Far East J. Math. Sci. (FJMS) **27**(1) (2007), 205-216
This paper is available online at http://www.pphmj.com

$C_p(2)$ CAN BE CHARACTERIZED BY ITS ORDER COMPONENTS

HUAGUO SHI[1,2] **and GUIYUN CHEN**[1]

[1]School of Mathematics and Statistics
Southwest China Normal University
Chongqing, 400715, P. R. China

[2]Sichuan Vocational and Technical College
Sichuan, 629000, P. R. China
e-mail: shihuaguo@126.com

Abstract

It is proved that if $M = C_p(2)$, p is an odd prime, and G is a finite group with the same order components of M, then $G \cong M$.

1. Introduction

If G is a finite group, then we define the prime graph $\Gamma(G)$ as follows: its vertices are the primes dividing the order of G, and two vertices p and q are joined by an edge if and only if there is an element in G of order pq. We denote the set of all connected components of graph $\Gamma(G)$ by $T(G) = \{\pi_i(G)$, for $i = 1, 2, \ldots, t(G)\}$, where $t(G)$ is the number of connected components of $\Gamma(G)$, and if G is of even order we always assume 2 in $\pi_1(G)$. We also denote the set of all primes dividing n by $\pi(n)$, where n is

2000 Mathematics Subject Classification: 20D05, 20D60.

Keywords and phrases: finite groups, prime graphs, order components.

Submitted by Pu Zhang

Received June 22, 2007

a natural number. Obviously $|G|$ can be expressed as a product of $m_1, m_2, \ldots, m_{t(G)}$, where m_i is a positive integer with $\pi(m_i) = \pi_i(G)$. All m_i are called the *order components* of G. Let $OC(G) = \{m_1, m_2, \ldots, m_{t(G)}\}$ be the set of order components of G. The order components of non-abelian simple groups having at least two prime graph components have been attained in [4].

Some simple groups can be characterized by their order components, such as a finite simple group with at least three prime graph components [3], sporadic simple groups [4], Suzuki-Ree groups [6], $G_2(q)$, where $q \equiv 0 \pmod 3$ [5], $E_8(q)$ [1], $PSL_2(q)$ [7], ${}^3D_4(q)$ [8], ${}^2D_n(3)$, $9 \le n = 2^m + 1 \ne p$ [9], ${}^2D_{p+1}(2)$, $5 \le p \ne 2^m - 1$ [24], A_p, where p and $p-2$ are primes [12], $PSL(5, q)$ [13], $PSL(3, q)$, where q is an odd prime power [14], $PSL(3, q)$ for $q = 2^n$ [15], $F_4(q)$, where q is even [16], $C_2(q)$, where $q > 5$ [17], $PSU_5(q)$ [18], $PSU(3, q)$ for $q > 5$ [19], ${}^2D_4(q)$ [20], ${}^2E_6(q)$ [22], $E_6(q)$ [21]. In this paper we continue this work and shall prove the following theorem:

Theorem. *Let $M = C_p(2)$, p be an odd prime. If a finite group G has the same order components of M, then $G \cong M$.*

2. Preliminary Results

Lemma 1 [4, Lemma 6]**.** *If $t(G) \ge 2$, H is a π_i subgroup of G, and $H \lhd G$, then $\prod_{j=1, j \ne i}^{t(G)} m_i \mid (|H| - 1)$.*

Lemma 2 [2, Theorem 2]**.** *Let G be a 2-Frobenius group of even order. Then $t(G) = 2$, G has a normal series $1 \unlhd H \unlhd K \unlhd G$ such that $|K/H| = m_2$, $|H| \cdot |G/K| = m_1$, $|G/K| \mid (|K/H| - 1)$, $|G/K| \mid \varphi(|K/H|)$, and H is nilpotent.*

Lemma 3 [25, Lemma 3]**.** *If M is a simple group with $t(M) = 2$, G is a finite group and $OC(G) = OC(M)$, then one of the following holds*:

(1) *G is a Frobenius group or 2-Frobenius group.*

(2) *G has a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that H is a nilpotent π_1-group, K/H is a non-abelian simple group, the odd order component of M is equal to one of those of K/H, G/K is a cyclic π_1-group, and $|G/K| \mid |Out(K/H)|$.*

Lemma 4 [11, Remark]**.** *The only solution of the equation $p^m - q^n = 1$, where p, q are primes and $m, n > 1$, is $3^2 - 2^3 = 1$.*

Lemma 5 [26]**.** *Let p be a prime and n be a natural number, $n \geq 2$. Then there exists a prime divisor r of $p^n - 1$ which does not divide $p^m - 1$ for any natural number $m \leq n$, except $n = 6$, $p = 2$ or $n = 2$, $p + 1$ is a power of 2. Such r is called a primitive prime divisor of $p^n - 1$.*

Of course a primitive prime divisor of $p^n - 1$ cannot divide $p^n + 1$ or $p^m - 1$ for $n \nmid m$.

Lemma 6 [23, Lemma 1]**.** *If $n \geq 6$ is a natural number, then there exist at least $s(n)$ primes p_i such that $\frac{n+1}{2} < p_i < n$:*

$s(n) = 6$ *for* $n \geq 49$;

$s(n) = 5$ *for* $42 \leq n \leq 47$;

$s(n) = 4$ *for* $38 \leq n \leq 41$;

$s(n) = 3$ *for* $18 \leq n \leq 37$;

$s(n) = 2$ *for* $14 \leq n \leq 17$;

$s(n) = 1$ *for* $6 \leq n \leq 13$.

Lemma 7. *Let p be a prime, $q > 1$ be a natural number $e = \min\{d : p \mid (q^d - 1)\}$, $q^e = 1 + p^r k$, $p \nmid k$, t be a natural number satisfying $t = p^s u$ and $p \nmid u$. If $p > 2$ or $r > 2$, then $p^{r+s} \parallel (q^{et} - 1)$.*

Proof.

$$q^{et} - 1 = (1 + p^r k)^t - 1 = tp^r k + \sum_{i=2}^{t} \binom{t}{i} (p^r k)^i.$$

If $s = 0$, then $p \nmid t$, $p^r \parallel q^{et} - 1$.

If $s > 0$, by calculation we can prove that $p^{r+s+1} \mid \binom{t}{i} (p^r k)^i$ for $2 \le i \le t$, hence $p^{r+s} \parallel (q^{et} - 1)$.

So we have that $p^{r+s} \parallel (q^{et} - 1)$. □

Lemma 8. *Set $q > 1$ is a natural number, $s = \prod_{i=1}^{n} (q^i - 1)$, p is a prime, $p \mid s$. We denote the power of p in the standard factorization of s by s_p. $e = \min\{d : p \mid q^d - 1\}$, $q^e = 1 + p^r k$, $p \nmid k$. If $p > 2$ or $r > 2$, then $s_p < q^{\frac{np}{p-1}}$.*

Proof. Set $a = \left[\frac{n}{e}\right]$, $w = \prod_{i=1}^{a} (q^{ei} - 1)$, hence, $s_p = w_p = p^{ra + \sum_{j=1}^{\infty} \left[\frac{a}{p^j}\right]}$ $\le p^{ra + \frac{a}{p-1}} < q^{\frac{np}{p-1}}$ by Lemma 7 and $p > 2$ or $r > 2$. □

Lemma 9. *Let q be an odd natural number, $s = \prod_{i=1}^{n} (q^i - 1)$. Then $s_2 < q^{1.5n}$.*

Proof. Set $2^r \parallel q - 1$. We divide the proof into two cases based on r is 1 or not.

Case 1. When $r = 1$,

$$s = \prod_{i=1}^{n} (q^i - 1) = \prod_{i=1,\, 2 \mid i}^{n} (q^i - 1) \cdot \prod_{j=1,\, 2 \nmid j}^{n} (q^j - 1).$$

For $2 \mid i$, set $v = q^2$, $v = 1 + 2^r k$, $2 \nmid k$, clearly $r \geq 2$, hence, the power of 2 in the standard factorization of $\prod_{i=1,\,2\mid i}^{n} (q^i - 1) = \prod_{i=1}^{\left[\frac{n}{2}\right]} (v^i - 1)$ is less than $q^{\left[\frac{n}{2}\right]\cdot 2}$ by Lemma 8. For $2 \nmid j$, we have that $2 \,\|\, q^j - 1$, so $2^{\left[\frac{n+1}{2}\right]} \,\|\, \prod_{j=1,\,2\nmid j}^{n} (q^j - 1)$.

Hence $s_2 < 2^{\left[\frac{n+1}{2}\right]} q^{\left[\frac{n}{2}\right]\cdot 2} < q^{1.5n}$.

Case 2. When $r \neq 1$, by Lemma 7, we have that $s_2 = 2^{rn + \sum_{j=1}^{\infty}\left[\frac{n}{2^j}\right]}$ $< 2^{rn} \cdot 2^n < q^n \cdot 2^n < q^{1.5n}$ since $r \neq 1$. □

Definition 1. Let a and f be expressions of integers with integral coefficients. If $f \mid a$ and $(f, a/f) = 1$, then we say that f is a *Hall factor* of a.

Lemma 10 [10, Theorem 1]**.** *If q is a power of a prime number, $c = \prod_{i=1}^{n} (q^{2i} - 1)$ or $(q^n \pm 1) \cdot \prod_{i=1}^{n-1} (q^{2i} - 1)$, then there exists a Hall factor f of c satisfying*:

(1) *If* $n \geq 23$, *then* $f > q^{8n}$;

(2) *If* $n = 22$, *then* $f > q^{7n}$;

(3) *If* $18 \leq n \leq 21$, *then* $f > q^{6n}$;

(4) *If* $16 \leq n \leq 17$, *then* $f > q^{5n}$;

(5) *If* $14 \leq n \leq 15$, *then* $f > q^{4n}$.

And if the standard factorization of $f = \prod_{k=1}^{t} r_k^{\delta_k}$, *then* $r_k^{\delta_k} \leq \dfrac{q^{n-1} - 1}{q - 1}$.

3. Proof of the Theorem

Proof. Because $M = C_p(2)$, and G has the same order components with M, so the even order component of G is

$$m_1 = 2^{p^2}(2^p + 1)\prod_{i=1}^{p-1}(2^{2i} - 1),$$

the odd order component of G is $m_2 = 2^p - 1$.

We divide the proof into 11 cases based on Lemma 3 and Tables 1-4 in [4].

Case 1. G cannot be a Frobenius group or a 2-Frobenius group.

Subcase 1.1. If G is a Frobenius group with Frobenius kernel H and complement K, then $|H| = m_1$ and $|K| = m_2$ since $|k| < |h|$. There exists a primitive prime divisor r of $2^{2p} - 1$ since p is an odd prime. Let $S_r \in Syl_r(H)$, obviously $|S_r| \mid (2^p + 1)$ and $S_r \trianglelefteq G$. Furthermore, $|S_r| \equiv 1 \pmod{m_2}$ by Lemma 1, which is impossible.

Subcase 1.2. If G is a 2-Frobenius group, then there is a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that H is a nilpotent π_1 group, $|K/H| = m_2$, $|G/K| \mid (|K/H| - 1) = 2^p - 2$, it follows that $(2^p + 1) \mid |H|$. Similarly to Subcase 1.1, we can show that it is impossible.

From Subcase 1.1, Subcase 1.2 and Lemma 3, we have the following properties:

(1) There is a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that K/H is a simple group, H and G/K are π_1 group and H is nilpotent.

(2) The odd order component of G is one of those of K/H, consequently $t(K/H) \geq 2$. Hence K/H may be one of the simple groups listed in Tables 1-4 in [4].

Case 2. $K/H \not\cong E_7(2)$, $E_7(3)$, $A_2(2)$, $A_2(4)$, ${}^2A_5(2)$, ${}^2E_6(2)$, ${}^2F_4(2)'$ or one of the sporadic simple groups.

If $p = 3$, only $A_2(2)$, $A_2(4)$, ${}^2A_5(2)$, M_{22}, J_1, HS or J_2 in the above-mentioned groups has an odd order component 7. If $K/H \cong A_2(2)$, $|G/K| \mid |Out(K/H)| = 2$, furthermore there exists a Sylow-5 subgroup of H denoted by S_5, and $|S_5| = 5$, $S_5 \lhd G$, hence $5 \equiv 1 \pmod 7$ by Lemma 1, which is a contradiction. Similarly we can prove that $K/H \not\cong A_2(4)$. And the order of ${}^2A_5(2)$, M_{22}, J_1, J_2 or HS cannot divides the order of $C_3(2)$.

Similarly we can prove that p cannot be 5 or 7.

If $p \geq 11$, then the odd order component of G is greater than any odd order components of any one of above-mentioned groups.

Case 3. $K/H \not\cong A_n$.

If $K/H \cong A_n$, then $2^p - 1 = n$, $n - 1$ or $n - 2$. Thus $|A_{2^p - 1}| \mid |A_n| \mid |C_p(2)|$. When $p \geq 5$, there exist at least three primes p_i satisfying $2^{p-1} < p_i < 2^p - 1$ by Lemma 6. But there exist at most two prime divisors of $|C_p(2)| = 2^{p^2} \prod_{i=1}^{p} (2^{2i} - 1)$ between 2^{p-1} and $2^p - 1$ a contradiction.

When $p = 3$, only A_7, A_8 or A_9 has an odd order component 7 equal to the odd order component of $C_3(2)$. If $K/H \cong A_7$, then $|G/K| \mid |Out(K/H)| = 2$, so there exists a Sylow-3 subgroup S_3 of H, and $|S_3| = 9$, $S_3 \lhd G$. Furthermore $9 \equiv 1 \pmod 7$ by Lemma 1, which is a contradiction. $|A_8|$ and $|A_9|$ cannot divides the order of $C_3(2)$.

Case 4. $K/H \not\cong A_n(q)$ and $K/H \not\cong {}^2A_n(q)$.

Subcase 4.1. If $K/H \cong A_1(q)$, then $2^p - 1 = q$, $q \pm 1$ or $(q \pm 1)/2$. Whenever in any case we have that $q \leq 2^{p+1}$, hence $|K/H| < 2^{3(p+1)}$. Assume $q = r^f$ we have that $|G/K| < 2p + 2$ since $|G/K| \mid |Out(K/H)| = 2f$. If $p + 1 \geq 14$, there exists a Hall factor g of $|G| =$

$2^{p^2}\prod_{i=1}^{p}(2^{2i}-1)$ such that $g > 2^{4p}$ and $g_{r'} \le 2^p - 1$ for any prime $r' \mid g$ by Lemma 10. Clearly $(g, |H|) \neq 1$. Let prime p' be satisfy $p' \mid (g, |H|)$ and $S_{p'} \in Syl_{p'}(G)$. $S_{p'}$ is a normal π_1-subgroup of G and $|S_{p'}| < 2^p - 1$, which contradicts Lemma 1.

By trivial calculation, we can show that p cannot be 3, 5, 7 or 11.

Subcase 4.2. If $K/H \cong A_{p'}(q)$, $q-1 \mid p'-1$, then $2^p - 1 = (q^{p'}-1)/(q-1)$, $q^{p'} \ge 2^p$.

If $p' \ge 7$, then $q^{p'(p'+1)/2} > 2^{3p}$, which implies q is a power of 2 by Lemmas 8 and 9. Suppose $q = 2^r$, hence $(2^{rp'}-1)/(2^r-1) = 2^p - 1$, $2^{rp'} = 2^{r+p} - 2^r - 2^p + 2$, $q = 2$, $p = p'$, so $|G/K| \cdot |H| = |G|/|K/H|$ $= \dfrac{2^{p(p-1)/2}\prod_{i=1}^{p}(2^i+1)}{2^{p+1}-1}$, which is impossible by Lemma 5 since $p' \ge 7$.

By calculation we can prove that p' cannot be 3 or 5.

Subcase 4.3. Similarly to Subcase 4.2 we can show that $K/H \not\cong A_{p'-1}(q)$ and $K/H \not\cong {}^2A_n(q)$.

Case 5. $K/H \not\cong D_n(q)$.

If $K/H \cong D_{p'}(5)$, $p' \ge 5$, then $(5^{p'}-1)/4 = 2^p - 1$, $5^{p'} = 2^{p+2} - 3 > 2^{p+1}$. Hence $5^{p'(p'-1)} > 2^{4p}$, which contradicts Lemma 8.

If $K/H \cong D_{p'}(3)$, $p' \ge 5$, then $2^p - 1 = (3^{p'}-1)/2$, which contradicts Lemma 4.

Similarly we can prove that $K/H \not\cong D_{p'+1}(3)$, $p' \ge 3$.

Case 6. $K/H \not\cong E_8(q)$, $E_6(q)$, $F_4(q)$, ${}^2E_6(q)$ or ${}^2F_4(q)$.

If $K/H \cong E_6(q)$, then $(q^6 + q^3 + 1)/(3, q-1) = 2^p - 1$, $q^9 > 2^p$, $q^{36} > 2^{4p}$, hence q is a power of 2 by Lemma 8. Let $q = 2^r$, we have that $2^{6r} + 2^{3r} = 3 \cdot 2^p - 4$ or $2^p - 2$, which is impossible.

Similarly we can prove that $K/H \not\cong E_8(q)$, $F_4(q)$, ${}^2E_6(q)$, ${}^2F_4(q)$ or ${}^2G_2(q)$.

Case 7. $K/H \not\cong {}^2B_2(q)$, $q = 2^{2k+1}$.

If $K/H \cong {}^2B_2(q)$, $q = 2^{2k+1}$, then $2^p - 1 = q \pm \sqrt{2q} + 1$ or $q - 1$.

If $2^p - 1 = q \pm \sqrt{2q} + 1$, then $0 \equiv q \pm \sqrt{2q} = 2^p - 2 \equiv 2 \pmod 4$, a contradiction.

If $2^p - 1 = q - 1$, then $q = 2^p$,

$$|G/K| \cdot |H| = \frac{2^{p^2 - 2p}(2^p + 1)\prod_{i=1}^{p-1}(2^{2i} - 1)}{2^{2p} + 1}.$$

Similarly to Subcase 4.2 we can get a contradiction.

Case 8. $K/H \not\cong {}^2G_2(q)$, $q = 3^{2k+1}$; $K/H \not\cong G_2(q)$, $K/H \not\cong {}^3D_4(q)$.

Subcase 8.1. If $K/H \cong G_2(q)$, $3 | q$, then $2^p - 1 = q^2 \pm q + 1$, $q(q \pm 1) = 2(2^{p-1} - 1)$. Let $q = 3^r$, from Lemma 7, we have that $2 \cdot 3^{r-1} | p - 1$, $2(2^{p-1} - 1) > 2^{p-1} > 2^{2.3^{r-1}} > 3^{3^{r-1}}$. If $r \geq 3$, then $2(2^{p-1} - 1) \geq 3^{3r} > q^3 > q^2 \pm q$, a contradiction. By calculation we have that r cannot be 1 or 2. Similarly we can prove $K/H \not\cong {}^2G_2(q)$, $q = 3^{2k+1}$.

Subcase 8.2. Similarly to Subcase 4.1, we can show that $K/H \not\cong G_2(q)$, $3 | q + 1$.

Subcase 8.3. If $K/H \cong G_2(q)$, $3 | q - 1$, then $2^p - 1 = q^2 - q + 1$, $q^2 > 2^p$. It follows that $q^6 > 2^{3p}$, hence q is a power of 2 by Lemma 8, which is impossible. Similarly we can prove that $K/H \not\cong {}^3D_4(q)$.

Case 9. $K/H \not\cong {}^2D_n(q)$.

Subcase 9.1. If $K/H \cong {}^2D_{p'+1}(2)$, $p' \neq 2^m - 1$, then $2^{p'} - 1 = 2^p - 1$, $p = p'$, $|{}^2D_{p+1}(2)| \mid |C_p(2)|$, which is impossible.

Subcase 9.2. If $K/H \cong {}^2D_{p'}(3)$, $5 \le p' = 2^m + 1$, then $2^p - 1 = (3^{p'-1} + 1)/2$ or $(3^{p'} + 1)/4$.

If $2^p - 1 = (3^{p'} + 1)/4$, then $3^{p'} > 2^p$, $3^{p'(p'-1)} > 2^{4p}$, which contradicts Lemma 8.

If $(3^{n-1} + 1)/2 = 2^p - 1$, then $3^{n-1} + 3 = 2^{p+1}$, which is impossible.

Similarly we can prove that $K/H \ncong {}^2D_{p'}(3)$, $5 \le p' \ne 2^n + 1$; $K/H \ncong {}^2D_n(3)$, $n = 2^m + 1$ is not a prime.

Subcase 9.3. If $K/H \cong {}^2D_{p'+1}(2)$, $3 \le p' = 2^n + 1$, then $2^p - 1 = 2^{p'} + 1$ or $2^{p'+1} + 1$, which is impossible.

Subcase 9.4. If $K/H \cong {}^2D_n(q)$, $4 \le n = 2^m$, then $(q^n + 1)/(2, q - 1) = 2^p - 1$. Clearly q cannot be a power of 2. Furthermore $q^n > 2^p$, $q^{n(n-1)} > 2^{3p}$, which contradicts Lemma 8.

Case 10. $K/H \ncong B_n(q)$.

Subcase 10.1. If $K/H \cong B_{p'}(3)$, then $(3^{p'} - 1)/2 = 2^p - 1$, $3^{p'} = 2^{p+1} - 1$, which contradicts Lemma 4.

Subcase 10.2. Similarly to Subcase 9.4, we have that $K/H \ncong B_n(q)$, $4 \le n = 2^m$.

Case 11. From Case 1 to Case 10 and Lemma 3, we have K/H isomorphic to one of $C_n(q)$.

Because $C_{p'}(3)$ has the same order components of $B_{p'}(3)$, so $K/H \ncong C_{p'}(3)$. Similarly we can prove that $K/H \ncong C_n(q)$, $4 \le n = 2^m$.

So $K/H \ncong C_{p'}(2)$, $2^p - 1 = 2^{p'} - 1$. Hence $p = p'$, $G/K = 1$, $H = 1$, which implies that $G \cong M$, this is the end of the proof. □

References

[1] Guiyun Chen, A new characterization of $E_8(q)$, J. Southwest China Normal Univ. 21(3) (1995), 215-217.

[2] Guiyun Chen, On Frobenius and 2-Frobenius group, J. Southwest China Normal Univ. 20(5) (1995), 485-487 (in Chinese).

[3] Guiyun Chen, On Thompson's conjecture, J. Algebra 15 (1996), 184-193.

[4] Guiyun Chen, A new characterization of sporadic groups, Algebra Colloq. 3(1) (1996), 49-58.

[5] Guiyun Chen, A new characterization of $G_2(q)$, $[q \equiv 0 \bmod (3)]$, J. Southwest China Normal Univ. 22(1) (1996), 47-51.

[6] Guiyun Chen, A new characterization of Suzuki-Ree group, Sci. China Ser. A 40(8) (1997), 807-812.

[7] Guiyun Chen, A new characterization of $PSL_2(q)$, Southeast Asian Bull. Math. 22 (1998), 257-263.

[8] Guiyun Chen, Characterization of ${}^3D_4(q)$, Southeast Asian Bull. Math. 25 (2001), 389-401.

[9] Guiyun Chen and Huaguo Shi, ${}^2D_n(3)$ $(9 \le n = 2^m + 1$ not a prime) can be characterized by its order components, J. Appl. Math. Comput. 19(1-2) (2005), 353-362.

[10] Guiyun Chen and Huaguo Shi, On Hall factors of several special integral expressions, J. Southwest China Normal Univ. 30(5) (2005), 763-770.

[11] P. Crescenzo, A diophantine equation which arises in the theory finite groups, Adv. Math. 17 (1975), 25-29.

[12] A. Iranmanesh and S. H. Alavi, A new characterization of A_p where p and $p - 2$ are primes, Korean J. Comput. Appl. Math. 8(3) (2001), 665-673.

[13] A. Iranmanesh and S. H. Alavi, A characterization of simple groups $PSL(5, q)$, Bull. Austral. Math. Soc. 65 (2002), 211-222.

[14] A. Iranmanesh, S. H. Alavi and B. Khosravi, A characterization of $PSL(3, q)$, where q is an odd prime power, J. Pure Appl. Algebra 170(2-3) (2002), 243-254.

[15] A. Iranmanesh, S. H. Alavi and B. Khosravi, A characterization of $PSL(3, q)$ for $q = 2^n$, Acta Math. Sinica (Engl. Ser.) 18(3) (2002), 463-472.

[16] A. Iranmanesh and B. Khosravi, A characterization of $F_4(q)$, where q is even, Far East J. Math. Sci. (FJMS) 2(6) (2000), 853-859.

[17] A. Iranmanesh and B. Khosravi, A characterization of $C_2(q)$ where $q > 5$, Comment. Math. Univ. Carolin. 43(1) (2002), 9-21.

[18] A. Iranmanesh and B. Khosravi, A characterization of $PSU_5(q)$, Int. Math. J. 3(2) (2003), 129-141.

[19] A. Iranmanesh, B. Khosravi and S. H. Alavi, A characterization of $PSU(3, q)$ for $q > 5$, Southeast Asian Bull. Math. 26(2) (2002), 33-44.

[20] B. Khosravi, A characterization of ${}^2D_4(q)$, Pure Math. Appl. 12(4) (2001), 415-424.

[21] Behrooz Khosravi and Bahman Khosravi, A characterization of $E_6(q)$, Algebras Groups Geom. 19(2) (2002), 225-243.

[22] Behrooz Khosravi and Bahman Khosravi, A characterization of ${}^2E_6(q)$, Kumamoto J. Math. 16 (2003), 1-11.

[23] A. S. Kondtratev and V. D. Mazurove, Recognition of alternating groups if prime degree from their element orders, Siberian Math. J. 41(2) (2000), 294-302.

[24] Huaguo Shi and Guiyun Chen, ${}^2D_{p+1}(2)$ $(5 \leq p \neq 2^m - 1)$ can be characterized by its order components, Kumamoto J. Math. 18 (2005), 1-8.

[25] J. S. Williams, Prime graph components of finite group, J. Algebra 69 (1981), 487-513.

[26] K. Zsigmody, Zur theories deb potencieses, Monarch. Math. Phys. 3 (1982), 265-284.