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ROBUST H_∞ CONTROLLER DESIGN OF UNCERTAIN FUZZY SYSTEMS WITH DISCRETE AND DISTRIBUTED DELAYS

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Abstract

In this paper, we consider the H_∞ disturbance attenuation of Takagi-Sugeno fuzzy systems with discrete and distributed delays. We employ a generalized Lyapunov functional to obtain delay-dependent conditions that guarantee the H_∞ disturbance attenuation of fuzzy systems with discrete and distributed delays. We introduce free matrices to such a Lyapunov functional in order to reduce the conservatism in H_∞ disturbance attenuation conditions. These techniques lead to generalized and less conservative conditions. Applying the same techniques made on the H_∞ disturbance attenuation conditions, we obtain delay-dependent conditions for the robust H_∞ disturbance attenuation of uncertain fuzzy systems with discrete and distributed delays. Moreover, we consider the state feedback that achieves the H_∞ disturbance attenuation. Based on the H_∞ disturbance attenuation conditions on the closed-loop system, we give design methods of the state feedback controllers for the fuzzy time-delay systems. Finally, we give two examples to illustrate our results.

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1. Introduction

Recently, research on nonlinear time-delay systems described by fuzzy system representation is very active. Time-delay systems often appear in many industrial fields and mathematical formulations. Thus, it is important to analyze time-delay systems and design controllers for them. Recent research has investigated stability conditions for linear time-delay systems based on Linear Matrix Inequalities (LMIs). Both delay-independent and delay-dependent stability conditions for linear systems with discrete delays have been obtained in [6, 14, 19]. Delay-independent conditions can be applied to systems with any size of time-delays, whereas delay-dependent conditions are less conservative. Distributed time-delay counterpart has been given in [3, 5, 8, 20]. Theory has been extended to fuzzy time-delay systems. Stability conditions for fuzzy time-delay systems with discrete delays have been obtained in [1, 2, 7, 22, 23, 25]. Robust stability analysis for uncertain fuzzy time-delay systems has also been investigated in [11, 12, 17, 21]. Also, the H_∞ disturbance attenuation has been considered in [4, 9]. However, no result on fuzzy systems with distributed delays has appeared in the literature, except [24].

In this paper, we consider the H_∞ disturbance attenuation of Takagi-Sugeno fuzzy systems with discrete and distributed time-delays. Here we extend the existing results to a class of fuzzy systems with discrete and distributed time-delays. The key technique to obtain delay-dependent H_∞ disturbance attenuation conditions for such systems is to select an appropriate Lyapunov functional and to introduce free matrices to it. We select a more generalized Lyapunov functional than that of Yoneyama [24] to obtain less conservative stability conditions. We also consider the robust H_∞ disturbance attenuation of fuzzy time-delay systems with uncertain parameters. Moreover, the design methods of the state feedback that achieves the robust H_∞ disturbance attenuation for uncertain fuzzy systems with discrete and distributed time-delays are proposed. Finally, two examples are given to illustrate the effectiveness of our results.

2. Time-delay Systems

In this section, we introduce Takagi-Sugeno fuzzy systems with discrete and distributed delays. Consider the Takagi-Sugeno fuzzy model with time-delay, described by the following IF-THEN rules:

IF

$$\xi_1 \text{ is } M_{i1} \text{ and } \cdots \text{ and } \xi_p \text{ is } M_{ip},$$

THEN

$$\dot{x}(t) = (A_i + \Delta A_i(t))x(t) + (A_{di} + \Delta A_{di}(t))x(t-\tau)$$

$$+ (D_i + \Delta D_i(t))\int_{t-\tau}^{t} x(s)ds + B_{1i}w(t) + (B_{2i} + \Delta B_i(t))u(t),$$

$$z(t) = C_i x(t) + C_{di}x(t-\tau) + D_{11i}w(t) + D_{12i}u(t), \quad i = 1, \dots, r,$$

$$x(\phi) = 0, \quad \phi \in [-\tau, 0],$$

where $\tau \geq 0$ is a time-delay, $x(t) \in \Re^n$ is the state, $w(t) \in \Re^{m_1}$ is the disturbance, $u(t) \in \Re^{m_2}$ is the control input and $z(t) \in \Re^q$ is the controlled output. The matrices $A_i, A_{di}, B_{1i}, B_{2i}, C_i, C_{di}, D_i, D_{11i}$ and D_{12i} are of appropriate dimensions. r is the number of IF-THEN rules. M_{ij} are fuzzy sets and $\xi_1, \dots, \xi_p$ are premise variables.

We set $\xi = [\xi_1 \cdots \xi_p]^T$ and $\xi(t)$ is assumed to be given or to be a measurable function. The uncertain matrices are of the form

$$[\Delta A_i(t) \quad \Delta A_{di}(t) \quad \Delta B_{2i}(t) \quad \Delta D_i(t)] = H_i F_i(t)[E_{1i} \quad E_{2i} \quad E_{bi} \quad E_{di}],$$
$$\forall i = 1, \dots, r, \qquad (1)$$

where $H_i, E_{1i}, E_{2i}, E_{bi}$ and E_{di} are known matrices of appropriate dimensions, and $F_i(t)$ are unknown real time varying matrices satisfying

$$F_i^T(t)F_i(t) \leq I, \quad i = 1, \dots, r.$$

The state and controlled output equations are defined as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} \lambda_i(\xi(t)) \Big\{ (A_i + \Delta A_i(t))x(t) + (A_{di} + \Delta A_{di}(t))x(t-\tau)$$

$$+ (D_i + \Delta D_i(t)) \int_{t-\tau}^{t} x(s)ds + B_{1i}w(t) + (B_{2i} + \Delta B_i(t))u(t) \Big\},$$

$$z(t) = \sum_{i=1}^{r} \lambda_i(\xi(t)) \{ C_i x(t) + C_{di}x(t-\tau) + D_{11i}w(t) + D_{12i}u(t) \}, \tag{2}$$

where

$$\lambda_i(\xi) = \frac{\beta_i(\xi)}{\sum_{i=1}^{r} \beta_i(\xi)}, \quad \beta_i(\xi) = \prod_{j=1}^{q} M_{ij}(\xi_j)$$

and $M_{ij}(\cdot)$ is the grade of the membership function of M_{ij}. We assume that

$$\beta_i(\xi(t)) \geq 0, \quad i = 1, \ldots, r, \quad \sum_{i=1}^{r} \beta_i(\xi(t)) > 0$$

for any $\xi(t)$. Hence $\lambda_i(\xi(t))$ satisfy

$$\lambda_i(\xi(t)) \geq 0, \quad i = 1, \ldots, r, \quad \sum_{i=1}^{r} \lambda_i(\xi(t)) = 1$$

for any $\xi(t)$. We say the system (2) achieves the robust H_∞ disturbance attenuation γ if it is robustly stable with $w(t) = 0$ and for a prescribed constant γ, it satisfies

$$\| z \|_2^2 < \gamma^2 \| w \|_2^2$$

for all $w(t) \neq 0$ and all admissible uncertainties.

When we consider the H_∞ disturbance attenuation γ of nominal fuzzy time-delay systems, we set $\Delta A_i = 0, \Delta A_{di} = 0, \Delta D_i = 0, \Delta B_i = 0, \forall i = 1, \ldots, r$ in (2).

3. H_∞ Disturbance Attenuation

This section gives sufficient conditions that guarantee Takagi-Sugeno fuzzy time-delay systems to achieve the robust H_∞ disturbance attenuation.

3.1. Nominal systems

We first give delay-dependent H_∞ disturbance attenuation conditions for the nominal fuzzy system (2) without uncertainties.

Theorem 3.1. *Given a scalar* $\tau \geq 0$ *and a prescribed constant* $\gamma > 0$, *the nominal fuzzy time-delay system* (2) *with* $u(t) = 0$ *and no uncertainties achieves the* H_∞ *disturbance attenuation* γ *if there exist common matrices*

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} > 0, \quad Q \geq 0, \quad R \geq 0, \quad W = \begin{bmatrix} W_{11} & W_{12} \\ W_{12}^T & W_{22} \end{bmatrix} \geq 0, \tag{3}$$

and some matrices N_{ij}, $i = 1, \ldots, 5$, $j = 1, \ldots, r$ *and* T_i, $i = 1, \ldots, 5$, *such that*

$$\Phi_i = \begin{bmatrix} \Phi_{11i} & \Phi_{12i} & \Phi_{13i} & \Phi_{14i} & -\tau N_{1i} & \Phi_{16i} \\ \Phi_{12i}^T & \Phi_{22} & \Phi_{23i} & \Phi_{24i} & -\tau N_{2i} & \Phi_{26i} \\ \Phi_{13i}^T & \Phi_{23i}^T & \Phi_{33i} & \Phi_{34i} & -\tau N_{3i} & \Phi_{36i} \\ \Phi_{14i}^T & \Phi_{24i}^T & \Phi_{34i}^T & \Phi_{44i} & \Phi_{45i} & \Phi_{46i} \\ -\tau N_{1i}^T & -\tau N_{2i}^T & -\tau N_{3i}^T & \Phi_{45i}^T & -\tau W_{22} & -\tau N_{5i}^T \\ \Phi_{16i}^T & \Phi_{26i}^T & \Phi_{36i}^T & \Phi_{46i}^T & -\tau N_{5i} & \Phi_{66i} \end{bmatrix} < 0,$$

$$\forall i = 1, \ldots, r, \tag{4}$$

where

$$\begin{aligned} \Phi_{11i} &= P_{12} + P_{12}^T + Q + \tau^2 R + \tau W_{11} \\ &\quad + N_{1i} + N_{1i}^T - T_1 A_i - A_i^T T_1^T + C_i^T C_i, \\ \Phi_{12i} &= P_{11} + \tau W_{12} + N_{2i}^T + T_1 - A_i^T T_2^T, \\ \Phi_{13i} &= -P_{12} - N_{1i} + N_{3i}^T - A_i^T T_3^T - T_1 A_{di} + C_i^T C_{di}, \\ \Phi_{14i} &= \tau P_{22} + \tau N_{4i}^T - \tau A_i^T T_4^T - \tau T_1 D_i, \end{aligned}$$

$$\Phi_{16i} = -T_1 B_{1i} - A_i^T T_5^T + N_{5i}^T + C_i^T D_{11i},$$

$$\Phi_{22} = \tau W_{22} + T_2 + T_2^T,$$

$$\Phi_{23i} = -N_{2i} + T_3^T - T_2 A_{di},$$

$$\Phi_{24i} = \tau P_{12} + \tau T_4^T - \tau T_2 D_i,$$

$$\Phi_{26i} = -T_2 B_{1i} + T_5^T,$$

$$\Phi_{33i} = -Q - N_{3i} - N_{3i}^T - T_3 A_{di} - A_{di}^T T_3^T + C_{di}^T C_{di},$$

$$\Phi_{34i} = -\tau P_{22} - \tau N_{4i}^T - \tau A_{di}^T T_4^T - \tau T_3 D_i,$$

$$\Phi_{36i} = -T_3 B_{1i} - A_{di}^T T_5^T - N_{5i}^T + C_{di}^T D_{11i},$$

$$\Phi_{44i} = -\tau W_{11} - \tau^2 T_4 D_i - \tau^2 D_i^T T_4^T - \tau^2 R,$$

$$\Phi_{45i} = -\tau W_{12} - \tau^2 N_{4i},$$

$$\Phi_{46i} = -\tau T_4 B_{1i} - \tau D_i^T T_5^T,$$

$$\Phi_{66i} = -T_5 B_{1i} - B_{1i}^T T_5^T - \gamma^2 I + D_{11i}^T D_{11i}.$$

Proof. We consider the following Lyapunov functional

$$V(t) = V_1(t) + V_2(t),$$

where

$$\begin{aligned} V_1(t) = {} & x^T(t) P_{11} x(t) + 2x^T(t) P_{12} \int_{t-\tau}^{t} x(s) ds \\ & + \left[\int_{t-\tau}^{t} x(s) ds\right]^T P_{22} \int_{t-\tau}^{t} x(s) ds + \int_{t-\tau}^{t} x^T(s) Q x(s) ds \\ & + \int_{-\tau}^{0} \int_{t+\theta}^{t} x^T(s) W_{11} x(s) ds d\theta + 2\int_{-\tau}^{0} \int_{t+\theta}^{t} x^T(s) W_{12} \dot{x}(s) ds d\theta \\ & + \int_{-\tau}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) W_{22} \dot{x}(s) ds d\theta, \end{aligned}$$

$$V_2(t) = \int_{t-\tau}^{t} \left[\int_{\theta}^{t} x^T(s)ds\right] R \left[\int_{\theta}^{t} x(s)ds\right] d\theta$$

$$+ \int_{0}^{\tau}\int_{t-\theta}^{t} (s - t + \theta)x^T(s)Rx(s)dsd\theta$$

and P is a positive definite matrix to be determined, and Q, R, W are positive semi-definite matrices to be determined. It follows from Leibniz-Newton formula that the following equation holds for any matrices $N_{ki,}$ $k = 1, \ldots, 5$, $i = 1, \ldots, r$:

$$2\sum_{i=1}^{r} \lambda(\xi(t))\Big[x^T(t)N_{1i} + \dot{x}^T(t)N_{2i} + x^T(t-\tau)N_{3i}$$

$$+ \int_{t-\tau}^{t} x^T(s)ds N_{4i} + w^T(t)N_{5i}\Big] \times \left[x(t) - \int_{t-\tau}^{t}\dot{x}(s)ds - x(t-\tau)\right] = 0. \quad (5)$$

It is easy to see from the definition of the nominal system (2) with $u(t) = 0$ and no uncertainties that the following equation also holds for any matrices T_i, $i = 1, \ldots, 5$:

$$2\left[x^T(t)T_1 + \dot{x}^T(t)T_2 + x^T(t-\tau)T_3 + \int_{t-\tau}^{t} x^T(s)dsT_4 + w^T(t)T_5\right]$$

$$\times \sum_{i=1}^{r}\lambda_i(\xi(t))\left[\dot{x}(t) - A_i x(t) - A_{di}x(t-\tau) - D_i\int_{t-\tau}^{t} x(s)ds - B_{1i}w(t)\right] = 0. \quad (6)$$

Note that

$$\frac{d}{dt}V_2(t) = 2\int_{t-\tau}^{t}(s - t + \tau)x^T(t)Rx(s)ds - \left(\int_{t-\tau}^{t} x^T(s)ds\right)R\left(\int_{t-\tau}^{t} x^T(s)ds\right)$$

$$+ \frac{1}{2}\tau^2 x^T(t)Rx(t) - \int_{t-\tau}^{t}(s - t + \tau)x^T(s)Rx(s)ds$$

$$\le \int_{t-\tau}^{t}(s - t + \tau)x^T(t)Rx(t)ds + \int_{t-\tau}^{t}(s - t + \tau)x^T(s)Rx(s)ds$$

$$- \left(\int_{t-\tau}^{t} x^T(s)ds\right)R\left(\int_{t-\tau}^{t} x^T(s)ds\right) + \frac{1}{2}\tau^2 x^T(t)Rx(t)$$

$$
\begin{aligned}
&-\int_{t-\tau}^{t}(s-t+\tau)x^T(s)Rx(s)ds \\
&\le \frac{1}{2}\tau^2 x^T(t)Rx(t)+\int_{t-\tau}^{t}(s-t+\tau)x^T(s)Rx(s)ds \\
&\quad -\left(\int_{t-\tau}^{t}x^T(s)ds\right)R\left(\int_{t-\tau}^{t}x(s)ds\right) \\
&\quad +\frac{1}{2}\tau^2 x^T(t)Rx(t)-\int_{t-\tau}^{t}(s-t+\tau)x^T(s)Rx(s)ds \\
&\le \tau^2 x^T(t)Rx(t)-\left(\int_{t-\tau}^{t}x^T(s)ds\right)R\left(\int_{t-\tau}^{t}x(s)ds\right).
\end{aligned}
$$

Thus, differentiation of $V(t)$ with respect to t along the solution of (2) and addition of the above zero quantities (5) and (6) give

$$
\begin{aligned}
&\frac{d}{dt}V(t)+\| z(t)\|^2-\gamma^2\| w(t)\|^2 \\
&\le 2x^T(t)P_{11}\dot{x}(t)+2\dot{x}^T(t)P_{12}\int_{t-\tau}^{t}x(s)ds+2x^T(t)P_{12}[x(t)-x(t-\tau)] \\
&\quad +2[x(t)-x(t-\tau)]^T P_{22}\int_{t-\tau}^{t}x(s)ds+x^T(t)Qx(t)-x^T(t-\tau)Qx(t-\tau) \\
&\quad +\int_{t-\tau}^{t}x^T(t)W_{11}x(t)ds-\int_{t-\tau}^{t}x^T(s)W_{11}x(s)ds \\
&\quad +2\int_{t-\tau}^{t}x^T(t)W_{12}\dot{x}(t)ds-2\int_{t-\tau}^{t}x^T(s)W_{12}\dot{x}(s)ds \\
&\quad +\int_{t-\tau}^{t}\dot{x}^T(t)W_{22}\dot{x}(t)ds-\int_{t-\tau}^{t}\dot{x}^T(s)W_{22}\dot{x}(s)ds \\
&\quad +\tau^2 x^T(t)Rx(t)-\left(\int_{t-\tau}^{t}x^T(s)ds\right)R\left(\int_{t-\tau}^{t}x(s)ds\right) \\
&\quad +2\sum_{i=1}^{r}\lambda_i(\xi(t))\Big[x^T(t)N_{1i}+\dot{x}^T(t)N_{2i}+x^T(t-\tau)N_{3i}
\end{aligned}
$$

$$+ \int_{t-\tau}^{t} x^T(s)ds N_{4i} + w^T(t)N_{5i}\Big]\Big[x(t) - \int_{t-\tau}^{t} \dot{x}(s)ds - x(t-\tau)\Big]$$

$$+ 2\Big[x^T(t)T_1 + \dot{x}^T(t)T_2 + x^T(t-\tau)T_3 + \int_{t-\tau}^{t} x^T(s)dsT_4 + w^T(t)T_5\Big]$$

$$\cdot \sum_{i=1}^{r} \lambda_i(\xi(t))\Big[\dot{x}(t) - Ax(t) - A_d x(t-\tau) - D_i \int_{t-\tau}^{t} x(s)ds - B_{1i}w(t)\Big]$$

$$+ \| z(t) \|^2 - \gamma^2 \| w(t) \|^2$$

$$= \frac{1}{\tau^2} \sum_{i=1}^{r} \lambda_i(\xi(t)) \int_{t-\tau}^{t} \int_{t-\tau}^{t} \zeta^T(t, s)\Phi_i \zeta(t, \theta) ds d\theta,$$

where $\zeta(t, s) = [x^T(t)\ \dot{x}^T(t)\ x^T(t-\tau)\ x^T(s)\ \dot{x}^T(s)\ w^T(t)]^T$ and Φ_i are defined in (4). If $\Phi_i < 0$, then we obtain

$$V(\infty) - V(0) + \| z(t) \|_2^2 - \gamma^2 \| w(t) \|_2^2 < 0$$

after integration from $t = 0$ to $t = \infty$, because $\lambda_i(\xi(t)) \geq 0$. Since $V(\infty) > 0$ and $V(0) = 0$, we have $\| z(t) \|_2^2 < \gamma^2 \| w(t) \|_2^2$.

For the stability, we let $w(t) = 0$ and $z(t) = 0$. It can be shown that if $\Phi_i < 0$ holds, then $\overline{\Phi}_i < 0$, where $\overline{\Phi}_i$ is a submatrix resulting from Φ, from which the sixth row and column are eliminated. In this case, with the similar argument above, we have $\dot{V}(t) < 0$. Thus, the system (2) is stable if the conditions (3) and (4) hold.

3.2. Uncertain systems

Next we consider the robust H_∞ disturbance attenuation of fuzzy time-delay system (2). Theorem 3.1 can be extended to a class of uncertain fuzzy time-delay systems. Applying Theorem 3.1 to (2) and making some mathematical manipulation with lemmas in Appendices of [13] give the following theorem. Complete proof is similar to that of Theorem 3.2 in [25], and is thus omitted.

Theorem 3.2. *Given a scalar* $\tau \geq 0$ *and a prescribed constant* $\gamma > 0$, *the uncertain fuzzy time-delay system* (2) *with* $u(t) = 0$ *achieves the robust* H_∞ *disturbance attenuation* γ, *if there exist common matrices* $P > 0$, $Q \geq 0$, $R \geq 0$, $W \geq 0$ *as in* (3), *some matrices* N_{ki}, $k = 1, \dots, 5$, $i = 1, \dots, r$, T_i, $i = 1, \dots, 5$ *and scalars* $\varepsilon_i > 0$, $i = 1, \dots, r$, *such that*

$$\Pi_i = \begin{bmatrix} \Pi_{11i} & \Phi_{12i} & \Pi_{13i} & \Pi_{14i} & -\tau N_{1i} & \Phi_{16i} & -T_1 H_i \\ \Phi_{12i}^T & \Phi_{22} & \Phi_{23i} & \Phi_{24i} & -\tau N_{2i} & \Phi_{26i} & -T_2 H_i \\ \Pi_{13i}^T & \Phi_{23i}^T & \Pi_{33i} & \Pi_{34i} & -\tau N_{3i} & \Phi_{36i} & -T_3 H_i \\ \Pi_{14i}^T & \Phi_{24i}^T & \Pi_{34i}^T & \Pi_{44i} & \Phi_{45i} & \Phi_{46i} & -\tau T_4 H_i \\ -\tau N_{1i}^T & -\tau N_{2i}^T & -\tau N_{3i}^T & \Phi_{45i}^T & -\tau W_{22} & -\tau N_{5i}^T & 0 \\ \Phi_{16i}^T & \Phi_{26i}^T & \Phi_{36i}^T & \Phi_{46i}^T & -\tau N_{5i} & \Phi_{66i} & -T_5 H_i \\ -H_i^T T_1^T & -H_i^T T_2^T & -H_i^T T_3^T & -\tau H_i^T T_4^T & 0 & -H_i^T T_5^T & -\varepsilon_i I \end{bmatrix}$$

$$< 0, \ \forall i = 1, \dots, r,$$

where Φ*'s are given in* (4) *and*

$$\Pi_{11i} = \Phi_{11i} + \varepsilon_i E_{1i}^T E_{1i},$$

$$\Pi_{13i} = \Phi_{13i} + \varepsilon_i E_{1i}^T E_{2i},$$

$$\Pi_{14i} = \Phi_{14i} + \varepsilon_i \tau E_{1i}^T E_{di},$$

$$\Pi_{33i} = \Phi_{33i} + \varepsilon_i E_{2i}^T E_{2i},$$

$$\Pi_{34i} = \Phi_{34i} + \varepsilon_i \tau E_{2i}^T E_{di},$$

$$\Pi_{44i} = \Phi_{44i} + \varepsilon_i \tau^2 E_{di}^T E_{di}.$$

4. State Feedback

Here we consider the design of state feedback controllers that achieve the robust H_∞ disturbance attenuation of fuzzy time-delay systems. We assume that the following rules are given

$$\text{IF } \xi_1(t) \text{ is } M_{1i} \text{ and } \cdots \text{ and } \xi_p(t) \text{ is } M_{pi},$$

$$\text{THEN } u(t) = K_i x(t) + K_{di} x(t - \tau), \quad i = 1, \dots, r,$$

where K_i and K_{di} are matrices to be determined. Then the natural choice of a state feedback controller is the following [15],

$$u(t) = \sum_{i=1}^{r} \lambda_i(\xi(t))\{K_i x(t) + K_{di}x(t-\tau)\}, \tag{7}$$

where the same weights $\lambda_i(\xi(t))$ as in (2) are used. The problem is to find state feedback gains K_i and K_{di} in the control law (7) that achieves the robust H_∞ disturbance attenuation of the system (2).

4.1. Nominal systems

We first consider the H_∞ disturbance attenuation problem for the nominal system (2) with no uncertainties. Applying Theorem 3.1 to the closed-loop system (2) and (7), we have the following result.

Theorem 4.1. *Given scalars* $\tau \geq 0, t_i, i = 1, \ldots, 4$ *and a prescribed constant* $\gamma > 0,$ *the* H_∞ *disturbance attenuation* γ *of the nominal fuzzy system* (2) *with no uncertainties is achievable by the control law* (7), *if there exist* $P > 0, Q \geq 0, R \geq 0, W \geq 0$ *as in* (3) *and some matrices* $N_{kij},$ $k = 1, \ldots, 5, i, j = 1, \ldots, r, S$ *and* $L_j, L_{dj}, j = 1, \ldots, r,$ *such that*

$$\Xi_{ij} = \begin{bmatrix} \Xi_{11ij} & \Xi_{12ij} & \Xi_{13ij} & \Xi_{14ij} & -\tau N_{1ij} & \Xi_{16ij} & \Xi_{17ij} \\ \Xi_{12ij}^T & \Xi_{22} & \Xi_{23ij} & \Xi_{24i} & -\tau N_{2ij} & -t_2 B_{1i} & 0 \\ \Xi_{13ij}^T & \Xi_{23ij}^T & \Xi_{33ij} & \Xi_{34ij} & -\tau N_{3ij} & \Xi_{36ij} & \Xi_{37ij} \\ \Xi_{14ij}^T & \Xi_{24i}^T & \Xi_{34ij}^T & \Xi_{44i} & \Xi_{45ij} & -\tau t_4 B_{1i} & 0 \\ -\tau N_{1ij}^T & -\tau N_{2ij}^T & -\tau N_{3ij}^T & \Xi_{45ij}^T & -\tau W_{22} & -\tau N_{5ij}^T & 0 \\ \Xi_{16ij}^T & -t_2 B_{1i}^T & \Xi_{36ij}^T & -\tau t_4 B_{1i}^T & -\tau N_{5ij} & -\gamma^2 I & D_{11i}^T \\ \Xi_{17ij}^T & 0 & \Xi_{37ij}^T & 0 & 0 & D_{11i} & -I \end{bmatrix}$$

$$< 0, \ \forall i, \ j = 1, \ldots, r, \tag{8}$$

where Φ*'s are given in* (4) *and*

$$\Xi_{11ij} = P_{12} + P_{12}^T + Q + \tau^2 R + \tau W_{11} + N_{1i} + N_{1i}^T - t_1(SA_i^T + L_j^T B_{2i}^T) - t_1(A_i S^T + B_{2i} L_j),$$

$$\Xi_{12ij} = P_{11} + \tau W_{12} + N_{2i}^T - t_2(A_i S^T + B_{2i}L_j) + t_1 S,$$

$$\Xi_{13ij} = -P_{12} - N_{1i} + N_{3i}^T - t_3(A_i S^T + B_{2i}L_j) - t_1(SA_{di}^T + L_{dj}^T B_{2i}^T),$$

$$\Xi_{14ij} = \tau P_{22} - \tau t_1 SD_i^T + \tau N_{4i}^T - \tau t_4(A_i S^T + B_{2i}L_j),$$

$$\Xi_{16ij} = N_{5i}^T - t_1 B_{1i},$$

$$\Xi_{17ij} = SC_i^T + L_j^T D_{12i}^T,$$

$$\Xi_{22} = \tau W_{22} + t_2(S + S^T),$$

$$\Xi_{23ij} = -N_{2i} + t_3 S^T - t_2(SA_{di}^T + L_{dj}^T B_{2i}^T),$$

$$\Xi_{24i} = \tau P_{12} - \tau t_2 SD_i^T + \tau t_4 S^T,$$

$$\Xi_{33ij} = -Q - N_{3i} - N_{3i}^T - t_3(SA_{di}^T + L_{dj}^T B_{2i}^T) - t_3(A_{di}S^T + B_{2i}L_{dj}),$$

$$\Xi_{34ij} = -\tau P_{22} - \tau t_3 SD_i^T - \tau N_{4i}^T - \tau t_4(A_{di}S^T + B_{2i}L_{dj}),$$

$$\Xi_{36ij} = -N_{5i}^T - t_3 B_{1i},$$

$$\Xi_{37ij} = SC_{di}^T + L_{dj}^T D_{12i}^T,$$

$$\Xi_{44i} = -\tau W_{11} - \tau^2 t_4(SD_i^T + D_i S^T) - \tau^2 R,$$

$$\Xi_{45ij} = -\tau W_{12} - \tau^2 N_{4ij}.$$

In this case, feedback gains in (7) *are given by*

$$K_i = L_i S^{-T}, \quad K_{di} = L_{di} S^{-T}, \quad i = 1, \dots, r. \tag{9}$$

Proof. For nominal case, the closed-loop system (2) with (7) is given by

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t)) \cdot \left\{(A_i + B_{2i}K_j)x(t) + (A_{di} + B_{2i}K_{dj})x(t-\tau) + D_i\int_{t-\tau}^{t} x(s)ds\right\},$$

$$z(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t))\{(C_i + D_{12i}K_j)x(t) + (C_{di} + D_{12i}K_{dj})x(t) + D_{11i}w(t)\}.$$

Then, we replace A_i, A_{di}, C_i and C_{di} in (4) with $A_i + B_{2i}K_j$, $A_{di} + B_{2i}K_{dj}$, $C_i + D_{12i}K_j$ and $C_{di} + D_{12i}K_{dj}$, respectively, and let $T_1 = t_1S, T_2 = t_2S$, $T_3 = t_3S, T_4 = t_4S$ and $T_5 = 0$. Furthermore, we make a congruent transformation with $\widetilde{S} = \text{diag}[S^{-1}\ S^{-1}\ S^{-1}\ S^{-1}\ S^{-1}I]$ to obtain $\Xi_{ij} = \widetilde{S}\Phi_i\widetilde{S}^T$. For simplicity of notation, we denote $S^{-1}P_{11}S^{-T}$, $S^{-1}P_{12}S^{-T}$, $S^{-1}P_{22}S^{-T}$, $S^{-1}QS^{-T}$, $S^{-1}RS^{-T}$, $S^{-1}W_{11}S^{-T}$, $S^{-1}W_{12}S^{-T}$, $S^{-1}W_{22}S^{-T}$, $S^{-1}N_{ki}S^{-T}$, $i = 1, \ldots, 5$, $S^{-1}N_{5j}$, $j = 1, \ldots, r$ and S^{-1} by P_{11}, P_{12}, P_{22}, Q, R, W_{11}, W_{12}, W_{22}, N_{kij}, $k = 1, \ldots, 5$ and S, respectively, and letting $L_j = K_jS^T$ and $L_{dj} = K_{dj}S^T$. Finally, by Schur complement, we obtain (8). If $\Xi_{ij} < 0$, then it follows that Ξ_{22} must be negative definite, which leads to S being nonsingular. Hence a stabilizing state feedback controller is given by (7) with feedback gains (9).

By using the techniques of relaxed conditions in [10], conditions in Theorem 4.1 can be relaxed as follows.

Theorem 4.2. *Given scalars* $\tau \geq 0$, t_i, $i = 1, \ldots, 4$ *and a prescribed constant* $\gamma > 0$, *the* H_∞ *disturbance attenuation* γ *of the nominal fuzzy system* (2) *with no uncertainties is achievable by the control law* (7), *if there exist* $P > 0, Q \geq 0, R \geq 0, W \geq 0$ *as in* (3) *and some matrices* N_{kij}, $k = 1, \ldots, 5$, $i, j = 1, \ldots, r$, S *and* L_j, L_{dj}, $j = 1, \ldots, r$, *such that*

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & \cdots & \Xi_{1r} \\ \Xi_{12}^T & \Xi_{22} & \cdots & \Xi_{2r} \\ \vdots & \vdots & \cdots & \vdots \\ \Xi_{1r}^T & \Xi_{2r}^T & \cdots & \Xi_{rr} \end{bmatrix} < 0, \tag{10}$$

where Ξ_{ij} *are given in* (8). *In this case, feedback gains in* (7) *are given by* (9).

4.2. Uncertain systems

Next, we consider the design of a robust H_∞ disturbance attenuation controller (7) for the system (2). Similar techniques used to obtain

Theorem 3.2 lead to Theorem 4.3. Complete proof follows similar lines of Theorem 4.3 in [25] and is thus omitted.

Theorem 4.3. *Given scalars* $\tau \geq 0$, t_i, $i = 1, \ldots, 4$ *and a prescribed constant* $\gamma > 0$, *the robust* H_∞ *disturbance attenuation* γ *of the uncertain fuzzy system* (2) *is achievable by the control law* (7), *if there exist* $P > 0$, $Q \geq 0$, $R \geq 0$, $W \geq 0$ *as in* (3) *and some matrices* N_{kij}, $k = 1, \ldots, 5$, $i, j = 1, \ldots, r$, S *and* L_j, L_{dj}, $j = 1, \ldots, r$ *and scalars* $\varepsilon_{1ij} > 0$, $\varepsilon_{2ij} > 0$, $\varepsilon_{3i} > 0$, $i, j = 1, \ldots, r$, *such that*

$$\Lambda_{ij} =$$

$$\begin{bmatrix}
\Lambda_{11ij} & \Xi_{12ij} & \Xi_{13ij} & \Xi_{14ij} & -\tau N_{1ij} & \Xi_{16ij} & \Xi_{17ij} & \Lambda_{18ij} & \Lambda_{19ij} & \tau t_1 S E_{di}^T \\
\Xi_{12ij}^T & \Xi_{22} & \Xi_{23ij} & \Xi_{24i} & -\tau N_{2ij} & -t_2 B_{1i} & 0 & \Lambda_{28ij} & \Lambda_{29ij} & \tau t_2 S E_{di}^T \\
\Xi_{13ij}^T & \Xi_{23ij}^T & \Lambda_{33ij} & \Xi_{34ij} & -\tau N_{3ij} & \Xi_{36ij} & \Xi_{37ij} & \Lambda_{38ij} & \Lambda_{39ij} & \tau t_3 S E_{di}^T \\
\Xi_{14ij}^T & \Xi_{24i}^T & \Xi_{34ij}^T & \Lambda_{44i} & \Xi_{45ij} & -\tau t_4 B_{1i} & 0 & \Lambda_{48ij} & \Lambda_{49ij} & \tau^2 t_4 S E_{di}^T \\
-\tau N_{1ij}^T & -\tau N_{2ij}^T & -\tau N_{3ij}^T & \Xi_{45ij}^T & -\tau W_{22} & -\tau N_{5ij}^T & 0 & 0 & 0 & 0 \\
\Xi_{16ij}^T & -t_2 B_{1i}^T & \Xi_{36ij}^T & -\tau t_4 B_{1i}^T & -\tau N_{5ij} & -\gamma^2 I & D_{11i}^T & 0 & 0 & 0 \\
\Xi_{17ij}^T & 0 & \Xi_{37ij}^T & 0 & 0 & D_{11i} & -I & 0 & 0 & 0 \\
\Lambda_{18ij}^T & \Lambda_{28ij}^T & \Lambda_{38ij}^T & \Lambda_{48ij}^T & 0 & 0 & 0 & -\varepsilon_{1ij} I & 0 & 0 \\
\Lambda_{19ij}^T & \Lambda_{29ij}^T & \Lambda_{39ij}^T & \Lambda_{49ij}^T & 0 & 0 & 0 & 0 & -\varepsilon_{2ij} I & 0 \\
\tau t_1 E_{di} S^T & -\tau t_2 E_{di} S^T & \tau t_3 E_{di} S^T & \tau^2 t_4 E_{di} S^T & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{3i} I
\end{bmatrix}$$

$$< 0, \tag{11}$$

where Ξ*'s are defined in* (8) *and*

$$\Lambda_{11ij} = \Xi_{11ij} + \varepsilon_{1ij} H_i^T H_i,$$

$$\Lambda_{18ij} = t_1 (S E_{1i}^T + L_j^T E_{bi}^T),$$

$$\Lambda_{19ij} = t_1 (S E_{2i}^T + L_{dj}^T E_{bi}^T),$$

$$\Lambda_{28ij} = t_2 (S E_{1i}^T + L_j^T E_{bi}^T),$$

$$\Lambda_{29ij} = t_2 (S E_{2i}^T + L_{dj}^T E_{bi}^T),$$

$$\Lambda_{33ij} = \Xi_{33ij} + \varepsilon_{2ij} H_i^T H_i,$$

$$\Lambda_{38ij} = t_3 (S E_{1i}^T + L_j^T E_{bi}^T),$$

$$\Lambda_{39ij} = t_3 (S E_{2i}^T + L_{dj}^T E_{bi}^T),$$

$$\Lambda_{44i} = \Xi_{44i} + \varepsilon_{3i} H_i^T H_i,$$

$$\Lambda_{48ij} = \tau t_4 (S E_{1i}^T + L_j^T E_{bi}^T),$$

$$\Lambda_{49ij} = \tau t_4 (S E_{2i}^T + L_{dj}^T E_{bi}^T).$$

In this case, feedback gains in (7) *are given by* (9).

Similar to Theorem 4.2, conditions in Theorem 4.3 can also be relaxed as follows.

Theorem 4.4. *Given scalars* $\tau \geq 0$, t_i, $i = 1, \ldots, 4$ *and a prescribed constant* $\gamma > 0$, *the robust* H_∞ *disturbance attenuation* γ *of the uncertain fuzzy system* (2) *is achievable by the control law* (7), *if there exist* $P > 0$, $Q \geq 0$, $R \geq 0$, $W \geq 0$ *as in* (3) *and some matrices* N_{kij}, $k = 1, \ldots, 5$, $i, j = 1, \ldots, r$, S *and* L_j, L_{dj}, $j = 1, \ldots, r$ *and scalars* $\varepsilon_{1ij} > 0$, $\varepsilon_{2ij} > 0$, $\varepsilon_{3i} > 0$, $i, j = 1, \ldots, r$, *such that*

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \cdots & \Lambda_{1r} \\ \Lambda_{12}^T & \Lambda_{22} & \cdots & \Lambda_{2r} \\ \vdots & \vdots & \cdots & \vdots \\ \Lambda_{1r}^T & \Lambda_{2r}^T & \cdots & \Lambda_{rr} \end{bmatrix} < 0, \tag{12}$$

where Λ_{ij} *are defined in* (11). *In this case, feedback gains in* (7) *are given by* (9).

Remark 4.1. Conditions (10) in Theorem 4.2 and conditions (12) in Theorem 4.4 are more relaxed ones than conditions (8) in Theorem 4.1 and (11) in Theorem 4.3, respectively. Teixiera et al. [16, 18] have considered relaxed stability conditions for fuzzy systems. The techniques in [16, 18] can be applied to the stability analysis of fuzzy time-delay systems, and conditions (10) and (12) can be further relaxed.

Remark 4.2. If a memoryless control law is preferred, then we let $L_{di} = 0$, $i = 1, \ldots, r$ in Theorems 4.1 to 4.4. Then, we obtain the control law (7) with $K_{di} = 0$, $i = 1, \ldots, r$.

5. Examples

The following two examples illustrate our results. The first one shows the robust H_∞ disturbance attenuation of fuzzy time-delay systems. The second one gives the design method of a state feedback controller for uncertain fuzzy time-delay systems.

Example 5.1. Consider the robust H_∞ disturbance attenuation of the following uncertain fuzzy time-delay systems

$$\dot{x}(t) = \sum_{i=2}^{r} \lambda_i(x_1(t)) \Big\{ (A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di})x(t-\tau)$$

$$+ \begin{bmatrix} \delta & 0 \\ 0 & \delta \end{bmatrix} \int_{t-\tau}^{t} x(s)ds + B_{1i}w(t) \Big\},$$

$$z(t) = \sum_{i=2}^{r} \lambda_i(x_1(t)) C_{1i} x(t),$$

where $\lambda(x_1(t)) = 1/(1 + \exp(-x_1(t)))$, $\lambda_2(x_1(t)) = 1 - \lambda_1(x_1(t))$ and

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix},\ A_2 = \begin{bmatrix} -2 & 0 \\ 0 & -1.4 \end{bmatrix},\ A_{d1} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} -1 & 0 \\ -1 & -1.5 \end{bmatrix},\ B_{11} = B_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ C_{11} = C_{12} = [1 \quad 0],$$

and ΔA_i and ΔA_{di} satisfy $\| \Delta A_i \| \le \beta$ and $\| \Delta A_{di} \| \le \beta$. The uncertain matrices ΔA_i and ΔA_{di} are described by (1) with $H_i = I$, $E_{1i} = E_{2i} = \beta I$.

First, we compare our results with other results on stability and robust stability when $B_{1i} = 0$, $C_{1i} = 0$. Since there is no result on fuzzy systems with distributed delays except for Yoneyama [24], we let $\delta = 0$ for comparison.

Table I. Maximum upper bound τ for $\delta = 0$

β	0	0.3
Cao & Frank [1]	-	N/A
Chen & Liu [4]	0.6666	0.4785
Guan & Chen [7]	1.1228	N/A
Li et al. [12]	2.2615	0.8168
Yoneyama [23]	2.3421	N/A
Tian & Peng [17]	2.3675	1.0658
Theorem 3.1/Theorem 3.2	2.3675	1.2636

Table I compares our results with the results in the literature. For the nominal fuzzy system when $\beta = 0$, Cao and Frank [1] do not guarantee the stability, whereas the others do guarantee the stability. Among them, Tian and Peng [17] and Theorem 3.1 give the maximum upper bound τ. For the robust stability of the uncertain fuzzy system when $\beta = 0.3$, Theorem 3.2 gives the maximum upper bound τ among Li et al. [12], Yoneyama [23] and Theorem 3.2, where the robust stability is guaranteed.

Table II. Maximum upper bound τ for $\beta = 0.1$

δ	0.1	0.3	0.5	0.7
Yoneyama [24]	1.4641	1.3525	1.2390	1.1459
Theorem 3.2	1.4822	1.4882	1.3904	1.2811

Table II lists the maximum upper bound τ for $\beta = 0.1$ and different δ's. Table II shows that Theorems 3.1 and 3.2 guarantee the higher maximum upper bound τ than that of Yoneyama [24].

Next, we consider the H_∞ disturbance attenuation for time-delay $\tau = 0.6$.

Table III. Minimum lower bound γ for $\delta = 0$

β	0	0.05	0.1	0.45
Jiang & Han [9]	0.0404	-	-	-
Chen & Liu [4]	0.0004	0.1561	-	-
Theorem 3.1/Theorem 3.2	0.0002	0.0431	0.0900	2.0269

Table III shows the minimum lower bound γ for $\delta = 0$. Jiang and Han [9] do not achieve the robust H_∞ disturbance attenuation for any $\beta \neq 0$. When $\beta \geq 0.1$, Chen and Liu [4] do not guarantee the robust H_∞ disturbance attenuation for any large γ. Obviously, our theorems give a better result than recent papers.

Example 5.2. Consider the state feedback controller design for following uncertain fuzzy time-delay systems

$$\dot{x}(t) = \sum_{i=2}^{r} \lambda_i(x_1(t)) \Big\{ (A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di})x(t-\tau)$$

$$+ (D_i + \Delta D_i) \int_{t-\tau}^{t} x(s)ds + B_{1i}w(t) + B_{2i}u(t) \Big\},$$

$$z(t) = \sum_{i=2}^{r} \lambda_i(x_1(t)) \{ C_{1i}x(t) + D_{11i}w(t) \},$$

where $\tau = 0.5$, $\lambda(x_1(t)) = 1 - \sin^2 x_1(t)$, $\lambda_2(x_1(t)) = 1 - \lambda_1(x_1(t))$ and

$$A_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1.5 \end{bmatrix}, A_{d1} = \begin{bmatrix} -2 & -0.5 \\ 0 & -1 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} -2 & -0.5 \\ 0 & -1.5 \end{bmatrix}, B_{11} = B_{12} = B_{21} = B_{22} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$D_1 = D_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, C_1 = C_2 = [1 \quad 0], D_{111} = D_{112} = 0.1,$$

and $\Delta A_i, \Delta A_{di}$ and ΔD_i are described by (1) with $H_i = I, E_{1i} = E_{2i} = E_{di} = 0.2I$. Theorem 4.3 gives the minimum lower bound $\gamma = 0.3040$

when $t_1 = 1$, $t_2 = 0.32$, $t_3 = -0.01$, $t_4 = 0.01$. In this case, state feedback gains in (7) are given by $K_1 = [-9.2161\ -13.2931]$, $K_2 = [-8.8235\ -13.7064]$ $K_{d1} = [-1.4075\ \ 0.6245]$ and $K_{d2} = [-1.4559\ 1.1051]$ If a memoryless feedback with $K_{di} = 0$, $i = 1, 2$ is employed, then we can find the minimum lower bound $\gamma = 0.3365$ by Theorem 4.3 when $t_1 = 1$, $t_2 = 0.33$, $t_3 = 0.01$, $t_4 = -0.01$. In this case, we have state feedback gains in (7) given by $K_1 = [-15.4422\ -21.5859]$ and $K_2 = [-14.8204\ -22.0289]$.

6. Conclusion

We have obtained delay-dependent conditions that guarantee the robust H_∞ disturbance attenuation for Takagi-Sugeno fuzzy systems with discrete and distributed delays. Based on such conditions, we have proposed design methods of H_∞ disturbance attenuation controllers for fuzzy time-delay systems as well as robust H_∞ controllers for uncertain fuzzy time-delay systems. Finally, we have shown examples to illustrate the effectiveness of our results.

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