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INDUCED GRAVITY WITH A RICCI-COUPLED SCALAR FIELD ON THE BRANE

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Abstract

We consider a DGP braneworld scenario where a non-minimally coupled scalar field is present on the brane. We study the effect of this non-minimally coupled scalar field on the shape of induced gravity in the weak field limit. As a result, DGP crossover scale is non-minimal coupling-dependent.

1. Introduction

Theories of extra spatial dimensions, in which the observed universe is realized as a brane embedded in a higher dimensional spacetime, have attracted a lot of attention in the last few years. In this framework, ordinary matter is trapped on the brane but gravitation propagates through the entire spacetime [3, 4, 20-22]. The cosmological evolution on the brane is given by an effective Friedmann equation that incorporates the effects of the bulk in a non-trivial manner [6, 7, 31]. From a cosmological point of view, the importance of brane models lies in the fact they can provide an alternative scenario to explain late-time expansion of the universe [16-18]. These theories usually yield correct Newtonian limit

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at large distances since the gravitational field is quenched on sub-millimeter transverse scales. This quenching appears either due to finite extension of the transverse dimensions [1, 3, 13, 28] or due to sub-millimeter transverse curvature scales induced by negative cosmological constants [2, 14, 26, 34-36]. A common feature of these type of models is that they predict deviations from the usual 4-dimensional gravity at short distances. The model proposed by Dvali et al. (DGP) [20-22] is different in this respect since it predicts deviations from the standard 4-dimensional gravity even over large distances. In this scenario, the transition between four and higher-dimensional gravitational potentials arises due to the presence of both the brane and bulk Einstein terms in the action. Even if there is no 4-dimensional Einstein-Hilbert term in the classical theory, such a term should be induced by loop-corrections from matter fields [12, 27, 37]. Generally one can consider the effect of an induced gravity term as a quantum correction in any brane-world model. The existence of a higher dimensional embedding space allows for the existence of bulk or brane matter which can certainly influence the cosmological evolution on the brane [15, 29]. A particular form of bulk or brane matter is a scalar field. Scalar fields play an important role both in models of the early universe and late-time acceleration. These scalar fields provide a simple dynamical model for matter fields in a brane-world model. In the context of induced gravity corrections, it is then natural to consider a non-minimal coupling of the scalar field to the intrinsic (Ricci) curvature on the brane that is a function of the field. The resulting theory can be thought of as a generalization of the Brans-Dicke type scalar-tensor gravity in a braneworld context [5, 8-10, 15, 23-25, 29, 32, 33]. A scalar field non-minimally coupled to the bulk or brane Ricci scalar has been studied recently [8-10, 23-25]. Some of these studies are concentrated on the bulk scalar field non-minimally coupled to the bulk Ricci scalar [8, 9, 23-25]. Some other authors have studied the minimally [5] or non-minimally [10] coupled scalar field to the induced Ricci scalar on the brane. However, the shape of gravity and its weak field limit in the presence of non-minimally coupled scalar field has not been studied in DGP scenario yet. The goal of this paper is to do this end. In the spirit of DGP scenario, we study the effect of an induced gravity term which contains an arbitrary function of a scalar field coupled to induced Ricci scalar on the brane. We will present four-dimensional equations on a DGP brane with a scalar field non-minimally coupled to the induced Ricci curvature, embedded in a five-dimensional Minkowski bulk. This is an extension to a brane-world context of scalar-tensor (Brans-Dicke) gravity. We study the weak field limit of induced gravity by calculating the gravitational potential of a static mass distribution on the brane. It has been shown that the mass density of ordinary matter on the brane should be modified by the addition of the mass density attributed to the scalar field on the brane. As a result of non-minimal coupling of scalar field to the induced Ricci scalar, the crossover scale of DGP braneworld is minimal coupling-dependent.

2. Weak Field Limit of DGP Scenario

The action for Dvali-Gabadadze-Porrati (DGP) model with codimension one is given as follows

$$S = \frac{m_4^3}{2}\int d^4x \int dy\sqrt{-g}\,\mathcal{R} + \left[\int d^4x\left(\frac{m_3^2}{2}\sqrt{-q}\,R - m_4^3\sqrt{-q}\,\overline{K} + \mathcal{L}\right)\right]_{y=0}, \quad (1)$$

where y is coordinate of fifth dimension and we assume brane is located at $y = 0$. g_{AB} is five dimensional bulk metric with Ricci scalar $\mathcal{R}$, while $q_{\mu\nu}$ is induced metric on the brane with induced Ricci scalar R. g_{AB} and $q_{\mu\nu}$ are related via $q_{\mu\nu} = \delta_\mu{}^A\delta_\nu{}^B g_{AB}$. $\overline{K}$ is trace of the mean extrinsic curvature of the brane defined as

$$\overline{K}_{\mu\nu} = \frac{1}{2}\lim_{\varepsilon\to 0}([K_{\mu\nu}]_{y=-\varepsilon} + [K_{\mu\nu}]_{y=+\varepsilon}), \quad (2)$$

and corresponding term in the action is York-Gibbons-Hawking term (see [19] and references therein). The Lagrangian $\mathcal{L}$ contains matter degrees of freedom. This action yields the following Einstein equations [20-22]

$$m_4^3\left(\mathcal{R}_{AB} - \frac{1}{2}g_{AB}\mathcal{R}\right) + m_3^2\delta_A{}^\mu\delta_B{}^\nu\left(R_{\mu\nu} - \frac{1}{2}q_{\mu\nu}R\right)\delta(y) = \delta_A{}^\mu\delta_B{}^\nu T_{\mu\nu}\delta(y), \quad (3)$$

where $T_{\mu\nu}$ is energy-momentum confined to the brane. This relation leads to the following equations

$$G_{AB} \equiv \mathcal{R}_{AB} - \frac{1}{2} g_{AB} \mathcal{R} = 0, \tag{4}$$

and

$$G_{\mu\nu} \equiv \left(R_{\mu\nu} - \frac{1}{2} q_{\mu\nu} R \right) = \frac{T_{\mu\nu}}{m_3^2} \tag{5}$$

for bulk and brane respectively. The Lanczos-Israel matching condition relates jump of the extrinsic curvature of the brane to its energy-momentum content:

$$\lim_{\varepsilon \to +0} [K_{\mu\nu}]_{y=-\varepsilon}^{y=+\varepsilon}$$

$$= \frac{1}{m_4^3} \left[T_{\mu\nu} - \frac{1}{3} q_{\mu\nu} q^{\alpha\beta} T_{\alpha\beta} \right]_{y=0} - \frac{m_3^2}{m_4^3} \left[R_{\mu\nu} - \frac{1}{6} q_{\mu\nu} q^{\alpha\beta} R_{\alpha\beta} \right]_{y=0}. \tag{6}$$

In this relation the second term can be described as energy-momentum of gravitational field [19]. To understand the shape of gravity in this model, we calculate the gravitational potential of a static mass distribution in weak field limit. We consider the perturbation $g_{AB} = \eta_{AB} + h_{AB}$ for background metric. Within the Gaussian normal coordinates one can impose a harmonic gauge on the longitudinal coordinates x^{μ} as follows [19-22]

$$\partial_{\alpha} h^{\alpha}{}_{\mu} + \partial_y h_{y\mu} = \frac{1}{2} \partial_{\mu} (h^{\alpha}{}_{\alpha} + h_{yy}), \tag{7}$$

this will lead us to a decoupled equation for the gravitational potential of a static mass distribution. The transverse equations in the gauge (7) are as follows

$$R_{yy} - R^{\alpha}{}_{\alpha} = \frac{1}{2} \partial_{\alpha} \partial^{\alpha} (h^{\beta}{}_{\beta} - h_{yy}) + \partial_y \partial_{\alpha} h^{\alpha}{}_{y} = 0, \tag{8}$$

$$R_{y\mu} = \frac{1}{2} (\partial_{\mu} \partial_{\alpha} h^{\alpha}{}_{y} - \partial_A \partial^A h_{y\mu}) + \frac{1}{4} \partial_{\mu} \partial_y (h_{yy} - h^{\alpha}{}_{\alpha}) = 0. \tag{9}$$

These equations can be solved and the results are $h_{y\mu} = 0$, $h_{yy} = h^{\alpha}{}_{\alpha}$. The remaining equations take the following form

$$m_4^3(\partial_\alpha\partial^\alpha + \partial_y^2)h_{\mu\nu} + m_3^2\delta(y)(\partial_\alpha\partial^\alpha h_{\mu\nu} - \partial_\mu\partial_\nu h^\alpha{}_\alpha)$$
$$= -2\delta(y)\left(T_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu}\eta^{\alpha\beta}T_{\alpha\beta}\right). \tag{10}$$

So, the gravitational potential of a mass density $\rho(\vec{r}) = M\delta(\vec{r})$ on the brane satisfies the following equation

$$m_4^3(\Box^{(4)} + \partial_y^2)U(\vec{r},\ y) + m_3^2\delta(y)\Box^{(4)}U(\vec{r},\ y) = \frac{2}{3}M\delta(\vec{r})\delta(y), \tag{11}$$

where $\Box^{(4)}$ is 4-dimensional (brane) d'Alembertian. We consider the following Fourier ansatz

$$U(\vec{r},\ y) = \frac{1}{(2\pi)^4}\int d^3\vec{p}\int dp_y U(\vec{p},\ p_y)\exp[i(\vec{p}\cdot\vec{r} + p_y y)]. \tag{12}$$

Insertion of this ansatz in equation (11) yields the following integral equation

$$m_4^3(\vec{p}^2 + p_y^2)U(\vec{p},\ p_y) + \frac{m_3^2}{2\pi}\vec{p}^2\int dp'_y U(\vec{p},\ p'_y) = -\frac{2}{3}M. \tag{13}$$

This equation can be solved to find [19-22]

$$U(\vec{p},\ p_y) = -\frac{4}{3}\frac{M}{(\vec{p}^2 + p_y^2)(2m_4^3 + m_3^2|\,\vec{p}\,|)}. \tag{14}$$

The resulting potential on the brane is

$$U(\vec{r}) = -\frac{M}{6\pi m_3^2 r}\left[\cos(\xi r) - \frac{2}{\pi}\cos(\xi r)\mathrm{Si}(\xi r) + \frac{2}{\pi}\sin(\xi r)\mathrm{Ci}(\xi r)\right], \tag{15}$$

where $\xi = \frac{2m_4^3}{m_3^2}$, $\mathrm{Si}(x) = \int_0^x d\omega\frac{\sin\omega}{\omega}$ and $\mathrm{Ci}(x) = -\int_x^\infty d\omega\frac{\cos\omega}{\omega}$. There is a transition scale $\xi^{-1} \equiv \ell = \frac{m_3^2}{2m_4^3}$ between four and five-dimensional behavior of the gravitational potential in this scenario:

$$r \ll \ell: \quad U(\vec{r}) = -\frac{M}{6\pi m_3^2 r}\left[1 + \left(\gamma - \frac{2}{\pi}\right)\frac{r}{\ell} + \frac{r}{\ell}\ln\left(\frac{r}{\ell}\right) + \mathcal{O}\left(\frac{r^2}{\ell^2}\right)\right], \tag{16}$$

and

$$r \gg \ell: \quad U(\vec{r}) = -\frac{M}{6\pi^2 m_4^3 r^2}\left[1 - 2\frac{\ell^2}{r^2} + \mathcal{O}\left(\frac{\ell^4}{r^4}\right)\right], \tag{17}$$

where $\gamma \simeq 0.577$ is Euler's constant. In which follows we extend this formalism to the case where there exists a scalar field non-minimally coupled to induced Ricci scalar on the brane.

3. Induced Gravity with Non-minimally Coupled Brane-scalar Field

Now we extend formalism of the preceding section to the case where a scalar field non-minimally coupled to induced Ricci scalar is present on the brane. The action of the problem can be written as

$$S = \int d^5x \frac{m_4^3}{2}\sqrt{-g}\,\mathcal{R}$$
$$+ \int d^4x\sqrt{-q}\left[\frac{m_3^2}{2}\alpha(\phi)R[q] - \frac{1}{2}q^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi) + m_4^3\overline{K} + \mathcal{L}_m\right], \tag{18}$$

where we have included a general non-minimal coupling $\alpha(\phi)$. The ordinary matter part of the action is shown by Lagrangian $\mathcal{L}_m \equiv \mathcal{L}_m(q_{\mu\nu}, \psi)$, where ψ is matter field and corresponding energy-momentum is $T_{\mu\nu}$. The pure scalar field Lagrangian, $\mathcal{L}_\phi = -\frac{1}{2}q^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi)$, yields the following energy-momentum tensor

$$\tau_{\mu\nu} = \nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}q_{\mu\nu}(\nabla\phi)^2 - q_{\mu\nu}V(\phi). \tag{19}$$

The Bulk-brane Einstein's equations calculated from action (1) are given by

$$m_4^3\left(\mathcal{R}_{AB} - \frac{1}{2}g_{AB}\mathcal{R}\right)$$
$$+ m_3^2\delta_A{}^\mu\delta_B{}^\nu\left[\alpha(\phi)\left(R_{\mu\nu} - \frac{1}{2}q_{\mu\nu}R\right) - \nabla_\mu\nabla_\nu\alpha(\phi) + q_{\mu\nu}\Box^{(4)}\alpha(\phi)\right]\delta(y)$$
$$= \delta_A{}^\mu\delta_B{}^\nu\Upsilon_{\mu\nu}\delta(y). \tag{20}$$

This relation can be written as follows

$$m_4^3\left(\mathcal{R}_{AB} - \frac{1}{2}g_{AB}\mathcal{R}\right) + m_3^2\alpha(\phi)\delta_A{}^{\mu}\delta_B{}^{\nu}\left(R_{\mu\nu} - \frac{1}{2}q_{\mu\nu}R\right)\delta(y)$$

$$= \delta_A{}^{\mu}\delta_B{}^{\nu}\mathcal{T}_{\mu\nu}\delta(y), \tag{21}$$

where $\mathcal{T}_{\mu\nu}$ is total energy-momentum of the system defined as follows

$$\mathcal{T}_{\mu\nu} = m_3^2\nabla_\mu\nabla_\nu\alpha(\phi) - m_3^2 q_{\mu\nu}\Box^{(4)}\alpha(\phi) + \Upsilon_{\mu\nu}, \tag{22}$$

and $\Upsilon_{\mu\nu} = \mathcal{T}_{\mu\nu} + \tau_{\mu\nu}$. From (21) we find

$$G_{AB} = \mathcal{R}_{AB} - \frac{1}{2}g_{AB}\mathcal{R} = 0 \tag{23}$$

and

$$G_{\mu\nu} = \left(R_{\mu\nu} - \frac{1}{2}q_{\mu\nu}R\right) = \frac{\mathcal{T}_{\mu\nu}}{m_3^2\alpha(\phi)} \tag{24}$$

for bulk and brane respectively. The corresponding junction conditions relating the extrinsic curvature to the energy-momentum tensor of the brane, have the following form

$$\lim_{\varepsilon\to+0}[K_{\mu\nu}]_{y=-\varepsilon}^{y=+\varepsilon} = \frac{1}{m_4^3}\left[\mathcal{T}_{\mu\nu} - \frac{1}{3}q_{\mu\nu}q^{\alpha\beta}\mathcal{T}_{\alpha\beta}\right]_{y=0}$$

$$- \frac{m_3^2\alpha(\phi)}{m_4^3}\left[R_{\mu\nu} - \frac{1}{6}q_{\mu\nu}q^{\alpha\beta}R_{\alpha\beta}\right]_{y=0}. \tag{25}$$

As previous section, we set $g_{AB} = \eta_{AB} + h_{AB}$ to investigate the weak field limit of the problem. We adapt the harmonic gauge (7) on longitudinal coordinates to obtain the following equations

$$m_4^3(\partial_\alpha\partial^\alpha + \partial_y^2)h_{\mu\nu} + m_3^2\alpha(\phi)(\partial_\alpha\partial^\alpha h_{\mu\nu} - \partial_\mu\partial_\nu h_\alpha^\alpha)\delta(y)$$

$$= -2\delta(y)\left[\mathcal{T}_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu}\eta^{\alpha\beta}\mathcal{T}_{\alpha\beta}\right]. \tag{26}$$

We suppose that non-minimally coupled scalar field has an effective mass M_ϕ. The gravitational potential of mass densities $\rho_\psi(\vec{r}) = M_\psi\delta(\vec{r})$ and $\rho_\phi(\vec{r}) = M_\phi\delta(\vec{r})$ on the brane satisfies the following equation

$$m_4^3(\partial_\alpha\partial^\alpha + \partial_y^2)U(\vec{r},\, y) + m_3^2\alpha(\phi)\delta(y)\partial_\alpha\partial^\alpha U(\vec{r},\, y)$$

$$= \frac{2}{3}(M_\psi + M_\phi)\delta(\vec{r})\delta(y). \tag{27}$$

This equation shows that the mass in standard DGP framework should be modified by the addition of the mass of the non-minimally coupled scalar field. Using the Fourier ansatz (12) in equation (28) we find

$$m_4^3(\vec{p}^2 + p_y^2)U(\vec{p},\, p_y) + \frac{m_3^2\alpha(\phi)}{2\pi}\vec{p}^2\int dp'_y U(\vec{p},\, p'_y) = -\frac{2}{3}(M_\psi + M_\phi). \tag{28}$$

This integral equation has the following solution

$$U(\vec{p},\, p_y) = -\frac{4}{3}\frac{M_\psi + M_\phi}{(\vec{p}^2 + p_y^2)(2m_4^3 + m_3^2\alpha(\phi)|\,\vec{p}\,|)}. \tag{29}$$

The resulting potential on the brane is

$$U(\vec{r}) = -\left(\frac{M_\psi + M_\phi}{6\pi m_3^2\alpha(\phi)r}\right)\left[\cos(\xi_\alpha r) - \frac{2}{\pi}\cos(\xi_\alpha r)\mathrm{Si}(\xi_\alpha r) + \frac{2}{\pi}\sin(\xi_\alpha r)\mathrm{Ci}(\xi_\alpha r)\right], \tag{30}$$

where $\xi_\alpha = \dfrac{2m_4^3}{m_3^2\alpha(\phi)}$. Now there is a modified transition scale

$$\xi_\alpha^{-1} \equiv \ell_\alpha = \frac{m_3^2\alpha(\phi)}{2m_4^3} \tag{31}$$

between four and five-dimensional behavior of the gravitational potential in this scenario:

$$r \ll \ell_\alpha: \quad U(\vec{r}) = -\frac{M_\psi + M_\phi}{6\pi m_3^2\alpha(\phi)r}\left[1 + \left(\gamma - \frac{2}{\pi}\right)\frac{r}{\ell_\alpha} + \frac{r}{\ell_\alpha}\ln\left(\frac{r}{\ell_\alpha}\right) + \mathcal{O}\left(\frac{r^2}{\ell_\alpha^2}\right)\right], \tag{32}$$

and

$$r \gg \ell_\alpha: \quad U(\vec{r}) = -\frac{M_\psi + M_\phi}{6\pi^2 m_4^3 r^2}\left[1 - 2\frac{\ell_\alpha^2}{r^2} + \mathcal{O}\left(\frac{\ell_\alpha^4}{r^4}\right)\right]. \tag{33}$$

Therefore, the mass density of ordinary matter on the brane should be modified by the addition of the mass density attributed to the scalar field on the brane. On the other hand, the DGP transition scale between four and five dimensional behavior of gravitational potential now is explicitly dependent on the strength of non-minimal coupling. If $\alpha(\phi)$ varies slightly from point to point on the brane, it can be interpreted as a spacetime dependent Newton's constant. The dynamics that control this variation are determined by the following equation

$$\nabla^{\mu}\phi\nabla_{\mu}\phi - \frac{dV}{d\phi} + \frac{m_3^2}{2}\left(\frac{d\alpha(\phi)}{d\phi}\right)R = 0. \tag{34}$$

Note that in the real world we do not want $\alpha(\phi)$ to vary too much since it will have observable effects in classic experimental test of general relativity and also in cosmological tests such as primordial nucleosynthesis [11]. This can be ensured either by choosing a large mass for scalar field ϕ or choosing $\alpha(\phi)$ so that large changes in ϕ give rise to relatively small changes in Newton's constant. So, when $\alpha(\phi)$ varies in DGP brane from point to point, the crossover scale will change and is no longer a constant. This feature would change previous picture of crossover scale in DGP scenario and my change some arguments on phenomenology of this scenario [30]. We can define a modified four-dimensional Planck mass as $m_3^{(\alpha)} = \sqrt{\alpha(\phi)}\,m_3$ and therefore the effect of non-minimal coupling can be attributed to the modification of four-dimensional Planck mass. This is equivalent to modification of four dimensional Newton's constant. If we use the reduced Planck mass for m_3, gravitational potential for small r limit will be stronger than the ordinary four-dimensional potential by a factor $\frac{4}{3}\alpha^{-1}$. In fact the coupling of the masses on the brane to the induced Ricci tensor on the brane is increased by this factor. This extra factor of $\frac{4}{3}\alpha^{-1}$ is in agreement with the tensorial structure of the graviton propagator due to additional helicity state of the five-dimensional graviton [20-22].

4. Summary and Conclusion

In the spirit of DGP braneworld scenario, we have studied the effect of an induced gravity term which contains an arbitrary function of a scalar field coupled to induced Ricci scalar on the brane. Four-dimensional equations on a DGP brane with a scalar field non-minimally coupled to the induced Ricci curvature, embedded in a five-dimensional Minkowski bulk have been presented. This is an extension to a brane-world context of scalar-tensor (Brans-Dicke) gravity. The weak field limit of induced gravity has been studied by calculating the gravitational potential of a static mass distribution on the brane. It has been shown that the mass density of ordinary matter on the brane should be modified by the addition of the mass density attributed to the scalar field on the brane. In the presence of non-minimally coupled scalar field on the brane, the crossover scale of DGP scenario will be modified by a factor of non-minimal coupling, α. This non-minimal coupling-dependent crossover scale may shed light on the phenomenology of DGP model.

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