Far East J. Dynamical Systems **9**(2) (2007), 211-221
This paper is available online at http://www.pphmj.com

SYNCHRONIZATION AND ANTI-SYNCHRONIZATION OF CHAOS IN PERMANENT MAGNET RELUCTANCE MACHINE

U. E. VINCENT and A. UCAR

Department of Physics
Olabisi Onabanjo University
P. M. B. 2002, Ago-Iwoye, Nigeria
e-mail: ue_vincent@yahoo.com

Department of Electrical and Electronics
Firat University
Elazig 23119, Turkey
e-mail: aucar@firat.edu.tr

Abstract

In this paper, we investigate chaos synchronization and anti-synchronization between two coupled permanent magnet reluctance machines (PMSM) exhibiting chaotic behaviour. Nonlinear controllers derived from nonlinear control theory are designed for a drive-response PMSM system so that the response system synchronizes and anti-synchronizes with the drive system. Numerical simulations are given to illustrate and verify the approach.

1. Introduction

Recently, considerable research activities have been devoted by some

2000 Mathematics Subject Classification: 37N35, 65P20, 37D45.

Keywords and phrases: synchronization, active control, permanent magnet reluctance machine.

Communicated by Jessie Park

Received March 18, 2007

researchers to the study of the dynamics of permanent magnet synchronous machine (PMSM) [1, 8, 9, 10, 12, 16]. The existing mathematical models that describes a PMSM system are multivariable, nonlinear and strongly coupled; and therefore exhibits various bifurcations and chaotic phenomena. For instance, some bifurcation and chaotic behaviours which includes limit-cycle oscillations, subcritical and supercritical Hopf bifurcations and different kinds of strange chaotic attractors were reported in [9, 10, 12]. In [8], Harb employed back-stepping nonlinear control and sliding mode control to eliminate the chaotic behaviour that could arise during a high speed performance of a PMSM system.

A PMSM is a kind of high-efficient and high-powered motor that is widely used in motor drive, various servo systems and household applications. It can offer a high-performance drive by utilizing the torques due to the magnets and reluctance variation. Thus, the study of synchronization phenomena in PMSM systems is of high practical importance. Synchronization of chaotic systems has been explored very intensively by many researchers in various fields ranging from physics, mathematics to engineering. Various forms of synchronization phenomenon have been investigated for coupled oscillators. In this paper, we investigate complete synchronization and anti-synchronization between two PMSM systems using the active control technique.

The use of active control method for the synchronization of chaotic systems was proposed by Bai and Lonngren [2, 3]. The active control has since then received considerable attention in the last few years. Applications to various systems abound, some of which includes the Lorenz, Chen and Lü system [5], nonlinear Bloch equations [19], Geophysical system [22], Spatiotemporal dynamical system [6], the so-called Unified chaotic attractor [20], electronic circuits which model a third-order "jerk" equation [4] and most recently in RCL-shunted Josephson junction [21].

2. System Description

For simplicity, we consider here a smooth-air-gap PMSM system. In dimensionless units, the system can be described by the following normalized nonlinear differential equations:

$$\dot{x}_1 = -x_1 + y_1 z_1 + u_d$$

$$\dot{y}_1 = -y_1 - x_1 z_1 + \gamma z_1 + u_q$$

$$\dot{z}_1 = \sigma(y_1 - z_1) - T_L, \tag{1}$$

where x_1, y_1 and z_1 are state variables corresponding to the currents and motor angular frequency respectively; u_d and u_q are the direct- and quadrature-axis stator voltage components, respectively, and T_L represents the external load torque. γ and σ are the system parameters. The dynamics of system (1) has been extensively studied by Li et al. [12] for a fixed subset of the system parameters (σ, γ, T_L) and for three cases of $(u_d = u_q = T_L = 0)$, $(u_q = T_L = 0, u_d \neq 0)$ and the general case $(u_d \neq u_q \neq T_L \neq 0)$. In the first case, the PMSM system is identical to the famous Lorenz equation and can be thought of as that, after an operating period, the external input are switched off. In this paper, we consider the general case exhibiting the chaotic attractor shown in Figure 1, for the parameters $(\sigma = 5.46, \gamma = 20$ and $u_d = -20, u_q = 1, T_L = 1.2)$.

3. Synchronization via Active Control

The most popular synchronization phenomenon is that the difference of the states variables of synchronized systems converges to zero and is called complete synchronization (CS). This was the original idea of synchronization as presented by Pecora and Carroll [15]. Almost all research reports on chaotic synchronization are directed to CS. To study complete synchronization in coupled PMSM system, let the drive PMSM system be given by equation (1) while the response system be given by

$$\dot{x}_2 = -x_2 + y_2 z_2 + u_d + \mu_x$$

$$\dot{y}_2 = -y_2 - x_2 z_2 + \gamma z_2 + u_q + \mu_y$$

$$\dot{z}_2 = \sigma(y_2 - z_2) - T_L + \mu_z, \tag{2}$$

where $\mu_i(t)$, $i = x, y, z$ are control functions to be determined. Subtracting (1) from (2) we obtain the error dynamics as

$$\begin{aligned}
\dot{e}_x &= -e_x + y_2 z_2 - y_1 z_1 + \mu_x \\
\dot{e}_y &= -e_y + \gamma e_z - x_2 z_2 + x_1 z_1 + \mu_y \\
\dot{e}_z &= \sigma e_y - \sigma e_z + \mu_z,
\end{aligned} \tag{3}$$

where $e_i = i_2 - i_1$, $i = x, y, z$.

We now re-define the control functions, to eliminate terms in (4) which cannot be expressed as linear terms in e_x, e_y and e_z, as follows:

$$\begin{aligned}
\mu_x &= -y_2 z_2 + y_1 z_1 + v_x \\
\mu_y &= x_2 z_2 - x_1 z_1 + v_y \\
\mu_z &= v_z.
\end{aligned} \tag{4}$$

Substituting (4) into (3), we have

$$\begin{aligned}
\dot{e}_x &= -e_x + v_x \\
\dot{e}_y &= -e_y + \gamma e_z + v_y \\
\dot{e}_z &= \sigma e_y - e_z + v_z.
\end{aligned} \tag{5}$$

We choose a constant matrix $\mathbf{A}$ which will control the error dynamics (5) such that

$$\begin{bmatrix} v_x(t) \\ v_y(t) \\ v_z(t) \end{bmatrix} = \mathbf{A} \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix} \tag{6}$$

with

$$\mathbf{A} = \begin{bmatrix} \lambda_1 + 1 & 0 & 0 \\ 0 & \lambda_2 + 1 & -\gamma \\ 0 & -\sigma & \lambda_3 + \sigma \end{bmatrix}. \tag{7}$$

In (7) the three eigenvalues λ_1, λ_2 and λ_3 must be negative to ensure a stable and synchronized identical PMSM systems is achieved.

To numerically verify the effectiveness of the designed controllers, we used the standard fourth-order Runge-Kutta algorithm to solve the drive-response systems (1) and (2). The parameters of the systems were selected such that the system is operated in the chaotic state as shown in Fig. 1. That is $u_q = 1$, $u_d = -20$, $T_L = 1.2$, $\sigma = 20$, and $\gamma = 20$. The initial conditions were taken as $x_1(0) = y_1(0) = z_1(0) = -0.01$, $x_2(0) = 0.4$, $y_2(0) = -0.3$ and $z_2(0) = 0.2$, respectively; resulting in the initial errors: $e_x = 0.39$, $e_y = -0.31$ and $e_z = 0.29$. The simulation results for e_x, e_y and e_z are shown in Fig. 2(a). It is evident that the desired synchronization is achieved. In Fig. 2(b), we also show the direct relationship between the state variables of drive-response system along the synchronization manifold (here $x_1 = x_2$, for example).

4. Anti-synchronization via Active Control

Anti-synchronization (AS) [7, 11, 13, 14, 23] is a phenomenon in which the state variables of the synchronized systems have the same absolute values but opposite signs [7, 11, 13, 14, 23]. We say that AS is achieved if $\lim_{t\to\infty}\| x_1 + x_2 \| \to 0$, where x_1 and x_2 are the state variables of the systems S_1 and S_2, respectively. AS phenomenon has been observed experimentally in the context of self-synchronization in salt-water oscillators [14]. To investigate AS in the PMSM system, we define the AS errors for the drive-response system as:

$$e_x = x_1 + x_2, \quad e_y = y_1 + y_2, \quad e_z = z_1 + z_2. \tag{8}$$

Adding (1) and (2) and using the definition (8), we obtain

$$\begin{aligned}
\dot{e}_x &= -e_x + e_y(e_z - z_2) + y_2(2z_2 - e_z) + 2u_d + \mu_x,\\
\dot{e}_y &= -e_y + \gamma e_z + e_x(z_2 - e_z) + x_2(e_z - 2z_2) + 2u_q + \mu_y,\\
\dot{e}_z &= \sigma e_x - \sigma e_z - 2T_L + \mu_z.
\end{aligned} \tag{9}$$

Redefining the control inputs as

$$\mu_x = -e_y(e_z - z_2) - y_2(2z_2 - e_z) - 2u_d + v_x,$$

$$\mu_y = -e_x(z_2 - e_z) - x_2(e_z - 2z_2) - 2u_q + v_y,$$

$$\mu_z = 2T_L + v_z, \tag{10}$$

equation (9) becomes

$$\dot{e}_x = -e_x + v_x,$$

$$\dot{e}_y = -e_y + \gamma e_z + v_y,$$

$$\dot{e}_z = \sigma e_x - \sigma e_z + v_z, \tag{11}$$

where v_x, v_y and v_z are new control inputs. Proceeding as before, we choose a constant matrix **B** such that

$$[v_x, v_y, v_z]^T = \mathbf{B}[e_x, e_y, e_z]^T, \tag{12}$$

where **B** is a 3×3 constant matrix chosen as

$$\mathbf{B} = \begin{bmatrix} \lambda_1 + 1 & 0 & 0 \\ 0 & \lambda_2 + 1 & -\gamma \\ 0 & -\sigma & \lambda_3 + \sigma \end{bmatrix}. \tag{13}$$

Here, we find that the feedback matrix **B** required to produce anti-synchronization is equivalent to the matrix **A** required to produce synchronization, due to the fact that the systems under consideration are identical. However, the controllers $[\mu_x, \mu_y, \mu_z]^T$ are not equivalent.

Selecting the eigenvalues as before (i.e., $-1, -1, -1$), we simulated the systems (1) and (2) using the same parameter settings as in Figure 1, the initial AS errors being $e_x = 0.41$, $e_y = 0.31$ and $e_z = 0.21$, respectively. In Figure 3(a), we show the asymptotic convergence of the AS errors (e_x, e_y, e_z). Figure 4 shows the temporal behaviour of the state variables and finally in Figure 3(b), we have ignored the initial transience and plotted x_1 vs x_2. Evidently, Figure 3(b) suggest that anti-synchronization is analogous to the phenomenon of inverse synchronization reported in [17, 18] wherein two systems can be synchronized on the synchronization manifold $x_1 = -x_2$ as the error signal $\Delta = x_1 - (-x_2) = x_1 + x_2 \to 0$.

5. Concluding Remarks

Conclusively, we have demonstrated in this paper, the synchronization behaviour of two identical permanent magnet reluctance machines using active control. In particular, we employed the technique of active control to achieve AS between two PMSM systems and finally show that AS is equivalent to inverse CS. Our numerical simulations also confirm the theoretical results.

References

[1] E. H. Abed, J. C. Alexander, H. Wang, A. H. Hamdan and H. C. Lee, Dynamic bifurcation in a power system model exhibiting voltage collapse, Proceedings of the 1992 IEEE International Symposium on Circuits and Systems, San Diego, CA, 1992.

[2] E. W. Bai and K. E. Lonngren, Chaos Solitons Fractals 8 (1997), 51.

[3] E. W. Bai and K. E. Lonngren, Chaos Solitons Fractals 11 (2000), 1041.

[4] E. W. Bai, K. E. Lonngren and J. C. Sprott, Chaos Solitons Fractals 13 (2002), 1515.

[5] H. K. Chen, Chaos Solitons Fractals 25 (2005), 1049.

[6] S. Codreanu, Chaos Solitons Fractals 15 (2003), 507.

[7] A. A. Emadzadeh and M. Haeri, Trans. Eng. Comp. & Tech. 6 (2005), 1305

[8] A. M. Harb, Chaos Solitons Fractals 19 (2004), 1217.

[9] A. Harb et al., Electric Mach. Power Syst. 28 (1999), 1019.

[10] N. Hemati and N. Kwatny, Bifurcation of equilibria and chaos in permanent magnet machines, Proceedings of IEEE the 32nd Conference on Decision and Control, San Antonio, Texas, December, 1993, p. 473.

[11] C.-M. Kim et al., Phys. Lett. A 320 (2003), 39.

[12] Z. Li et al., IEEE Trans. Circ. Syst. 49 (2002), 383.

[13] C. Li and X. Liao, Int. J. Bifurc. Chaos 16 (2006), 1041.

[14] S. Nakata et al., Physica D 115 (1998), 313.

[15] L. M. Pecora and T. L. Carroll, Phys. Rev. Lett. 64 (1990), 821.

[16] C. Rajagopalan, P. W. Sauer and M. A. Pai, Analysis of voltage control systems exhibiting Hopf bifurcation, Proceedings of IEEE the 28th Conference on Decision and Control, Tampa, FL, 1989.

[17] E. M. Shahverdiev et al., Phys. Rev. 66 (2002), 017204

[18] E. M. Shahverdiev et al., Chaos Solitons Fractals (2006), in press.

[19] A. Ucar et al., Phys. Lett. A 314 (2003), 96.

[20] A. Ucar et al., Chaos Solitons Fractals 27 (2006), 1292.

[21] A. Ucar et al., Chaos Solitons Fractals 31 (2007), 105.

[22] U. E. Vincent, Phys. Lett. A 343 (2005), 133.

[23] Y. Zhang and J. Sun, Phys. Lett. A 330 (2004), 442.

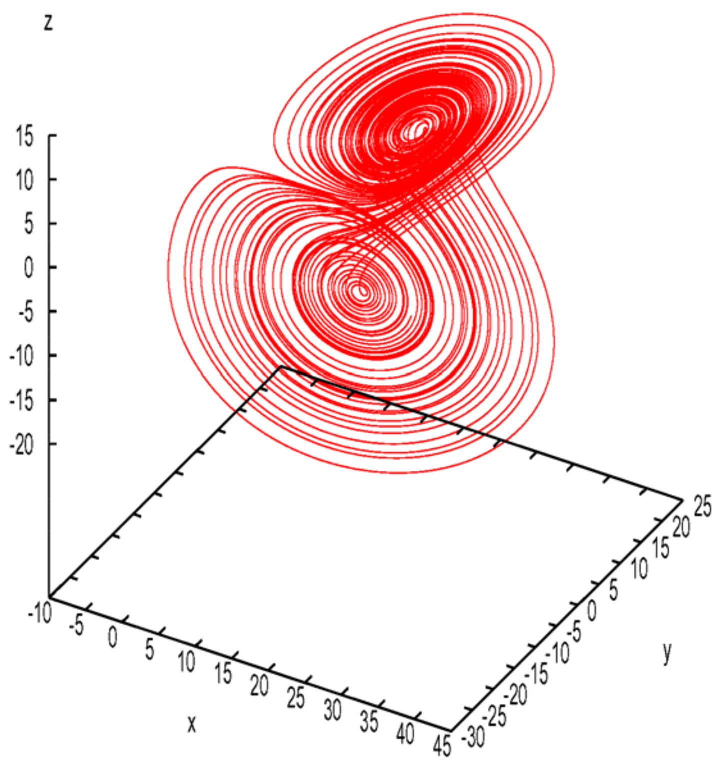
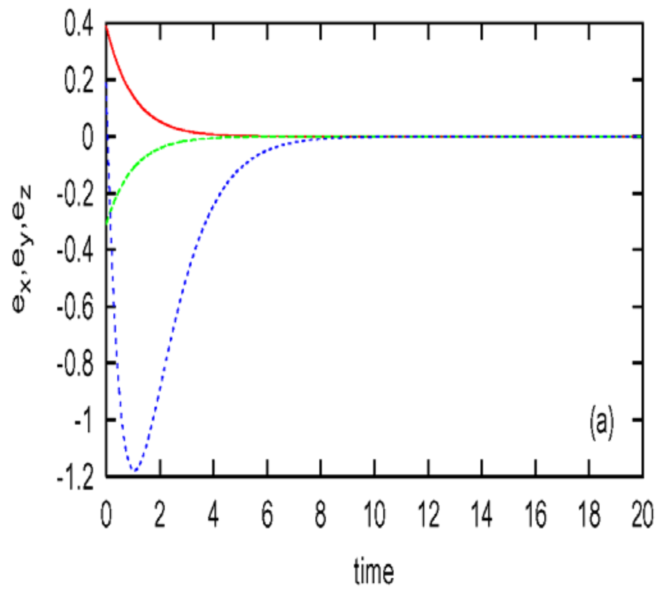


Figure 1. Three-dimensional view of the chaotic attractor of the PMSM system for $\sigma = 5.46$, $\gamma = 20$ and $u_d = -20$, $u_q = 1$, $T_L = 1.2$.



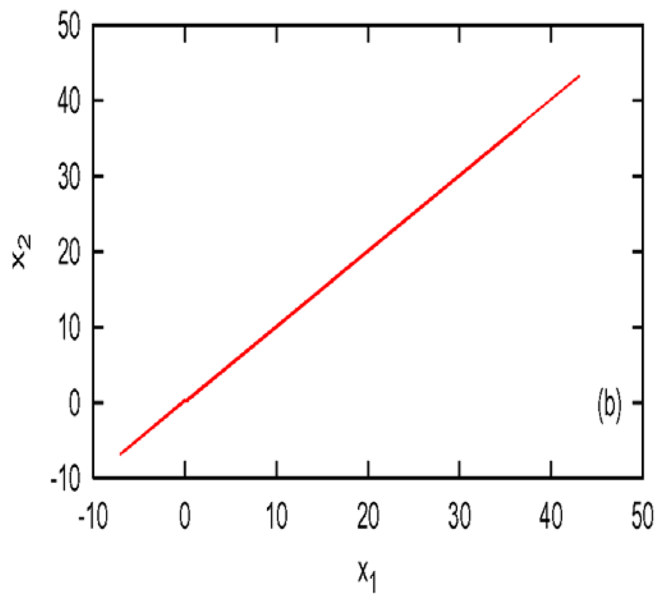


Figure 2. Synchronization dynamics (a) Errors states e_x (red), e_y (blue), e_z (green), (b) x_1 vs x_2 with control activated.

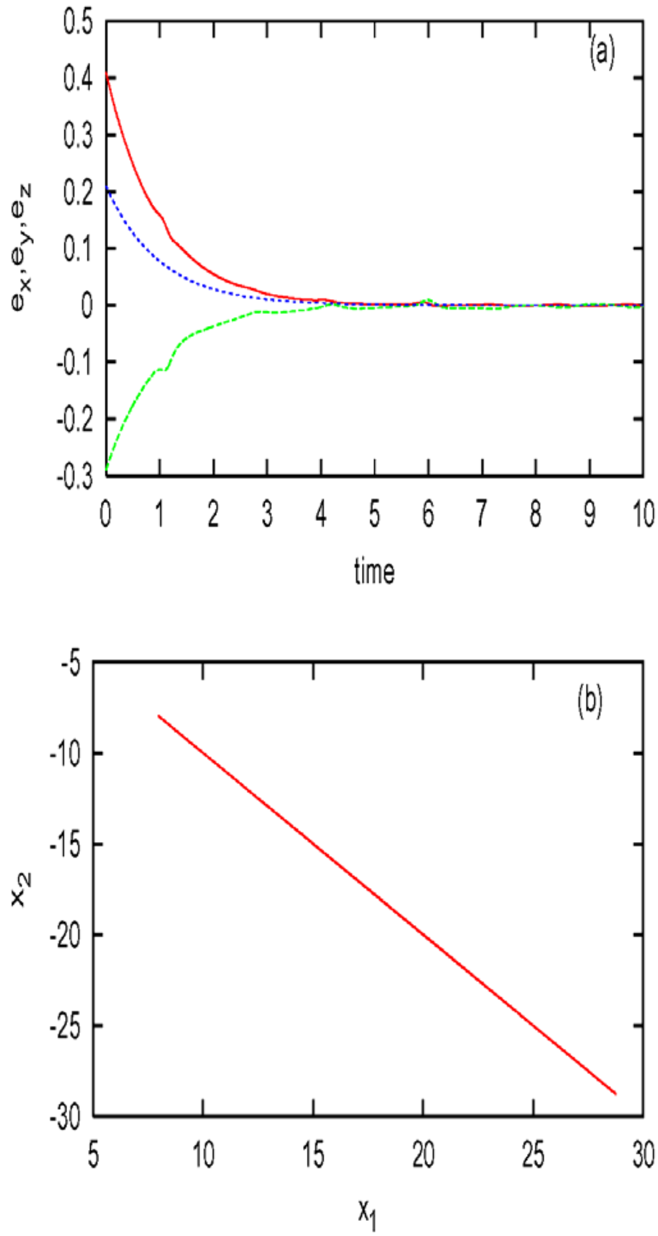


Figure 3. Anti-synchronization dynamics (a) Error states e_x (red), e_y (blue), e_z (green), (b) $(x_1$ vs $x_2)$ showing inverse synchronization with control activated.

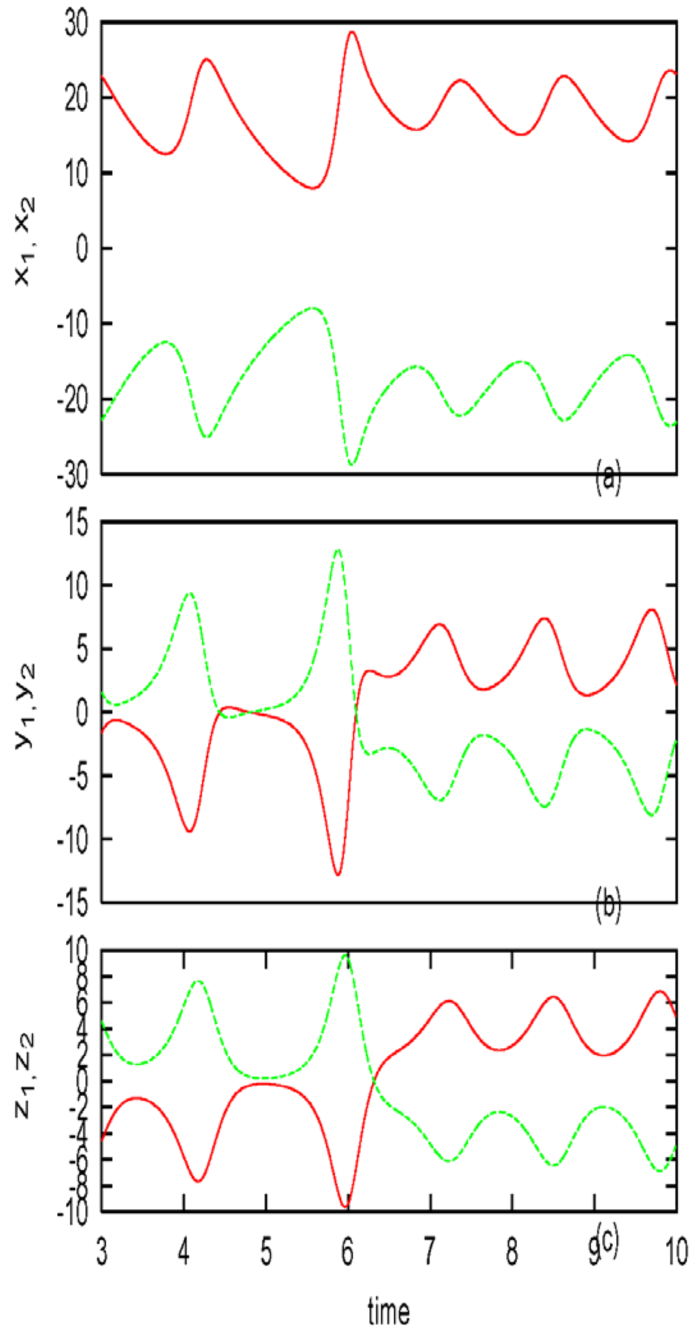


Figure 4. Time evolution of the state variables in the AS state (a) x_1, x_2 (b) y_1, y_2 (c) z_1, z_2.