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A MODEL OF ENERGY SPATIAL-COLLOCATION

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Abstract

An energy spatial-collocation optimization model based on the equilibrium of marginal benefits is built to maximize the benefits of energy consumption under the circumstances of the state being the investment subject and a market economy. To get more energy each region must boost the efficiency of energy consumption and the marginal profits of resources consumption.

1. Introduction

With the 20 year's high-speed development after the reform and opening up, the shortage of energy in east China, which is becoming more and more severe, is the "bottle-neck" of the economic development of the eastern region. Energy in east China is deficient while in west China it is abundant. Consequently, to develop the rich energy resources in west China in order to solve the shortage of energy in the east is an important subject of the win-win strategy research of the economic cooperation, reciprocal advantage-patching, linkage between the east and the west, and also an essential part of the strategy of the west development. The large-scale energy projects of transmitting natural gas and electricity from China's west to east are being put into practice. For the sake of

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convenience, we call the eastern regions of China the *recipients of energy*. When these important energy projects are completed, another important problem confronts China: how is energy collocated reasonably and efficiently to those recipients in east China? After the regions gain their resources, they also have a big problem: how can they make reasonable use of this limited energy so as to maximize the total benefits?

The collocation of energy resources is generally considered temporally, spatially and departmentally. Yang et al. [1] made a study of the optimal trans-temporal collocation of energy. In China, spatial collocation of energy is generally performed according to actual demands and previous consumption of each region. In recent years, under the extensive economic growth mode, a non-economic energy supplies structure has been implemented [2]. An important characteristic of this structure is that the energy supplied to every province and region by energy productive sectors is collocated in accordance with the administrative plans, and in this structure, no restrictions from the price means unlimited energy demands of consumers. With the transformation of economic growth mode and the establishment and further perfection of the market economic structure, the non-economic collocating structure has gradually been replaced by the economic one. The “invisible hand”—the market—has been playing a more and more decisive role in the energy collocation, the price of energy begins to restrict the demands, and the energy situation inside China is experiencing great changes. We must be aware of the scarcity of energy and the serious situation facing China’s energy. Our energy amount per capital is comparatively small. We have an unequal collocation and unfair structure. The technical level of energy industry is low, and the energy consumption is large while its utility is low. The problem of energy in the countryside and environmental problems caused by energy are more and more serious, which restricts the development of economy and society. Therefore, under the governmental subject and market economy the collocation of energy, as a scarce resource, should maximize the economic benefits after satisfying the demands of basic uses in every region. In this paper we use marginal utility theory to build an optimized energy spatial-collocating model based on the principle of marginal benefit equilibrium.

2. Building a Model

Energy consumed in a region is divided into energy for basic use and energy for economic development. The energy for basic use is that used in a region which maintains production in each industry and residents' daily life, and this demand for energy must be satisfied, and can be deducted directly from the total amount of energy. The rest is regarded as energy for economic developments whose collocation should aim at maximizing the economic and social benefits and get optimized collocation from the viewpoint of economic growth of the country and each region. In this paper, we only consider the collocation of energy for economic development.

According to the concept of marginal output in economics, the marginal benefit of energy consumption refers to the added value of production when one unit of energy is added to the present energy while all the other productive factors are constant. Suppose that the total gross energy transmitted from the west to the east at time t is E, and the economic benefits the transferred energy produces vary from one collocation program to another. From the perspective of the state, the programs should maximize the total economic benefits.

Suppose that in east China there are N recipient regions (in practice the number of recipients equals that of administrative regions), each of which can get Q_{i0}, $i = 1, 2, \ldots, N$ energy for economic development. Let us suppose that each region always first makes use of the native energy, and then energy transmitted from the west. Suppose the marginal benefit of energy consumption of each recipient region is $f_i = f_i(Q_{i0})$, it can be fitted by quadratic function with relevant data about energy; then re-number each region in terms of the amount of marginal benefits of energy use. To be universally applicable, we assume $f_i > f_j$, $\forall i < j$.

Now the problem is how to calculate the amount of received energy E_i with the known total amount of energy input E to maximize the economic benefit after energy collocation. Let us suppose the economic benefit of each recipient region after receiving the energy is

$$U_i = \int_{Q_{i0}}^{Q_i} f_i(Q_i)dQ_i = \int_0^{E_i} f_i(Q_{i0} + E_i)dE_i.$$

Then we have the following optimization problem:

$$\mathrm{Max} U = \sum_{i=1}^{N} U_i = \sum_{i=1}^{N} \int_{Q_{i0}}^{Q_i} f_i(Q_i) dQ_i = \sum_{i=1}^{N} \int_0^{E_i} f_i(Q_{i0} + E_i) dE_i, \tag{1}$$

$$\text{s.t. } E = \sum_{i=1}^{N} E_i, \tag{2}$$

$$\frac{df_i(Q_i)}{dQ_i} = \frac{df_i(Q_{i0} + E_i)}{dE_i} < 0, \tag{3}$$

where $Q_i = Q_{i0} + E_i$ is the total amount of energy consumed in the ith recipient region, formulae (3) indicate the degression of marginal benefit of energy consumption.

3. The Solution of the Model

The above optimization problem is one of conditional extreme values. Thus, a Lagrangian function is structured as follows:

$$L = U + \lambda\left(E - \sum_{i=1}^{N} E_i\right). \tag{4}$$

Then we have

$$\frac{dL}{dE_i} = \frac{d\left(U + \lambda\left(E - \sum_{i=1}^{N} E_i\right)\right)}{dE_i} = f_i(Q_{i0} + E_i) - \lambda = 0 \tag{5}$$

and

$$\frac{d^2 L}{dE_i^2} = \frac{df_i(Q_{i0} + E_i)}{dE_i} < 0. \tag{6}$$

From (5) we can get

$$f_i(Q_{i0} + E_i) = \lambda, \ \forall i = 1, 2, \ldots, N. \tag{7}$$

When (7) is true, that is, the marginal benefit of energy consumption in every region equals λ, the economic benefit arrives at maximum.

From (7) we have

$$Q_i = Q_{i0} + E_i = f_i^{-1}(\lambda), \tag{8}$$

where $f_i^{-1}(\cdot)$ is the inverse function of $f_i(\cdot)$. For convenience, we can suppose $f_i^{-1}(\cdot) = g_i(\cdot)$. Then

$$E_i = g_i(\lambda) - Q_{i0}. \tag{9}$$

λ in the above formula can be obtained in the following way: from (2) and (9) we can get

$$\sum_{i=1}^{N} g_i(\lambda) - \sum_{i=1}^{N} Q_{i0} = E. \tag{10}$$

According to the above analysis, λ can be explained as the equilibrium marginal benefit of energy consumption in each region.

Explanations of the above results are as follows: first we supply energy to No. 1 recipient region which has the greatest marginal energy benefits. From the marginal benefit degression principle we know that with the added energy supplies in No. 1 recipient region, its marginal benefit of energy consumption begins to decrease; when its marginal benefit equals that of No. 2 recipient, that is, $f_1(Q_{10} + E_1^2) = f_2(Q_{20})$, we begin to supply energy to both the regions, where E_1^2 is the energy that No. 1 recipient has got when No. 2 recipient begins to receive energy, while $Q_{10} + E_1^2$ means that the energy marginal benefit of No. 2 recipient corresponds to the total energy consumption of No. 1 recipient. When the marginal benefits of both No. 1 and No. 2 recipients degress to the marginal benefit of No. 3 recipient, we begin to supply energy to the three regions at the same time. Then we go on doing like this, until E is collocated. That is to say, the energy collocation between the regions should guarantee marginal benefit equilibrium of energy consumption in each region, which can maximize the total economic benefit produced by energy in each region.

Thus, the marginal benefit equilibrium principle of energy spatial-collocation can be stated as: when the marginal benefit of energy consumption in each recipient region arrives at equilibrium, the total economic benefit of energy consumption in the east maximizes.

Generally, $\forall i < j$, from

$$f_i(Q_{i0} + E_i^j) = f_j(Q_{j0}). \tag{11}$$

We can decide when to supply energy to the jth recipient region. E_i^j is the received amount of energy in the ith recipient region when the jth begins to receive the collocated energy. $Q_{i0} + E_i^j$ is the energy consumed by ith when the jth's marginal benefit equals that of the ith, which is noted as $Q_{i0}^j = Q_{i0} + E_i^j$. If $f_j(Q_{j0})$ is noted as y_{j0}, then we can get the following from (11):

$$Q_{i0}^j = f_i^{-1}(y_{j0}) = g_i(y_{j0}). \tag{12}$$

4. Further Discussion of the Model

If energy transmitted from the west to the east is sufficient, that is, $E \geq \sum_{i=1}^{N} E_i^N = \sum_{i=1}^{N} (Q_i - Q_{i0})$, then all the recipient regions can get energy. If so, given the equilibrium marginal benefit corresponding to the energy sum after the collocation is y, then we have $y = f_N(Q_{N0})$, and

$$\sum_{i=1}^{N} E_i = \sum_{i=1}^{N} (g_i(y) - Q_{i0}) = E, \tag{13}$$

from (13) we can calculate and get y, then put it into (9), then the amount of energy that can be supplied to each region is clear

$$E_i = g_i(y) - Q_{i0},\ i = 1, 2, \ldots, N. \tag{14}$$

In the empirical analysis, $g_i(y)$ can be fitted by quadratic function.

Suppose

$$g_i(y) = a_i y^2 + b_i y + c_i. \tag{15}$$

Then from (13), we get

$$\sum_{i=1}^{N} (a_i y^2 + b_i y + c_i) = E + \sum_{i=1}^{N} Q_{i0}, \tag{16}$$

that is,

$$\sum_{i=1}^{N} a_i \cdot y^2 + \sum_{i=1}^{N} b_i \cdot y + \left(\sum_{i=1}^{N} c_i - \sum_{i=1}^{N} Q_{i0} - E \right) = 0. \tag{17}$$

The result is

$$y(E) = \frac{-\sum_{i=1}^{N} b_i \pm \sqrt{\left(\sum_{i=1}^{N} b_i\right)^2 - 4\left(\sum_{i=1}^{N} a_i\right)\left(\sum_{i=1}^{N} c_i - \sum_{i=1}^{N} Q_{i0} - E\right)}}{2\sum_{i=1}^{N} a_i}. \tag{18}$$

Plus or minus in formula (18) can be determined according to the following rules. According to the degression principle of marginal benefit, the selection of plus or minus should make $y = y(E)$, a decreasing function of E, and should guarantee the marginal benefit $y \geq 0$. When $y = 0$, from (16), we know that the maximum consumption of energy in the east is $\sum_{i=1}^{N} c_i$, so $\sum_{i=1}^{N} c_i - \sum_{i=1}^{N} Q_{i0} > 0$, and the maximum energy-input $E < \sum_{i=1}^{N} c_i - \sum_{i=1}^{N} Q_{i0}$. For the sake of convenience, we can regard $a = \sum_{i=1}^{N} a_i$, $b = \sum_{i=1}^{N} b_i$, $c = \sum_{i=1}^{N} c_i - \sum_{i=1}^{N} Q_{i0} - E$.

Obviously, a, b are constants, $c > 0$ is also the decreasing function of E. Then (18) can be rewritten as:

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{19}$$

The above analysis shows that when a, b are constants, the selection of plus or minus should make y an increasing function of c, and $y \geq 0$. Then the selection of plus or minus is illustrated as follows:

(1) when $a > 0, b > 0,$ we choose a plus;

(2) when $a > 0, b < 0,$ we choose a minus;

(3) when $a < 0, b > 0,$ we choose a minus;

(4) when $a < 0, b < 0,$ we choose a plus.

If the energy transmitted from the west to the east is limited, that is, $E < \sum_{i=1}^{N}(Q_i - Q_{i0}),$ i.e., $y > f_N(Q_{N0}).$ Now only some of recipient regions can take part in the collocation, and we suppose it to be $N_1,$ $N_1 < N.$ The energy-collocation model is the same as the above one, only N being replaced by $N_1.$ Here the key question is, how to calculate N_1 when the amount of energy-input is $E.$ It can be calculated as follows. From the actual data about energy of every recipient region, (12) can help us to get $Q_{i0}^{j}.$ Thus, from $Q_{i0}^{j} = Q_{i0} + E_i^j$ we can get $E_i^j,$ $i, j = 1, 2, \ldots, N,$ $i \leq j.$ Then, $\forall j,$ $E_0^j = \sum_{i=1}^{j} E_i^j$ can be given. E_0^j expresses the amount of energy received by the previous recipient regions when jth region begins to receive energy. That is to say, when the energy input is $E_0^j,$ the equilibrium marginal benefit of energy consumption equals the marginal benefit of the original energy consumption in the jth recipient region. According to the marginal benefit equilibrium principle of energy spatial collocation, when the amount of input is $E \geq E_0^j,$ the jth recipient region can get collocated energy. Therefore, for E and $\forall j,$ we judge whether $E_0^j \leq E < E_0^{j+1}$ is sound, when the above formula is right, the corresponding j is the N_1 we aim to get.

5. Conclusions

The unbalanced distribution of China's energy sources and the status quo of the economic development require that the energy resources in China be transmitted from the west to the east. When the basic demand of energy sources in each region is satisfied, the state is the investment subject and the market economy is becoming more and more mature, energies for economic development should be consumed to maximize the economic and environmental benefits. On the basis of the marginal benefit equilibrium principle of energy consumption, this paper proposes an optimizing model of energy spatial collocation which aims to maximize economic profits of energy consumption. In the model, when the marginal benefits of energy consumption in every energy recipient region arrive at equilibrium, the total economic profits of energy consumption in the entire eastern region are the maximum. This model has a great practical significance for the reasonable collocation of energy which is transmitted through networks or pipelines, especially such energy as electricity, oil, and natural gas and so on.

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