JP Journal of Biostatistics
Volume 1, Number 2, 2007, Pages 123-128
Published online: October 18, 2007
This paper is available online at http://www.pphmj.com

RELATIVE RISK, ODDS RATIO, RELATIVE AND ABSOLUTE RISK REDUCTION - SOME COMMENTS ABOUT THEIR RELATIONS AND INTERPRETATION

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Abstract

Associations of two dichotomous variables can be expressed by a variety of measures. Relative risk, odds ratio and risk reductions belong to the commonly used ones. By illustrations it is shown that the odds ratio differs markedly from the relative risk when two conditions are fulfilled: One rate is close to 50% while the other is not. The odds ratio generally is higher than the relative risk, hence taking it as an approximation for the relative risk may overestimate effects under certain circumstances. This may particularly affect meta analysis of binary data, where the odds ratio is a frequently used measure. On the other hand, relative risk, relative and absolute risk reduction are dependent on the way the four fold table was arranged – while the odds ratio is not. A formula for transposing the odds ratio into the relative risk is provided.

Introduction

There are various measures available for describing the relation between two dichotomous variables, i.e., a four fold table. Some of the most commonly used are Relative Risk (RR) and Odds Ratio (OR) [3].

2000 Mathematics Subject Classification: 62-07.

Keywords and phrases: relative risk, odds ratio, relative risk reduction, absolute risk reduction.

Received July 21, 2007

Some Cochrane reviews make use of two additional measures, i.e., Relative Risk Reduction (RRR) and Absolute Risk Reduction (ARR: EBM Notizbuch, 1997). In some cases, researchers or clinicians have to compare different indicators from different studies. The aim of this paper is (a) to demonstrate the advantages and limitations of these measures in different situations, (b) to propose a simple formula for estimating the bias when only OR is given, and (c) to allow comparisons between OR and RR.

	Outcome		
	Pos.	Neg.	
Treatment	a	b	$a + b$
Baseline	c	d	$c + d$
	$a + c$	$b + d$	

Risk at Treatment: $$R_T = \frac{a}{a+b}$$

Risk at Baseline: $$R_B = \frac{c}{c+d}$$

Relative Risk: $$RR = \frac{R_T}{R_B} \text{ or } \frac{a/(a+b)}{c/(c+d)}$$

Relative Risk Reduction: $$RRR = \frac{R_B - R_T}{R_B}$$

Absolute Risk Reduction: $$ARR = R_B - R_T$$

Odds Ratio: $$OR = \frac{a/b}{c/d} \text{ or } \frac{ad}{bc}$$

Figure 1. Abbreviations and Formulas.

Features of the Measures in Two Examples

Imagine a disease having probabilities for positive and negative outcomes both of 50% without any treatment. A table on 1000 subjects showing results of different treatments is displayed in Table 1 on the left hand side. Treatment A4, for instance, would reduce the total amount of negative outcome from 500 without treatment to 50. The risk for a single subject to have a negative outcome decreases from 50% to 5% under treatment A4. RR is 10 and gives an intuitively interpretable numerical indicator for the effect size of treatment A4. $\mathrm{RRR} = 90\%$ describes the reduction of the initial risk by the treatment for a single observation, some researchers prefer this to RR. $\mathrm{ARR} = 45\%$ indicates the overall reduction of negative outcomes in the population. ARR considers that some subjects who would have a positive outcome without treatment have to be treated, too. It is an important indicator for the calculation of benefit/cost analysis in economics. So far, different measures give different information but all three make sense in the given example. OR, even if widely used, does not provide an interpretable result in this situation - OR for treatment A4 is 19. On the first sight, this estimate even appears to be wrong, because the number 19 does not seem to make any sense in the four fold table defined by the baseline and treatment A4. It is calculated in that way that under treatment A4, the odds decreases from 1:1 at baseline to 19:1.

RR and OR are often regarded to be more or less the same. Wise [6] for example announced that a review is 88 times more likely to deny any harm of passive smoking when the author as an affiliation to tobacco industry compared to when the author does not. Wise took the OR instead of the RR, the latter was 7, still a strong effect but far smaller than 88 [1]. Even in good statistical textbooks OR is recommended as an estimate for RR with giving a small warning on the necessity to check base rates only (e.g., Fleiss [3]). Well-known statistical programmes report OR as an estimate for RR without any comment [5]. That both are not always interchangeable is demonstrated in the situation above. The effect size of the treatment can be overestimated by far using OR instead of RR.

Table 1. Various measures in different treatments for two baseline probabilities

Situation A: Baseline risk = .5								Situation B: Baseline risk = .1							
	Outcome (N)								Outcome (N)						
Treatment	Pos.	Neg.	R	RR	OR	RRR	ARR	Treatment	Pos.	Neg.	R	RR	OR	RRR	ARR
A1	400	600	.40	1.25	1.5	.20	.10	B1	80	920	.08	1.25	1.28	.20	.02
A2	250	750	.25	2	3	.50	.25	B2	50	950	.05	2	2.11	.50	.05
A3	100	900	.10	5	9	.80	.40	B3	20	980	.02	5	5.44	.80	.08
A4	50	950	.05	10	19	.90	.45	B4	10	990	.01	10	11	.90	.09
A5	25	975	.025	20	39	.95	.475	B5	5	995	.005	20	22.11	.95	.095
A6	10	990	.01	50	99	.98	.49	B6	2	998	.002	50	55.44	.98	.098
A7	5	995	.005	100	199	.99	.495	B7	1	999	.001	100	111	.99	.099
Baseline	500	500	.5					Baseline	100	900	.1				

Note: R = Risk, RR = Relative Risk, OR = Odds Ratio, RRR = Relative Risk Reduction, ARR = Absolute Risk Reduction.

What we can learn from the example: Not to take OR as an estimate for RR? Sackett et al. [4] proposed this a decade ago. But before following him, let us consider situation B, where the proportion of negative outcomes is only 10% without treatment. Table 1 displays on the right hand side the results for different treatments with identical RR's to situation A. Treatment B4 decreases the risk from 10% at baseline to 1%, $\text{RR} = 1:10$, $\text{RRR} = 90\%$. So far, measures give the same results. Because the treatments are working on different base rates, ARR differs. $\text{ARR}_{\text{B4}} = 9\%$ is low in comparison to situation A. In situation B4, OR has a value of 11, which is far closer to RR than in situation A4. In situation B still there is an overestimation of RR by OR, but the bias is small.

Which factors contribute to the magnitude of the bias? First, as demonstrated by the differences between situations A and B, it is the base rate. This effect is well known and warnings have been published. Altman [1] recommends not to take OR for RR when base rates exceed 20-30%. But a second factor contributes to the magnitude of bias. The larger the RR and OR become, i.e., the better the treatment, the more they differ (see treatments A1 to A7). OR is always larger than RR, but as long as effects are small there is only a small bias, too. This means that there is no risk to detect a false positive effect when taking OR for RR – at least as long as theoretical considerations and statistical decision are appropriate.

On the other hand, is there any reason to rely on a measure that is misleading in some cases? The answer is Yes, because OR obviously has some advantages over other measures. (1) It is easy to compute and (2) has a simple relation to coefficient ß in logistic regressions, what makes OR particularly suitably for multivariate analysis. (3) There are exactly two OR's for a four fold table, with one being the reciprocal of the other. This is not the case with respect to RR. Taking a four fold table in a different arrangement leads to different RR's [2]. It only makes sense to compute RR when the rare event is regarded, i.e., the probability that is smaller than 50%, as done in examples above. Strong treatment effects are not perceivable when the frequent event is taken as numerator. In situation B6 for example, the values calculated for the positive outcome would be RR ≈ .90, RRR ≈ .01. This will usually not become a problem as long as a single table has to be analysed, but when a series of tables has to be presented in a similar way, it may be impossible to report adequate results on bases of RR, RRR and ARR.

Hence it is helpful to estimate the magnitude of the overestimation of OR when it is interpreted as RR. We present a formula for transposing OR to RR where only the base rate is necessary as additional information. Obviously, we emphasise to compare results on the basis of RR rather than OR, whenever possible. However, when running multiple logistic regression analyses, we only obtain ß's or OR's. The former are not illustrative at all, and the latter are misleading if one subgroup has similar proportions. By simply correcting OR for baseline proportions, it becomes possible with a pocket calculator to estimate the bias. A note of caution should be added at this place. In case of multiple explanatory variables, the estimate is close to RR, but not precisely RR. If the difference between the observed OR and the estimated RR is large, i.e., would be of relevance in a given context, computation intensive procedures, such as Poisson regression, cannot be avoided [7]. If the difference is small, no more crosscheck is needed.

$$RR = \frac{a + b^{*}OR}{a + b} \quad \text{or} \quad RR = \frac{(d + c)^{*}OR}{d + c^{*}OR}.$$

Conclusion

RR, RRR and ARR are content related measures. They allow directly content related interpretations. However, there is one main restriction with these measures: The risks on which further computation are based upon must focus on the rare events. OR has no such restriction, but OR is a formal measure and does not allow content related interpretations under some circumstances. Using it will sometimes overestimate the size of an effect in comparison to RR or RRR. When one of the simple risks is higher than 20% and there is a (large) effect, OR is not roughly the same as RR. This may concern meta analysists for example, who are at risk overestimate effects. But we should be careful to abandon OR too fast: in many situations we have to rely on it for the analysis of data. But before reporting the results to an audience unfamiliar with the difference between OR and RR, statisticians should crosscheck their results – for instance by the formula given above. Therefore, we need to be aware when problems with OR may occur. The examples given above deal with a treatment. This was done only to have an easy to understand and interpretable framework to present different series of four fold tables. Of course, in any observational study the same problems occur.

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