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A NEW SCHEME FOR THE DECENTRALIZED STABILIZATION OF LINEAR LARGE SCALE SYSTEMS

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Abstract

This paper is concerned with the decentralized stabilization of input-decentralized linear large-scale systems. Three concepts called strength of stability of subsystems, strength of connection between subsystems and aggregating parameter matrix of overall system are formulated, then two criteria for the existence of a decentralized state feedback controller which guarantees the asymptotical stability of closed-loop system are derived by using Lyapunov theory, and a algorithm for designing such controller is proposed. A numerical example is given to illustrate the application of the results obtained in this paper.

1. Introduction and Problem Formulation

Let $\sum$ be an input-decentralized linear large-scale system composed of N interconnected subsystems $\sum_i$ described by

$$\textstyle\sum_i : \dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + \sum_{j=1,\, j \neq i}^{N} A_{ij} x_j(t), \tag{1}$$

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where $x_i \in R^{n_i}$ and $u_i \in R^{m_i}$ represent the state and input of the subsystem $\sum_i$ respectively, A_i, B_i and A_{ij} are constant matrices of appropriate dimensions. We denote $\sum_{i=1}^{N} n_i = n$, $\sum_{i=1}^{N} m_i = m$, and assume that all pairs (A_i, B_i) are controllable, our general goal is to stabilize the overall system (1) by employing a decentralized state feedback controller

$$u_i(t) = K_i x_i(t), \quad i = 1, 2, \ldots, N \tag{2}$$

for each subsystem $\sum_i$, where $K_i \in R^{m_i \times n_i}$ $(i = 1, 2, \ldots, N)$ represent the matrices of decentralized gains.

Applying the controller (2) to the system (1) results in the closed-loop system

$$\dot{x}_i(t) = (A_i + B_i K_i) x_i(t) + \sum_{j=1,\, j \neq i}^{N} A_{ij} x_j(t). \tag{3}$$

Definition 1. The system (1) is called *decentrally stabilizable* if there exists a decentralized state feedback controller (2) for each subsystem $\sum_i$ such that the closed-loop system (3) is asymptotically stable.

The problem above has been one of the must popular research topics in control systems during the last decades because it is very important in theory and application. Generally speaking, in spite of the controllable hypothesis for each subsystem, it is not always possible to find the decentralized controller with the desired stabilizing property. Therefore it is necessary to make some additional conditions about the interconnection matrix A_{ij}, and many results were given [2-8, 10]. The majority of these works are restricted to the system with a certain structure about A_{ij}, which limits very much their applications. So it is natural to study right along on this problem.

On the other hand, the decentralized stabilization for a large-scale system is evidently depended on the strength of stability of each subsystem and the strength of connection between subsystems. One way to address the strength problem is to consider the magnitudes of subsystem matrix A_i and interconnection matrix A_{ij}. Up to date, unfortunately, this approach has been received very little attention.

In this paper, we use the strength of stability and connection of system (1) to study its decentralized stabilization. Based on the magnitudes of subsystem matrix A_i and the interconnection matrix A_{ij}, we first obtain a scalar matrix Γ (called *aggregating parameter matrix*) with lower order by the solution of the Riccati equation, then derive the decentralized stabilization of (1) from the condition that the Γ is M-matrix, develop a new scheme for designing the controller (2) which guarantees the asymptotic stability of the closed-loop system (3). Finally a typical example is given to show the feasibility of this scheme. Compared with the corresponding results in the literature, our results have the advantage of less restriction, simplicity of numerical computation and easier to be applied.

Notations. Through the paper, $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and minimum eigenvalues of symmetric matrix P, respectively. For $x \in R^n$, $A = (a_{ij}) \in R^{m\times n}$, A^T denotes the transpose of A and $\| A \|_2 = [\lambda_{\max}(A^T A)]^{\frac{1}{2}}$, $\| A \|_E = \left(\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^2\right)^{\frac{1}{2}}$, $\| x \| = (x^T x)^{\frac{1}{2}}$. The notation $x > 0$ $(x < 0)$ means that all components of vector x are positive (negative). For $X \in R^{n\times n}$, the notation $X > 0$ $(X < 0)$ means that matrix X is symmetric and positive-definite (negative-definite). I_n is an identity matrix of size n.

2. The Aggregating Parameter Matrix

Because (A_i, B_i) is controllable, the Riccati equation of matrix

$$A_i^T P_i + P_i A_i - P_i B_i B_i^T P_i + I_{n_i} = 0, \quad i = 1, 2, \ldots, N \tag{4}$$

exists unique the symmetric positive definite solution P_i. Let

$$\gamma_{ii} = \inf_{x_i \neq 0, x_i \in R^{n_i}} \frac{x_i^T x_i}{2x_i^T P_i x_i}, \quad i = 1, 2, \ldots, N. \tag{5}$$

$$\gamma_{ij} = \sup_{x_i \neq 0, x_j \neq 0} \frac{| x_i^T P_i A_{ij} x_j |}{\sqrt{(x_i^T P_i x_i)(x_j^T P_j x_j)}}, \quad i \neq j, \quad i, j = 1, 2, \ldots, N. \tag{6}$$

Definition 2. We call γ_{ii} the *strength of stability* of the subsystem $\sum_i$, γ_{ij} the *strength of connection* between subsystems $\sum_i$ and $\sum_j$ $(i \neq j)$.

Definition 3. The matrix $\Gamma = (\bar{\gamma}_{ij})$ is called *aggregating parameter matrix* of system (1), where

$$\bar{\gamma}_{ij} = \begin{cases} \gamma_{ii} & i = j \\ -\gamma_{ij} & i \neq j \end{cases} \quad i, j = 1, 2, \ldots, N. \tag{7}$$

Before proceeding further, we give three lemmas which will be used in the proof of main results.

Lemma 1. *If square matrix* $A = (a_{ij})$ *of size* n *satisfies* $a_{ij} \leq 0$ $(i \neq j)$, *then the following four statements are equivalent in the sense that each implies the other three*

(1) *A is an M-matrix*;

(2) *All leading principal minors of matrix A are positive*;

(3) *There exists a positive diagonal matrix* $D > 0$ *such that* $DA + A^T D > 0$;

(4) *There exists a positive n-dimensional vector* $x > 0$ *such that* $Ax > 0$.

This lemma can be found in Siljak [9] and Arki [1].

Lemma 2. *Suppose* $x \in R^n$, $y \in R^m$, $A \in R^{m \times n}$. *Then*

$$\| A \|_2 = \max_{\| x \| = \| y \| = 1} | y^T Ax |. \tag{8}$$

Proof. Consider the following optimization problem

$$\max J = (y^T Ax)^2,$$

$$\text{subject to} \quad \| x \| = \| y \| = 1. \tag{9}$$

By the Lagrange multiplier method, we have $Ax^* = (y^{*^T} Ax^*)y^*$ at the optimal point $(x^{*^T}, y^{*^T})^T$, it follows that $x^{*^T} A^T Ax^* = (y^{*^T} Ax^*)^2 = \max J$, so the problem (9) is equivalent to

$$\max J_1 = (x^T A^T Ax),$$

$$\text{subject to} \quad \| x \| = 1. \tag{10}$$

Letting $0 \le \lambda_1 \le \lambda_2 \le \cdots \le \lambda_n$ be all eigenvalues of nonnegative matrix $A^T A$. It is obvious that

$$\lambda_1 \le J_1 \le \lambda_n,$$

which leads to

$$\max J_1 = \lambda_n = \lambda_{\max}(A^T A) = \| A \|_2^2.$$

Due to $\max J = \max J_1$, this implies that equation (8) holds. The proof is completed.

Lemma 3. *Suppose that* $A = (a_{ij})$ *and* $B = (b_{ij})$ *are two square matrices of size n, and satisfy*

(i) $a_{ii} \ge b_{ii} > 0, \quad i = 1, 2, \ldots, n;$

(ii) $b_{ij} \le a_{ij} \le 0, \quad i \ne j, i, j = 1, 2, \ldots, n.$

If B is an M-matrix, then A is an M-matrix also.

Proof. It is easy to conclude the fact that $(A - B)x \ge 0$ holds for any $x > 0$ under the conditions (i) and (ii). Because B is an M-matrix, there exists an $x_0 > 0$ such that $Bx_0 > 0$ by Lemma 1, so $Ax_0 \ge Bx_0 > 0$, which implies that A is an M-matrix by Lemma 1 again. The proof is completed.

3. Main Results

In this section, we will apply the aggregating parameter matrix defined in Section 2 to study the decentralized stabilization of system (1). Two criteria for the existence of a decentralized state feedback controller (2) will be derived by using Lyapunov theory. Our main results are stated in following theorems.

Theorem 1. *If the aggregating parameter matrix* $\Gamma = (\bar{\gamma}_{ij})$ *of system* (1), *defined by equations* (5), (6) *and* (7), *is an M-matrix, then the unique positive definite symmetric solution* P_i *of Riccati equation* (4) *provides us the decentralized gain matrices*

$$K_i = -B_i^T P_i, \quad i = 1, 2, \ldots, N, \tag{11}$$

which guarantee the asymptotical stability of the closed-loop system (3).

Proof. Consider $v_i = x_i^T P_i x_i$ as a Lyapunov function for subsystem $\sum_i$. Taking its time derivative along the solution of (3) and by using (4), (5) and (6) together, we have

$$\begin{aligned}
\dot{v}_i &= \dot{x}_i^T P_i x_i + x_i^T P_i \dot{x}_i \\
&= -x_i^T x_i - x_i^T P_i B_i B_i^T P_i x_i + \sum_{j=1,\, j\neq i}^{N} (x_i^T P_i A_{ij} x_j + x_j^T A_{ij}^T P_i x_i) \\
&\leq -x_i^T x_i + 2 \sum_{j=1,\, j\neq i}^{N} x_i^T P_i A_{ij} x_j \\
&\leq -2\gamma_{ii} v_i + 2 \sum_{j=1,\, j\neq i}^{N} \gamma_{ij} v_i^{\frac{1}{2}} v_j^{\frac{1}{2}}.
\end{aligned} \tag{12}$$

Now we take a weighting sum $v = \sum_{i=1}^{N} d_i v_i$ as a candidate of Lyapunov function for the closed-loop overall system (3), and denote $V = (v_1^{\frac{1}{2}}, v_2^{\frac{1}{2}}, \ldots, v_N^{\frac{1}{2}})^T$. Summing both sides of above inequality (12) from 1 to

N combined with (7) leads to

$$\dot{v} \leq -2\sum_{i=1}^{N} d_i \gamma_{ii} v_i + 2\sum_{i=1}^{N}\sum_{j=1,\, j\neq i}^{N} d_i \gamma_{ij} v_i^{\frac{1}{2}} v_j^{\frac{1}{2}} = -V^T (D\Gamma + \Gamma^T D)V, \quad (13)$$

where $d_i > 0$ is weighting parameters, $D = \operatorname{diag}\{d_1, d_2, ..., d_N\}$. Due to the condition that Γ is an M-matrix, there exists a group of $d_i > 0$ $(i = 1, 2, ..., N)$ such that $D\Gamma + \Gamma^T D > 0$ by Lemma 1, thus $\dot{v} \leq 0$, this assures the stability of the system (3). To prove the asymptotical stability, it is sufficient to note the fact that $V = 0$ if and only if $x = 0$, where $x = (x_1^T, x_2^T, ..., x_N^T)^T$, it follows that $\dot{v}(x) < 0$ for $x \neq 0$ and $\dot{v}(0) = 0$, this implies that the closed-loop overall system (3) is asymptotically stable. The proof is completed.

Theorem 1 presents a criterion for the existence of a decentralized state feedback controller (2) for the decentralized stabilization of system (1). However, in order to apply Theorem 1, we should calculate strength indices γ_{ii} and γ_{ij}, which is very difficult by directly using (5) and (6). The following result presents a method of calculating γ_{ii} and γ_{ij}.

Theorem 2. *Suppose that γ_{ii} and γ_{ij} are defined by equations* (5) *and* (6) *respectively, P_i is the unique symmetric positive definite solution of the Riccati equation* (4), *C_i and C_j are two nonsingular matrices such that $C_i^T P_i C_i = I_{n_i}$, $C_j^T P_j C_j = I_{n_j}$, then*

(1) $\gamma_{ii} = \frac{1}{2}\lambda_{\min}(P_i^{-1}), \quad i = 1, 2, ..., N,$ (14)

(2) $\gamma_{ij} = \| C_i^T P_i A_{ij} C_j \|_2, \quad i \neq j, \quad i, j = 1, 2, ..., N.$ (15)

Proof. (1) Because of $P_i > 0$, there exists an orthogonal matrix O_i such that

$$O_i^T P_i O_i = \operatorname{diag}\{\lambda_1^i, \lambda_2^i, ..., \lambda_{n_i}^i\},$$

where, $0 < \lambda_1^i \leq \lambda_2^i \leq \cdots \leq \lambda_{n_i}^i$ are all eigenvalues of matrix P_i. Applying orthogonal transformation $x_i = O_i \xi_i$ to equation (5), we first have $\| x_i \| = \| \xi_i \|$, then it follows that

$$\gamma_{ii} = \inf_{x_i \neq 0} \frac{x_i^T x_i}{2x_i^T P_i x_i} = \inf_{\| x_i \| = 1} \frac{1}{2x_i^T P_i x_i} = \inf_{\| \xi_i \| = 1} \frac{1}{2\xi_i^T (O_i^T P_i O_i) \xi_i}$$

$$= \inf_{\| \xi_i \| = 1} \frac{1}{2\xi_i^T \operatorname{diag}\{\lambda_1^i, \lambda_2^i, \ldots, \lambda_{n_i}^i\} \xi_i} = \min\left\{ \frac{1}{2\lambda_1^i}, \frac{1}{2\lambda_2^i}, \ldots, \frac{1}{2\lambda_{n_i}^i} \right\}$$

$$= \frac{1}{2} \lambda_{\min}(P_i^{-1}), \quad i = 1, 2, \ldots, N,$$

which completes the proof of part 1 of Theorem 2.

(2) Because of $P_i > 0$, $P_j > 0$, there exist two nonsingular matrices such that $C_i^T P_i C_i = I_{n_i}$, $C_j^T P_j C_j = I_{n_j}$. Applying nonsingular transformations $x_i = C_i \xi_i$, $x_j = C_j \xi_j$ to equation (6), we have

$$\gamma_{ij} = \sup_{\xi_i \neq 0, \xi_j \neq 0} \frac{| \xi_i^T C_i^T P_i A_{ij} C_j \xi_j |}{\sqrt{(\xi_i^T \xi_i)(\xi_j^T \xi_j)}} = \sup_{\| \xi_i \| = \| \xi_j \| = 1} | \xi_i^T C_i^T P_i A_{ij} C_j \xi_j |,$$

which leads to $\gamma_{ij} = \| C_i^T P_i A_{ij} C_j \|_2$ by Lemma 2, and this completes the proof of part 2 of Theorem 2. The proof of Theorem 2 is completed.

Remark 1. It is easy to conclude the fact from (14) and (15) that the aggregating parameter matrix $\Gamma = (\bar{\gamma}_{ij})$ should be an M-matrix if all interconnection magnitudes $\| A_{ij} \|_2$ $(i \neq j)$ are enough small. This means in practice that the decentralized stabilization of system (1) can be reached when strength of connection between subsystems is weaker.

Remark 2. Consider a special case that all inputs of system (1) are identically equal to zero $(u_i \equiv 0, i = 1, 2, \ldots, N)$, then the system (1) becomes

$$\dot{x}_i(t) = A_i x_i(t) + \sum_{j=1, j \neq i}^{N} A_{ij} x_j(t), \quad i = 1, 2, \ldots, N. \tag{16}$$

In this case, taking the Lyapunov equations

$$A_i^T P_i + P_i A_i + I_{n_i} = 0, \quad i = 1, 2, \ldots, N \tag{17}$$

instead of the Riccati equations (4) and following the same arguments with Theorem 1, we have

Corollary. *If all isolated subsystems of* (16) *described by*

$$\dot{x}_i(t) = A_i x_i(t), \quad i = 1, 2, \ldots, N \tag{18}$$

are asymptotically stable and the aggregating parameter matrix $\Gamma = (\bar{\gamma}_{ij})$ *of* (16) (*also defined by equations* (5), (6) *and* (7)) *is an M-matrix, then the system* (16) *is asymptotically stable.*

Based on the results of Theorems 1 and 2, a new scheme for the decentralized stabilization of the system (1) by means of decentralized state feedback controller (2) can be performed as follows:

Step 1. For each i $(1 \leq i \leq N)$, solve matrix Riccati equation (4) for P_i.

Step 2. Compute the invertible matrix C_i such that $C_i^T P_i C_i = I_{n_i}$.

Step 3. Using the matrices P_i and C_i obtained above, compute parameters γ_{ij} by equations (14) and (15), and construct aggregating parameter matrix $\Gamma = (\bar{\gamma}_{ij})$ of system (1) by Definition 3.

Step 4. Verify whether matrix $\Gamma = (\bar{\gamma}_{ij})$ is an M-matrix or not. If yes, then turn to step 5, and otherwise, to stop.

Step 5. Take the decentralized state feedback controller (2) for system (1) as following

$$u_i(t) = -B^T P x_i(t), \quad i = 1, 2, \ldots, N.$$

In the algorithm above, we should solve the Riccati equation (4) and compute the invertible matrix C_i which may involve very difficult numerical calculation. In order to get the criterion that is easier to be applied, let us first consider a class of input-decentralized large-scale system with single-input subsystems described by

$$\dot{x}_i(t) = A_i x_i(t) + b_i u_i(t) + \sum_{j=1,\, j\neq i}^{N} A_{ij} x_j(t), \quad i = 1, 2, \ldots, N, \tag{19}$$

where (A_i, b_i) are given in the companion form:

$$A_i = \begin{pmatrix} 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ a_1^i & a_2^i & \cdots & a_{n_i-1}^i & a_{n_i}^i \end{pmatrix}, \; b_i = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}.$$

Now given a group of positive numbers

$$\{\sigma_k^i > 0 \,|\, \sigma_r^i \neq \sigma_l^i \text{ if } r \neq l,\, i = 1, 2, \ldots, N,\, k = 1, 2, \ldots, n_i\}.$$

Denote

$$\sigma_m^i = \min_{1\leq k\leq n_i}\{\sigma_k^i\},\ \sigma_M^i = \max_{1\leq k\leq n_i}\{\sigma_k^i\},\ \rho_k^i = -\sigma_k^i,\ \xi_{ij} = \frac{(\sigma_M^i \sigma_M^j)^{\frac{1}{2}}}{\sigma_m^i}\| W_i^{-1} \|_2 \| W_j \|_2,$$

where $i, j = 1, 2, \ldots, N$, and W_i is a Vandermonde matrix

$$W_i = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ \rho_1^i & \rho_2^i & \cdots & \rho_{n_i}^i \\ (\rho_1^i)^2 & (\rho_2^i)^2 & \cdots & (\rho_{n_i}^i)^2 \\ \vdots & \vdots & \ddots & \vdots \\ (\rho_1^i)^{n_i-1} & (\rho_2^i)^{n_i-1} & \cdots & (\rho_{n_i}^i)^{n_i-1} \end{pmatrix}.$$

As straightforward application of Theorems 1, 2 and Corollary, we have

Theorem 3. *Let* $A_{ij} = (a_{pq}^{ij})$. *Then the system* (19) *is decentrally stabilizable if there exist real numbers* $\beta_1, \beta_2, \ldots, \beta_N$ *and* $\alpha > 1$ *such that* $\Omega = (\omega_{ij}) \in R^{N\times N}$ *is an M-matrix, where*

$$\omega_{ij} = \begin{cases} \sigma_m^i \alpha^{\beta_i}, \ i = 1, 2, \ldots, N, \\ -\alpha^{\frac{1}{2}(\beta_j - \beta_i)} \xi_{ij} \| \widetilde{A}_{ij} \|, \ i \neq j = 1, 2, \ldots, N. \end{cases} \tag{20}$$

$$\widetilde{A}_{ij} = (\widetilde{a}_{pq}^{ij}),$$

$$\widetilde{a}_{pq}^{ij} = a_{pq}^{ij} \alpha^{(1-p)\beta_i + (q-1)\beta_j}, \quad p = 1, 2, \ldots, n_i, \quad q = 1, 2, \ldots, n_j.$$

Proof. Since (A_i, b_i) is controllable, there exists k_i such that $A_i + b_i k_i$ has a set of distinct real eigenvalues

$$\{\alpha^{\beta_i} \rho_1^i, \alpha^{\beta_i} \rho_2^i, \ldots, \alpha^{\beta_i} \rho_{n_i}^i\}. \tag{21}$$

Applying the nonsingular transformations

$$x_i(t) = R_i W_i \bar{x}_i(t), \quad i = 1, 2, \ldots, N \tag{22}$$

to the closed-loop system

$$\dot{x}_i(t) = (A_i + b_i k_i) x_i(t) + \sum_{j=1, j \neq i}^{N} A_{ij} x_j(t), \ \ i = 1, 2, \ldots, N, \tag{23}$$

we have

$$\dot{\bar{x}}_i(t) = \Lambda_i \bar{x}_i(t) + \sum_{j=1, j \neq i}^{N} \overline{A}_{ij} \bar{x}_j(t), \ \ i = 1, 2, \ldots, N, \tag{24}$$

where

$$R_i = \mathrm{diag}\{1, \alpha^{\beta_i}, \ldots, \alpha^{(n_i - 1)\beta_i}\}, \ \ \Lambda_i = \{\alpha^{\beta_i} \rho_1^i, \alpha^{\beta_i} \rho_2^i, \ldots, \alpha^{\beta_i} \rho_{n_i}^i\},$$

and $\overline{A}_{ij} = W_i^{-1} R_i^{-1} A_{ij} R_j W_j = W_i^{-1} \widetilde{A}_{ij} W_j$.

It is clear that the system (19) is decentrally stabilizable if and only if the system (24) is asymptotically stable. In the following, we will use Corollary to study the asymptotical stability of (24).

First, the Lyapunov equations of (24) can be written as

$$\Lambda_i P_i + P_i \Lambda_i + I_{n_i} = 0, \quad i = 1, 2, \ldots, N, \tag{25}$$

which gives a unique symmetric positive definite solution $P_i = -\frac{1}{2} \Lambda_i^{-1}$.

Then, by using equations (14) and (15), it is straightforward to verify that the strength indices γ_{ii} and γ_{ij} of (24) satisfy

(1) $\gamma_{ii} = \sigma_m^i \alpha^{\beta_i}, \quad i = 1, 2, \ldots, N,$

(2) $0 \le \gamma_{ij} \le \alpha^{\frac{1}{2}(\beta_j - \beta_i)} \xi_{ij} \| \widetilde{A}_{ij} \|, \quad i \ne j, \quad i, j = 1, 2, \ldots, N,$

which, together with the conditions of Theorem 3 and Lemma 3, implies that the aggregating parameter matrix of the system (24) is an M-matrix. By Corollary, it follows that the system (24) is asymptotically stable, this, combined with the argument above, completes the proof of Theorem 3.

Remark 3. The result obtained in Theorem 3 can be easily generalized to the system with multi-input. One way, as pointed out in [4], transforms A_i and B_i into Luenberger's canonical form, and by applying the method treated in Theorem 3, the same result can be performed. The details are omitted here.

Remark 4. By well-known inequality $\| A \|_2 \le \| A \|_E$ and Lemma 3, it follows that the result still holds in Theorem 3 if we take $\| \bullet \|_E$ instead of the $\| \bullet \|_2$. This substitution is quite interesting and useful because the calculation of $\| \bullet \|_2$ is more difficult than that of $\| \bullet \|_E$.

4. Illustrative Example

To illustrate the application of methods obtained in this paper, we present the following example.

Example. Consider an input-decentralized large-scale system which is composed of following two interconnected subsystems

$$(S_1) \quad \dot{x}_1(t) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 2 & 3 & 2 & -1 \\ 0 & 0 & 0 & 1 \\ 2 & 1 & -1 & -2 \end{pmatrix} x_1(t) + \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 1 & 0 \\ 2 & 1 \end{pmatrix} x_2(t) + \begin{pmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 1 & 1 \end{pmatrix} u_1(t),$$

$$(S_2) \quad \dot{x}_2(t) = \begin{pmatrix} 0 & 1 \\ 3 & 4 \end{pmatrix} x_2(t) + \begin{pmatrix} 3 & 0 & 1 & 5 \\ 2 & 1 & 4 & 6 \end{pmatrix} x_1(t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u_2(t).$$

Because the subsystem S_1 is multi-inputting, we first apply the state feedback

$$u_1(t) = \begin{pmatrix} 2 & 1 & -2 & 1 \\ -2 & -1 & 0 & 0 \end{pmatrix} x_1(t) + \begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix} \overline{u}_1(t) \tag{26}$$

to the S_1 and obtain two single-input subsystems

$$(S_{11}) \quad \dot{x}_{11}(t) = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix} x_{11}(t) + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} x_2(t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \overline{u}_{11}(t),$$

$$(S_{12}) \quad \dot{x}_{12}(t) = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix} x_{12}(t) + \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} x_2(t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \overline{u}_{12}(t),$$

where $x_1 = (x_{11}^T, x_{12}^T)^T$, $\overline{u}_1 = (\overline{u}_{11}, \overline{u}_{12})^T$. Substituting $x_1 = (x_{11}^T, x_{12}^T)^T$ into S_2, we can rewrite S_2 as

$$(\overline{S}_2) \quad \dot{x}_2(t) = \begin{pmatrix} 0 & 1 \\ 3 & 4 \end{pmatrix} x_2(t) + \begin{pmatrix} 3 & 0 \\ 2 & 1 \end{pmatrix} x_{11}(t) + \begin{pmatrix} 1 & 5 \\ 4 & 6 \end{pmatrix} x_{12}(t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u_2(t).$$

Now consider a large-scale system $\overline{S}$ composed of three single-input subsystems S_{11}, S_{12} and $\overline{S}_2$. According to Theorem 3, let $\sigma_1^i = 2$, $\sigma_2^i = 3$ $(i = 1, 2, 3)$, $\beta_1 = \beta_2 = \beta_3 = 1$ and α to be determined later. By simple calculation, the corresponding matrix Ω in Theorem 3 about the system $\overline{S}$ is found as

$$\Omega = \begin{pmatrix} 2\alpha & 0 & -44.8\alpha \\ 0 & 2\alpha & -22.4 \\ -67.2 & -112\alpha & 2\alpha \end{pmatrix}.$$

It is easy to verify that Ω is an M-matrix for sufficiently large positive number α (for example, $\alpha > 1400$), which implies by Theorem 3 that system $\overline{S}$ (i.e., the system considered in example) is decentrally stabilizable.

To get the gain matrices of controller (2) for each subsystem, we denote

$$\overline{u}_{11}(t) = k_{11}^T x_{11}(t), \quad \overline{u}_{12}(t) = k_{12}^T x_{12}(t), \quad u_2(t) = k_2^T x_2(t), \tag{27}$$

where k_{11}, k_{12} and k_2 can be approached by the procedure of the pole assignment for control system (see (21) in the proof of Theorem 3)

$$k_{11} = \begin{pmatrix} -2-6\alpha^2 \\ -3-5\alpha \end{pmatrix}, \quad k_{12} = \begin{pmatrix} 1-6\alpha^2 \\ 2-5\alpha \end{pmatrix}, \quad k_2 = \begin{pmatrix} -3-6\alpha^2 \\ -4-5\alpha \end{pmatrix}. \tag{28}$$

Substitute (27) and (28) into (26), obtain

$$\begin{cases} u_1(t) = \begin{pmatrix} -6\alpha^2 & -2-5\alpha & -3+6\alpha^2 & -1+5\alpha \\ -2 & -1 & 1-6\alpha^2 & 2-5\alpha \end{pmatrix} x_1(t), \\ u_2(t) = (-3-6\alpha^2, -4-5\alpha) x_2(t). \end{cases} \quad (\alpha > 1400)$$

5. Conclusion

In this paper, we have studied the decentralized stabilization of input-decentralized linear large-scale systems. Based on three concepts called strength of stability of subsystems, strength of connection between subsystems and aggregating parameter matrix of overall system, the existence and designing problem of a decentralized state feedback controller which guarantees the asymptotical stability of closed-loop system have been investigated by using Lyapunov theory. Time-delays, uncertainties and discrete case in the system (1) are not considered for simplicity. However, the results obtained can be easily generalized to the system with time-delays, uncertainties and discrete case.

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