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COMPARISON OF MINIMIZING THE NORM OF FEEDBACK CONTROLLER MATRIX IN EIGENVALUE ASSIGNMENT BY DIFFERENT METHODS: A CASE STUDY

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Abstract

In the design of linear multi-variable feedback control systems, performance specifications can frequently be met by assigning appropriate closed-loop eigenstructure. Parametrized controller matrix with linear or non-linear parameters and genetic algorithms may be used for minimizing the norm of the feedback controller matrix. As a case study a comparison of the results obtained by different methods on an illustrative example is presented.

1. Introduction

Minimization of the norm of the feedback controller matrix which assigns prescribed eigenvalues to a linear time-invariant multi-variable system requires the determination of a parametric state feedback matrix with the property such that the closed-loop state matrix has the desired

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eigenvalues. If the parametrized controller is linear in its free parameters then the minimization of the norm of the feedback controller matrix can be obtained by analytical approach [3]. Furthermore, the solution of this problem is considered in [1, 2], when the parameters are non-linear but the major problem is that the explicit solution could not be obtained in this case and implicit methods are used. However, it is shown in [4] that genetic algorithms (GAs) can be an effective replacement for the traditional optimization techniques. As a case study, the effect of these three methods which are based on entirely different mathematical approaches, on the norm of the feedback controller matrix of a given problem is investigated.

2. Problem Statement

Consider a controllable linear time-invariant system defined by the state equation

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

or its discrete-time version

$$x(k+1) = Ax(k) + Bu(k), \tag{2}$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$ and the matrices A and B are real constant matrices of dimensions $n \times n$ and $n \times m$ respectively, with $\text{rank}(B) = m$. The aim of eigenvalue assignment is to design a linear state feedback controller,

$$u(t) = Kx(t), \tag{3}$$

where K is the state feedback controller matrix, producing a closed-loop system

$$\dot{x}(t) = (A + BK)\,x(t) \tag{4}$$

with a satisfactory response by shifting controllable poles from actual to desirable locations. Karbassi and Bell [3] have introduced an algorithm obtaining an explicit parametric controller matrix K_α by performing elementary similarity operations which transforms the controllable pair (B, A) into primary vector companion form. The parametric feedback matrix with linear parameters is, in general, of the form

$$K_{\alpha} = K_p + F_{\alpha}, \tag{5}$$

where K_p is the primary state feedback matrix such that the closed-loop matrix $(A + BK_p)$ has the required eigenvalues and F_{α} is the parametric state feedback matrix with linear parameters such that $(A + BF_{\alpha})$ has zero eigenvalues [3]. The minimization of the norm of the K_{α} is achieved by direct differentiation and solving a set of equations for α. It is evident that when the parameters are non-linear K_{α} cannot be obtained explicitly [1, 2]. These difficulties motivate the use of genetic algorithms for minimization of the norm of the feedback control matrix. In order to use genetic algorithms in this way, it is only necessary to encode the *nm* elements of the feedback matrix in accordance with the system of concatenated, multi-parameter, mapped, fixed point coding [4]. Then following a random initial choice, complete generation of such parameters can be readily produced with the basic genetic operations of cross-over, selection, and mutation. Indeed, by successive generations of state feedback controllers produced by genetic algorithms, the norm of the feedback matrix will be minimized and the actual eigenvalues approach the desired eigenvalues.

3. A Case Study

The effectiveness of the proposed methodologies can be conveniently presented for the system considered by Fahmy and O'Reilly [2]:

$$B = \begin{bmatrix} 1 & 2 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 & 2 \\ -2 & 3 & 0 \\ -2 & -1 & 0 \end{bmatrix}.$$

It is desired to locate the closed-loop eigenvalues at $\{-1, -1, -2\}$. The results of implementation of proposed methods are as follows:

(a) The state feedback controller matrix obtained by implementing explicit methods with linear parameters in the manner of Karbassi and Bell [3] is

$$K_1 = \begin{bmatrix} -2 & -2 & 0.667 \\ -1.5 & 1.5 & -1 \end{bmatrix}$$

with the Frobenius norm 3.734. This controller matrix produces a closed-loop matrix with eigenvalues located at $\{-1.999, -1.001, -1.000\}$.

(b) The controller matrix obtained by implicit methods with non-linear parameters in the manner of Fahmy and O'Reilly [2] is

$$K_2 = \begin{bmatrix} -1.791 & -2.127 & 1.142 \\ -1.541 & 1.303 & -0.944 \end{bmatrix}$$

with the Frobenius norm 3.741. This controller matrix produces a closed-loop matrix with eigenvalues located at $\{-1.994, -1.059, -0.946\}$.

(c) The state feedback controller matrix obtained by implementing genetic algorithms in the manner of Porter and Borairi [4] is

$$K_3 = \begin{bmatrix} -0.254 & -2.117 & 1.597 \\ -2.293 & 1.164 & -0.594 \end{bmatrix}$$

with the Frobenius norm 3.749. This controller matrix produces a closed-loop matrix with eigenvalues located at $\{-1.999, -1.011, -0.946\}$. It is evident that the proposed methods are capable of satisfying the above objective. It is interesting to note that many feedback matrices with the same minimum norm may exist, obviously the one obtained by implementation of linear parameters is more robust than the others. It also must be noted that as the size of system increases the differentiation process for linear case becomes tedious and time consuming. Moreover, in case of non-linear parameters the explicit formula does not exist which raises difficulties.

4. Conclusion

In this paper, three different techniques are used for eigenvalue assignment while minimizing the norm of the feedback controller matrix. Different feedback controller matrices obtained from different approaches achieve the desired objectives. It may be noted that the ability of GAs to automate the tuning of the parameters of the controller matrix can be valuable as the size of the system matrix increases, however, for more robust controllers the method involving linear parameters is prescribed, because perturbations in parameters when they are linear, produce similar perturbations in the actual eigenvalues of the closed-loop matrix.

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