\partial H(x^*, w^*, u^*)}{\partial w_i} \partial w_i \right) \right) dt \\ &+ \frac{1}{2} \int_0^T \left(\sum_i \partial \left(\frac{\partial H}{\partial x_i} \right) \partial x_i \right) dt \\ &= \int_0^T \partial H dt \end{aligned}$$

$$dQ = \sum_i \frac{\partial Q(x^*)}{\partial x_i} \partial x_i = -\int_0^T \partial H dt + \int_0^T \left(\sum_i \frac{\partial H}{\partial y_i} \partial y_i \right) dt + \int_0^T \left(\sum_i \partial \left(\frac{\partial H}{\partial y_i} \right) \partial y_i \right) dt$$

with $y = (x_1, x_2, ..., x_n, w_1, w_2, ..., w_n)$.

$$\begin{aligned} \int_0^T \partial H dt &= \int_0^T [H(y^* + \partial y, u^* + \partial u) - H(y^*, u^*)] dt \\ &= \int_0^T [H(y^*, u^* + \partial u) - H(y^*, u^*)] dt \\ &\quad + \int_0^T [H(y^* + \partial y, u^* + \partial u) - H(y^*, u^* + \partial u)] dt \end{aligned}$$

$$= \int_0^T \partial H dt + \int_0^T \left(\sum_{i=1}^{2n} \frac{\partial H(y^*, u^*)}{\partial y_i} - \partial y_i \right) dt$$

$$+ \frac{1}{2} \int_0^T \sum_{i=1}^{2n} \sum_{j=1}^{2n} \frac{\partial 2H(y^* + \partial y, u^* + \partial u)}{\partial y_i \partial y_j} \partial y_i \partial y_j dt$$

$$dQ = \sum_i \frac{\partial Q(x^*(T))}{\partial x_i} \partial x_i$$

$$= -\int_0^T \partial Hudt + \frac{1}{2} \int_0^T \left[\left(\sum_{i=1}^{2n} \frac{\partial H(y^* + \partial y, u^* + \partial u)}{\partial y_i} - \frac{\partial H(y^*, u^*)}{\partial y_i} \right) \partial y_i \right] dt$$

$$+ \frac{1}{2} \int_0^T \left[\left(\sum_{i=1}^{2n} \frac{\partial H(y^*, u^* + \partial u)}{\partial y_i} - \frac{\partial H(y^*, u^*)}{\partial y_i} \right) \partial y_i \right] dt$$

$$= -\frac{1}{2} \int_0^T \left[\sum_{i=1}^{2n} \sum_{j=1}^{2n} \left(\frac{\partial 2H(y^* + \partial y, u^* + \partial u)}{\partial y_i \partial y_j} \right) \partial y_i \partial y_j \right] dt$$

$$dQ = -\int \partial Hudt + \frac{1}{2} \int_0^T \left[\sum_{i=1}^{2n} \left(\frac{\partial H(y^*, u^* + \partial u)}{\partial y_i} - \frac{\partial H(y^*, u^*)}{\partial y_i} \right) \partial y_i \right] dt$$

$$+ \frac{1}{2} \int_0^T \left[\sum_{i=1}^{2n} \sum_{j=1}^{2n} \left(\frac{\partial 2H(\theta_1)}{\partial y_i \partial y_j} \right) \partial y_i \partial y_j \right] dt$$

$$- \frac{1}{2} \int_0^T \left[\sum_{i=1}^{2n} \sum_{j=1}^{2n} \left(\frac{\partial 2H(\theta_2)}{\partial y_i \partial y_j} \right) \partial y_i \partial y_j \right] dt$$

$$x(t) = B + A\int_0^t x(s) ds \Leftrightarrow \dot{x}(t) = Ax \text{ with } x(0) = B \Rightarrow x(t) = Be^{At}.$$

$$dQ = \sum_i \frac{\partial Q(x^*(T))}{\partial x_i} \partial x_i = -\int_0^T \partial H dt + L.$$

To show that L is small enough, it will suffice to appreciate ∂y_i that is to say, $\partial x_i, \partial w_i$.

$$\dot{x}_i^*(t) = f_i(x^*(t), u^*(t)),$$

$$\frac{d}{dt}(x_i^*(t) + \partial x(t)) = f_i(x_i^*(t) + \partial x(t), u^*(t) + \partial u(t)),$$

$$\begin{cases} \partial \dot{x}_i^* = f_i(x^*(t) + \partial x, u^*(t) + \partial u) - f_i(x^*(t), u^*(t)) \\ i = 1, n \quad \partial x(0) = 0. \end{cases} \tag{26}$$

Lemma [2], [3]

For all $\varepsilon > 0$, there exists $\eta(\varepsilon) > 0$ such that $\|\partial u(t)\| \leq \eta(\varepsilon)$ $\Rightarrow |\partial x_i(t)| < \varepsilon$ for all $t \in [0, T]$.

Justifications

Suppose that $f_i \in C_1[0, T]$. Then, by (26)

$$\int_0^T [f_i(x^* + \partial x, u^* + \partial u) \pm f_i(x^* + \partial x, u) - f_i(x^*, u^*)]dt$$

$$|\partial x_i(t)| \leq \int_0^Y [|f_i(x^* + \partial x, u^* + \partial u) - f_i(x^* + \partial x, u)|$$

$$+ |f_i(x^*, u^*) - f_i(x^* + \partial x, u^*)|]dt,$$

$$|\partial x_i(t)| \leq M\int_0^T \left[\sum_{j=1}^m |\partial u_j(s)| + \sum_{j=1}^n |\partial x_k(s)|\right]ds,$$

$$x(t) = \sum_{i=1}^n |\partial x_i(t)|; \quad i = 1, n,$$

$$0 \leq x(t) \leq M \cdot n\int_0^T \left[\sum_{j=1}^m |\partial u_j(s)| + x(s)\right]ds,$$

$$0 \leq x(t) \leq M \cdot n\int_0^T \left[\sum_{j=1}^m |\partial u_j(s)| + n \cdot M\int_0^t x(s)\right]ds = B + A\int_0^t x(s)$$

with $B = M \cdot n\int_0^T \sum_{j=1}^m |\partial u_j(s)|ds$ and $A = n \cdot M$,

$$0 \leq x(t) \leq B + A\int_0^t x(t)ds. \tag{27}$$

From (27), it follows that $x(t) \leq B \cdot e^{At}$.

If $x(s) = d \cdot e^{As}$, then d is continuous on $[0, t]$.

Consider the interval $[0, t]$ such that $d(s^*) = \max d(s)$ with $s \in [0, T]$.

If $t = s^*$, then by the replacement in (27), we obtain

$$d(s^*) \cdot e^{As^*} \leq B + A\int_0^{s^*} d(s) \cdot e^{As} ds \leq B + Ad(s^*) \cdot \left[\frac{1}{A} e^{As}\right]_0^{t^*}$$

$$B + d(s^*)[e^{As^*} - 1] = B + d(s^*) \cdot e^{As^*} - d(s^*)$$

$$\Rightarrow d(s^*) \leq B \Rightarrow d(t) \leq d(s^*) \leq B \Rightarrow d(t) \cdot e^{At} \leq B \cdot e^{At} \Rightarrow x(t) \leq B \cdot e^{At}.$$

Therefore

$$0 \leq x(t) \leq e^{At} M \cdot n\int_0^T \left[\sum_{j=1}^{m} |\partial u_j(s)|\right] ds,$$

$$|\partial x_i(t)| \leq C\|\partial u(t)\|_{L_1}.$$

Therefore for all $\varepsilon > 0$, taking $\eta = \varepsilon/c$,

$$\|\partial u(t)\|_{L_1} \leq \eta \Rightarrow |\partial x_i(t)| \leq c \cdot \eta = \varepsilon,$$

$$\begin{cases} \dot{w}_i(t) = -\sum w_j(t) \cdot \dfrac{\partial f_j(x, u)}{\partial x_i} \\ i = 1, n. \end{cases}$$

$$\Rightarrow |\partial x_i(t)| \leq C\|\partial u(t)\|_{L_1} \Rightarrow |\partial y_i(t)| \leq C \cdot \int_0^T \left[\sum_{j=1}^{m} |\partial u_j(t)|\right] dt,$$

$$N_1 = \frac{1}{2}\int_0^T \left[\sum_{i=1}^{2n} \left(\frac{\partial H(y^*, u^* + \partial u)}{\partial y_i} - \frac{\partial H(y^*, u^*)}{\partial y_i}\right)\partial y_i\right] dt,$$

$$| N_1 | \leq \frac{1}{2} C \int_0^T \sum_{j=1}^{m} |\partial u_j(t)| dt \int_0^T M \cdot \sum_{k=1}^{2n} \left(\sum_{k=1}^{m} | \partial u_k(t) | \right) dt$$

$$= \frac{1}{2} C \cdot M \cdot 2n \left[\int_0^T \left(\sum_{j=1}^{n} | \partial u_j(t) | dt \right) \right]^2 = P$$

$$= \frac{1}{2} \int_0^T \left[\frac{\partial^2 H(\theta_1)}{\partial y_i \partial y_j} \partial y_i \partial y_j \right] dt \leq P$$

$$= \frac{1}{2} \int_0^T \left[\frac{\partial^2 H(\theta_2)}{\partial y_i \partial y_j} \partial y_i \partial y_j \right] dt \leq bP$$

$$| N_1 | \leq D \left[\int_0^T \left(\sum_{j=1}^{n} | \partial u_j(t) | \right) dt \right]^2,$$

with $u^*(t) + \partial u = u_1(t)$. If $u^*(t)$ is an optimal control, then we have

$$dQ = -\int_{t_1}^{t_2} [H(x^*(t), w(t), u) - H(x^*(t), w^*(t), u^*(t))] dt + N \leq -\int_{t_1}^{t_2} \alpha dt + | N |$$

$$\leq -\int_{t_1}^{t_2} \alpha dt + D(t_2 - t_1) \int_{t_1}^{t_2} \left[\sum_{j=1}^{m} | \nu_j(t) - u_j(t) | \right]^2 dt,$$

$$dQ = Q(u) - Q(u^*) < 0.$$

This insures the contradiction that u^* and u_1 are optimal controls.

The model built allows us to calculate optimal orders for variable regimes and succession of its moment by an algorithm.

Under constraints of environment conditions, we choose moments of commutations $t_1, t_2, \ldots,$ the power of the engine, angles $\alpha_1, \alpha_2, \ldots,$ that allow to minimize (18).

3. Results of Calculation [2], [3]

First, we have analysis profiles of the road; the calculation of slopes; the representative part distinction for the optimal order calculation:

maximum slopes, minimum average. We have then undertaken a distribution of potential energy and kinetic energy in dependence of the profile of the road.

Table 1. Distribution of energies in dependence of the profile of the road.

$D_k(m) \cdot 10^3$	P_k	$F(x_k)[m/s^2]$	$D_k(m) \cdot 10^3$	P_k	$F(x_k)[m/s^2]$
0-40	+0.0030	+0.00300	295-350	0.0000	0.00000
78-145	+0.0059	+0.05879	400-780	–0.0024	–0.023544
148-155	+0.0036	+0.035316	145-148	–0.0250	–0.245250
155-167	+0.0125	+0.122625	155-165	–0.0055	–0.053955
220-295	+0.0007	+0.006867	167-220	–0.0038	–0.037964
350-365	+0.0083	+0.081423	365-375	–0.0025	–0.024525
375-384	+0.0083	+0.817173	395-415	–0.0015	–0.014715
384-395	+0.0086	+0.084758	420-480	–0.0025	–0.024525
415-420	+0.0020	+0.01962	500-508	–0.0100	–0.09810
480-500	+0.0005	+0.004905			

The comparison of the consumption of energy for usual management and optimal management has given us 60% to 80% of the gain of energy according to the typical CC (1750 kw) or the type 4B (2600 kw). This comparison has allowed us to make the analysis of the economic efficiency of optimal management of the train.

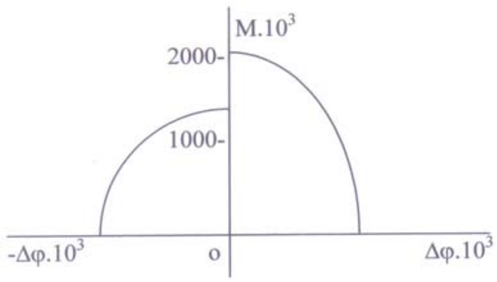

Figure 7. Dependence between the slope j and the mass [2].

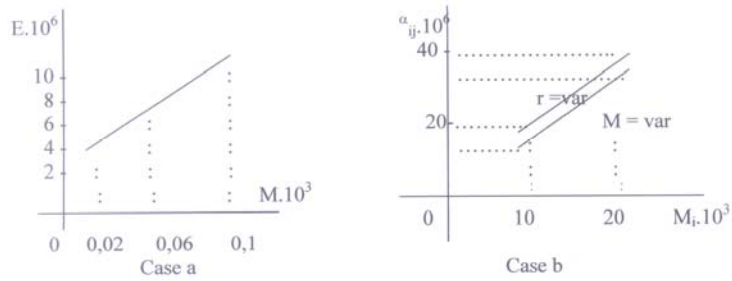

Figure 8. Dependence between energy and mass (a) and between power and mass (b).

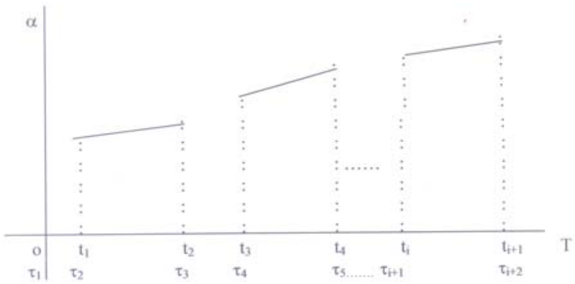

Figure 9. Partial result of the model [2].

Table 2. Consumption of energy by journey

Optimal strategy (m)		41008.3	166178.4	479357.7
Type CC (7L/km)		997686.600	4042937.500	11662241
Type 4B(8L/km)		1140213.300	4620500.100	13323276
Optimal strategy Variable mass	$M_1 = 10$	779158	3157391.5	9107798
	$M_2 = 12$	934989.600	3788869.500	10929358
Comparison consumption of energy usual management and optimal management				
Type CC (1750 kw) Optimal management	$M_1 = 10$	+218528 (76%)	+885546 (78%)	+25544443 (78%)
	$M_2 = 12$	+62696 (93%)	+254068 (93%)	+732883 (93%)
	$M_3 = 15$	–171050	–693149	–1999457

Type 4B (2600 kw) Optimal management	$M_1 = 10$	+361055 (68%)	+1463108 (68%)	+4220478 (68%)
	$M_2 = 12$	+205223 (82%)	+831630 (82%)	+2398918 (82%)
	$M_3 = 15$	–28523	–115586	–333422

4. Conclusion and Perspective Manner of Works

The problem of the minimization of the consumption of energy by a dynamic object is posed on the basis of the analysis of the existing state. The procedure for the optimal function of the commutation is formalized. Algorithm for optimal order calculation taking into account the influence of perturbations is determined on the basis of the detailed analysis of real conditions of the train journey.

Methods asymptotic in problems of optimal order can be approached by the utilization of finished elements. The quality of the approximation by functions regular enough is surely an interesting question, especially by comparing them with usual methods in optimal order.

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