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THE BRADLEY-TERRY MODEL FOR HANDLING CATEGORICAL RESPONSE VARIABLES FROM FARMER PARTICIPATORY TRIALS

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Abstract

This paper looks at responses from participatory on-farm farmer participatory trials that are often measured as ratings (farmers score each given treatment on a scale that is ordered but arbitrary) or rankings (where farmers arrange the treatments in order from most preferred to least preferred). Simple methods such as the preference statistic that uses the proportion of responses, Kruskal-Wallis test which is a one-way analysis of variance by ranks and the Friedman test that is a two-way analysis of variance by ranks are outlined. The Bradley-Terry model for ranks which is a logit model for paired comparisons is described and used to fit models for plot level covariates.

1. Introduction

On-farm farmer participatory trials are becoming increasingly common in research as one of the ways of bringing all stakeholders for testing and adopting new technologies [9]. The stakeholders include the

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farmers, researchers, extension officers, community, governments and donor agencies. Farmer participation ensures identification of constraints to food security and together with the researcher solutions are sought. In a participatory on-farm trial each farmer is required to compare a given number of treatments. In this paper the objectives are to identify, describe and contrast simple methods of analyzing ranking and rating data; suggest practical approaches on the use of ordinal regression for scores; and explore and implement Bradley-Terry models in statistical software.

Ranking and rating data are often used in a trial to assist the researcher in ascertaining the reasons that may lead a farmer to adopt/not adopt a particular variety. Rating is done by farmers assigning numerical values on a rating scale for instance of 1-5 for each given treatment. The labels are arbitrary and start with the least favored to most favored. When a number of treatments are arranged in order according to some quantity which they all possess to a certain degree, such data are said to be *ranked*. Ranked data can arise in different ways: According to some measurement or countable quality for instance tress ranked according to height; according to some quality which is thought to be measurable but cannot be measured for practical or theoretical reasons for instance ranking of the 'cob size' of given maize varieties; and according to some quality which cannot be measured on any measured on any objective scale for instance taste, texture and attractiveness.

Rating data is preferred when the response of interest can best be described in terms of words instead of continuous measurements for instance "poor", "good", "very good", "excellent". In a study where there are a large number of treatments for instance more than six then use of rating data is recommended. Rating data can also be used where the objective of the study is to identify good treatments rather than the most preferred treatment. Ranking data is employed when the treatments are all similar and one wants to unsure differentiation. Ranking data can also be applied when the objective is to prioritize treatments rather than to find good treatments and if the data required is in the form of preferences.

The problem of intervals between ranks can be resolved using a restricted scoring or open scoring where the following possibilities were described by Bart and Maxwell [3]. First one could use a restricted scoring by column/row for instance by allowing a fixed number of points per column or row, for example, 10 points for a given characteristic to give an interval between choices. Secondly open scoring by column/row be done where farmers have open-ended number of points per column or row.

This paper will identify, describe and contrast simple methods of analyzing ranking and rating data. Methods such as the preference statistic that uses the proportion of responses where treatment A is preferred to B, Kruskal-Wallis test which is a one-way analysis of variance by ranks [11] and the Friedman test that is a two-way analysis of variance by ranks are described and the shortcomings related to these methods that use linear model based analyses of ranking and rating data are outlined. Suggestions on practical approaches on the ordinal regression for rates have also been made. Further the paper will explore and implement the Bradley-Terry model in statistical software.

Logistic regression has been identified as the standard approach of analyzing binary and ordered categorical outcome data given that regression coefficients from logistic models have simple interpretation in terms of odds ratios that are easily understood. To analyze ranks, the Bradley-Terry model for ranks which is a logit model for paired comparisons shall be used to fit models. The cumulative logit model has been given as the appropriate model for analyzing rating data and models will be fitted to the data. The data is from experiments done by International Center for Research in Agroforestry (ICRAF) in Malawi to test the effect of gliricidia and sesbania in improving soil fertility.

2. Methodology

This section describes the binomial distribution model, the log likelihood and deviance function. The Bradley-Terry model for ranks is outlined and its case as a linear model is shown together with its application as a model of quasi symmetry, quasi independence. The ordinal regression model is also defined.

2.1. Binomial distribution model

The binomial distribution arises where the observations are non-negative counts bounded above by fixed value. Suppose Z_i is the ith binary random variable, where $Z_i = 1$ if the ith outcome is a success and $Z_i = 0$, otherwise with

$$\Pr(Z_i = 1) = \pi_i \quad \text{and} \quad \Pr(Z_i = 0) = 1 - \pi_i.$$

If there are n such random variables $Z_1, \ldots, Z_n$ which are independent with $\Pr(Z_i = 1) = \pi_i$, then their joint probability distribution is

$$\prod_{i=1}^{n} \pi_i^{z_i}(1-\pi_i)^{1-z_i} = \exp\left[\sum_{i=1}^{n} z_i \ln\left(\frac{\pi_i}{1-\pi_i}\right) + \sum_{i=1}^{n} \ln(1-\pi_i)\right], \tag{2.1}$$

which is a member of the exponential family. For the case where the π_i's are all equal, we can define

$$Y = \sum_{i=1}^{n} Z_i,$$

so that Y is the number of successes in n trials. The random variable Y has the binomial distribution $b(n, \pi)$ and

$$\Pr(Y = y) = \binom{n}{y} \pi^y (1-\pi)^{n-y}; \quad y = 0, 1, \ldots, n.$$

The moment generating function is used to find the mean and variance of Y, so that

$$M_Y(t) = E[e^{ty}] = (1 - \pi + \pi e^t)^n \tag{2.2}$$

and therefore the mean and variance of Y are

$$E(Y) = n\pi \quad \text{and} \quad Var(X) = n\pi(1-\pi). \tag{2.3}$$

2.1.1. Log likelihood function for binomial distribution

Suppose $y_1, \ldots, y_n$ are independent random variables of $Y_1, \ldots, Y_n$ such that $Y_i \sim B(m_i, \pi_i)$. The log likelihood is considered as a function of the coefficients appearing in the model and is expressed as

$$l(\pi,\ y) = \sum_{i=1}^{n}\left[y_i \ln\left(\frac{\pi_i}{1-\pi_i}\right) + m_i \ln(1-\pi_i)\right], \tag{2.4}$$

where the constant function of y not involving π, namely, $\sum_{i=1}^{n} \ln\binom{m_i}{y_i}$ has been omitted because it plays no role. The maximum achievable log likelihood is attained when we obtain the maximum likelihood estimates (mle) for the maximal model. This leads to the estimate as

$$\hat{\pi} = y_i / m_i.$$

The systematic part of the model specifies the relation between the vector π and the observational conditions as summarized by the $n \times p$ matrix. For the generalized linear model this relationship takes the form

$$g(\pi_i) = \eta_i = \sum_{j=1}^{n} X_{ij}\beta_j, \quad i = 1, \ldots, n, \tag{2.5}$$

so that the log likelihood can be expressed as a function of unknown parameters $\beta_1, \ldots, \beta_p$. In the case of linear logistic model, we have

$$g(\pi_i) = \eta_i = \ln\left(\frac{\pi_i}{1-\pi_i}\right) = \sum_{j=1}^{n} X_{ij}\beta_j, \quad i = 1, \ldots, n. \tag{2.6}$$

Hence, the log likelihood function

$$l(\hat{\pi}_{\max};\ y) = \sum_{i=1}^{N}\left[y_i \ln(y_i/m_i) + (m_i - y_i)\ln(1 - y_i/m_i) + \ln\binom{m_i}{y_i}\right] \tag{2.7}$$

and so the deviance function becomes

$$D = 2\sum_{i=1}^{N}\left[y_i \ln\left(\frac{y_i}{m_i\hat{\pi}_i}\right) + (m_i - y_i)\ln\left(\frac{m_i - y_i}{m_i - m_i\hat{\pi}_i}\right)\right]. \tag{2.8}$$

The deviance function can be likened to the residual sum of squares or weighted residual sum of squares in ordinary linear model. The smaller the deviance the better the fit of the logistic model. A large value for the deviance is an indication that there is a significant lack of fit for the logistic model and some other model may be appropriate. Notice that D does not involve nuisance parameters and so the goodness of fit can be assessed and the hypotheses can be tested directly by using the asymptotic distribution $D \sim \chi^2_{N-p}$, where p is the number of β parameters estimated under the null hypothesis.

2.2. The Bradley-Terry model for ranks

The Bradley-Terry model is useful in establishing the overall ranking of items through paired comparisons. For instance, it may be difficult for a farmer to rank all the treatments at the same occasion; rather it is preferable to compare the treatments in a pairwise manner.

Suppose t treatments are to be compared in pairs by a group of farmers. The Bradley-Terry model as defined by Agresti [1] is a logit model for paired comparison experiments. Let P_{ij} denote the probability that treatment T_i is preferred to treatment T_j selected from the pair (T_i, T_j), $j \neq i$.

Suppose $P_{ij} + P_{ji} = 1$ for all pairs and that there is no chance for a tie. The Bradley-Terry assumes non-negative parameters $\{\pi_i\}$ exist such that

$$P_{ij} = \Pr(T_i > T_j) = \frac{\pi_i}{(\pi_i + \pi_j)}. \tag{2.9}$$

Let $\pi = \exp(\phi_i)$, we have

$$P_{ij} = \frac{\exp(\phi_i|)}{\exp(\phi_i) + \exp(\phi_j)}. \tag{2.10}$$

Then

$$\ln P_{ij} = \phi_i| - \ln[\exp(\phi_i) + \exp(\phi_j)] \tag{2.11}$$

and it follows that

$$\ln(1 - P_{ij}) = \phi_j| - \ln[\exp(\phi_i) + \exp(\phi_j)], \tag{2.12}$$

so that

$$\ln\left(\frac{P_{ij}}{1-P_{ij}}\right) = \phi_i| - \phi_j, \tag{2.13}$$

where $\phi_i|$ is the score for treatment i.

Thus P_{ij} is a monotonic function of $\phi_i - \phi_j$, with $P_{ij} = 0.5$ when $\phi_i = \phi_j$. The odds of ranking one treatment higher than the other is a function of the difference $\phi_i| - \phi_j$; $(i \neq j)$; $i, j = 1, \ldots, t$.

The model accepts the existence of treatment parameters π_i for T_i whenever $\pi_i \geq 0$ and $\sum_{i=1}^{t} \pi_i = 1$. The model assumes the independence of the ratings of the same pair by different farmers and different pairs by the same farmer.

For the treatments, $T_1, \ldots, T_t$ are compared with N_{ij} comparisons of T_i and T_j, where

$$(N_{ij};\ i < j,\ j = 1, \ldots, t).$$

Then we have a total of $N_{ij} = \sum_{i<j} n_{ij}$ paired comparisons, where n_{ij} are the number of times T_i is preferred to T_j in the N_{ij} comparisons. Thus

$$n_{ij} + n_{ji} = N_{ij},$$

where the N_{ij} comparisons are independent with the probability P_{ij} applying to each and n_{ij} has a binomial distribution with parameters (N_{ij}, P_{ij}) and the overall likelihood function for the N_{ij} paired comparisons is

$$\prod_{i<j} \binom{N_{ij}}{n_{ij}} \left(\frac{\pi_i}{\pi_i + \pi_j}\right)^{n_{ij}} \left(\frac{\pi_j}{\pi_i + \pi_j}\right)^{N_{ij}-n_{ij}}. \tag{2.14}$$

Let m_{ij} be the expected number of times T_i is preferred to T_j. Then

$$m_{ij} = E[n_{ij}] = \frac{N_{ij}\pi_i}{(\pi_i + \pi_j)}. \tag{2.15}$$

For the ith pair of comparison, if T_i is preferable to T_j and let the vector $d_1 = (d_{11}, \ldots, d_{1n})$ be such that

$$d_{1k} = \begin{cases} 1, & k = i, \\ -1, & k = j, \\ 0, & \text{otherwise.} \end{cases}$$

Then it can be seen from the above argument that the likelihood of the Bradley-Terry model is identical to the binary logistic model with d_1 as covariates, no intercept and a constant response.

A rare and unique feature of the Bradley-Terry model $\operatorname{logit}(P_{ij}) = \phi_i| - \phi_j$ according to [8] is that the model matrix corresponding to the formula $\phi_i| - \phi_j$ does neither include the intercept nor does the constant vector lie in the column space of the variables because the null model of no preference has $P_{ij} = 0.5$ corresponding to the $\operatorname{logit}(P_{ij}) = 0$.

2.3. Bradley-Terry model and quasi symmetry

Fienberg and Larntz [7] showed that the Bradley-Terry model is a logit formulation of the quasi symmetry model. The results which are summarized in a square contingency table, in which cells on the main diagonal are empty. Table 1 below illustrates quasi symmetry when given four treatments.

Table 1. Layout for data in quasi symmetry with $t = 4$

	1	2	3	4
T_1	-	X_{12}	X_{13}	X_{14}
T_2	X_{21}	-	X_{23}	X_{24}
T_3	X_{31}	X_{32}	-	X_{34}
T_4	X_{41}	X_{42}	X_{43}	-

For quasi symmetry, given that an observation is in cell (i, j) or (j, i), the logit of the conditional probability that it is in cell (i, j) equals

$$\ln\left(\frac{m_{ij}}{m_{ji}}\right) = (\mu + \lambda_i^x + \lambda_i^y + \lambda_{ij}^{xy}) - (\mu + \lambda_j^x + \lambda_j^y + \lambda_{ji}^{xy})$$

$$= (\lambda_i^x + \lambda_i^y) - (\lambda_j^x + \lambda_j^y) = \phi_i - \phi_j, \tag{2.16}$$

where $\phi_i| = \lambda_i^x + \lambda_i^y$.

Estimates of $\{\lambda_i^x\}$ and $\{\lambda_i^y\}$ for quasi symmetry yield estimates of $\{\phi_i|\}$ for the Bradley-Terry model, and hence of $\{\pi_i = \exp(\phi_i)\}$ and $\{\pi_{ij}\}$.

The usual constraints of $\{\lambda_i^x\}$ and $\{\lambda_i^y\}$ imply a similar constraint on $\{\phi_i|\}$, such as $\sum_i \phi_i = 0$. Any constraint multiple of the $\{\pi_i\}$ estimates also satisfy $P_{ij} = \frac{\pi_i}{\pi_i + \pi_j}$, so the estimates can be scaled to satisfy a constraint such as $\sum_i \pi_i = 1$.

2.3.1. Bradley-Terry model and quasi independence

Table 2 below shows a layout for data in paired comparisons with $t = 4$. In this case, the binomial samples are 2 of the 4 categories. This layout was developed by Fienberg and Larntz [7] to show the relationship of the Bradley-Terry model a contingency table with log linear structure. The dashes are structural zeros.

Table 2. Layout for data in quasi independence with $t = 4$

	1	2	3	4	5	6
T_1	X_{12}	X_{13}	X_{14}	-	-	-
T_2	X_{21}	-	-	X_{23}	X_{24}	-
T_3	-	X_{31}	-	X_{32}	-	X_{34}
T_4	-	-	X_{41}	-	X_{42}	X_{43}

We have the data in a 4×6 table of counts and this fits the general format which is a $t \times \frac{1}{2}t(t-1)$ contingency table. The Bradley-Terry model relates to the model of quasi independence proportions in the $t \times \frac{1}{2}t(t-1)$ table of expected counts $\{m_{ij}\}$ laid out as in the above table.

3. Data Analysis and Results

The data that will be used was collected from an experiment carried out by International Center for Research in Agroforestry (ICRAF) in Malawi, beginning from November 1994 to April 1998. The research comprised of forty one farmers whose control was continuous maize planting in the four seasons versus the gliricidia treatment which involved mixed intercropping of maize and gliricidia and the sesbania treatment where there was relay planting of maize. The experiment was to check on whether the treatments improved the soil fertility. Actual yields were measured in tones per hectare during the harvest seasons of April 1995, April 1996, April 1997 and April 1998.

Continuous variates such as nitrogen in soil, phosphorus in soil, soil pH and soil cec (cation exchange capacity) were recorded as plot level covariates. The slope index whether flat, gentle or sloping was recorded, including how weeding was carried out in the 97/98 season and whether it was done at the proper time, or was done late or was never done. Fertilizer manure application in the 97/98 season was recorded on the 'yes' or 'no' alternatives. The data was entered on Microsoft Excel and analyzed using General Statistical Software (Genstat) version 6.0.

3.1. Bradley-Terry model fitted to data from on-farm cropping with sesbania and gliricidia in Malawi

The Bradley-Terry model fitted had a discrete $(0, 1)$ response variable where a success was labeled 1 and a failure was labeled 0, for each paired comparison of treatments. There were three treatments which were compared, namely, mixed intercropping of maize and gliricidia which was labeled *g*, secondly relay planting of maize and sesbania that was labeled *s* and the control of continuous maize that was labeled *c*. Forty one farmers were involved in the study and they compared the control c with either g or s. According to the fitted Bradley-Terry model, treatment s was expected to perform best followed by g and finally c. That was not always the case and that is why there are success and failures where the yields did not match the argument.

3.1.1. Bradley-Terry model fitted to treatments only

The fitted model is

$$\text{logit}(P_{ij}) = \text{treatment}_i - \text{treatment}_j, \quad i, j = 1, 2, 3.$$

This model is overparameterized because the model contains more unknown parameters than is justified by the number of observations. The simplest solution to this problem is to omit one of the parameters. For our case treatment c has been set to zero by Genstat statistical software.

The table below shows the coefficient of the estimates, their standard errors, t-statistics and the antilog of the estimates which are the odds ratios. It can be seen, from the table, that g performed better when compared to the other two treatments, namely, g and c. The challenge will be to compare the treatments and to investigate whether farm or plot level covariates and regressors can be used to help explain the reasons for variation across the fields.

Table 3. Estimates of treatment parameters

Treatments	estimate	Standard error	$t(*)$	Antilog of estimate
g	1.997	0.429	4.65	7.365
s	1.358	0.426	3.19	3.890
c	reference	0	0	1.000

Treatments g and s were ranked significantly higher than treatment c. The estimates shown in Table 3 show that treatment g is preferred by more than 7 times to treatment c with a 95% confidence interval of [3.177, 17.08]. The estimates for treatment s to treatment c also inform us that the former is ranked higher by over 3 times with a 95% confidence interval of [1.687, 8.962]. Treatment g is almost twice as preferred as treatment s.

Table 4 below compares the observed and fitted model and it can be observed that the observed treatment probabilities are well fitted by the model.

Table 4. Comparison of observed and fitted probabilities

Pair	Number of comparisons	Number with first pair ranked higher than second	Proportion of first pair ranked higher than second	Fitted probabilities p_{ij}
$g - c$	31	28	0.903	0.880
$s - c$	21	16	0.762	0.795
$g - s$	24	16	0.667	0.655

The observed and fitted probabilities can be obtained from the table of parameter estimates under the fitted model. These probabilities reveal that treatment g was ranked better than treatment c more times than treatment s was ranked higher than c.

The model explained that the difference between the treatments was not by chance. The mean deviance of treatments was 15.22 on 2 degrees of freedom which is significant at 0.01 level of significance with a critical value of 9.210 on 2 degrees of freedom from a chi-squared distribution. The model is a well fitting model since the residual deviance of 74.91 is approximately equal to its number of degrees of freedom which is 74 with a mean deviance of 1.012. This implies that the observed proportions adequately describe the fitted model.

The following sections will investigate the interactions of other factors that were recorded such as slope level, weeding, application of fertilizer and a continuous variate such as nitrogen and phosphorus in the soil.

3.1.2. Slope and treatment interaction

We further investigate whether the difference in the treatments may have been caused by the differences in the slope. The fields were classified into 3 levels, namely, (0 = flat, 1 = gentle, 2 = sloping). The assumption was that treatment *g* would perform relatively less well on sloping land. The fitted model was to test the effect of the three levels of slope on the treatments. The model can be written as

$$\text{logit}(P_{ij}) = (\text{treatment}_i - \text{treatment}_j) \cdot \text{slope}_k; \quad i, j, k = 1, 2, 3.$$

The change in deviance as a result of the slope/treatment interaction is 0.96 on 4 degrees of freedom which is not significant in any conventional significance level. Therefore, the conclusion is that different levels of slope did not bring about a significant change in the ranking of the treatments. The change in deviance is conspicuously small, suggesting that there was no consistent difference in the way treatments *g*, *s* and *c* are ranked on flat, gentle and sloping land. The residual deviance was 73.96 on 70 degrees of freedom with a mean deviance of 1.057 which shows that the observed proportions were well fitted for this model.

From Table 5 below the standard errors are very high and the estimates of the various levels of interaction are also high. The high values of standard errors may be as a result of multicollinearity. The estimates for slope = 2 are zero because number of plots planted on this slope level for treatments *g*, *s* and *c* are 4, 4 and 2, respectively.

Table 5. Estimates of parameters for slope and treatments

Treatment	Slope = 0	Std Error	Slope = 1	Std Error	Slope = 2	Std Error
g	1.8	15.9	2.0	15.9	7.2	-
s	1.1	16.0	1.5	16.0	7.2	-
c	reference	0	reference	0	reference	0

All the *t*-values are not significantly different from zero. Therefore, the conclusion is that the different levels of slope cannot explain the differences in ranking of the treatments. The hypothesis that treatment *g* would perform less well on sloping land cannot be explained further given that none of the treatments showed a significant change.

Further, it can be seen from the above table that the scores for treatment g are very close for slope = 0 and slope = 1 which can help explain why they were not significantly different. The same can be said about treatment s, the distance between the two levels of slope is also very small to allow for a significant difference in the slope.

3.1.3. Interaction between fertilizer and treatments

The table below shows the distribution of the treatments for the two fertilizer levels, namely, (level 0 = no and level 1 = yes).

Table 6. Table of fertilizer and treatments

	Fertilizer		
Treatment	0	1	Grand Total
g	32	6	38
s	21	4	25
c	30	8	38
Grand Total	83	18	101

From Table 6 it can be seen that only 18% of the farmers applied fertilizer on their plots and the majority did not put fertilizer on their plots.

The fitted model is

$$\operatorname{logit}(P_{ij}) = (\text{treatment}_i - \text{treatment}_j) \cdot \text{fertilizer}_l; \quad l = 1, 2.$$

The change in deviance as a result of bring in the two fertilizer levels to the model is 0.653 on 2 degrees of freedom with a mean deviance of 3.265 which is significant at 5% level of significance with a critical value of 5.991 on 2 degrees of freedom from a chi-squared distribution. Therefore, the conclusion is that the application of fertilizer contributed significantly in the ranking of the treatments.

Table 7. Table of scores for fertilizer and treatments

Treatment	Fertilizer = 0	Std Error	Fertilizer = 1	Std Error
g	2.498	0.966	0.377	-
s	1.43	1.04	1.43	-
c	reference	0	reference	-

The residual deviance was 68.12 on 71 degrees of freedom with a mean deviance of 0.9595 which is less than 1, so it can be deduced that the model was well fitted. The estimates are combined in Table 7 of scores for the fertilizer treatment interaction. From the above table the interaction between treatment *g* and fertilizer level 0 is significant at 5% level of significance with a critical value of 1.96, this implies application of fertilizer to gliricidia has a significant effect on the ranking of treatment *g*. The other interactions did not significantly increase the yield.

4. Discussion Summary

The fitted Bradley-Terry model for treatments showed that there was a significant difference at 5% level of significance in the way treatments gliricidia, sesbania and control of continuous maize were ranked. The plot level continuous covariates recorded for each plot such as nitrogen and phosphorus did not significantly affect the rankings of the treatments. Regressors such as weeding and slope of the land did not bring out a significant difference in the ranking of the treatments. The application of fertilizer was significantly different for the ranking of gliricidia treatment when compared to the control of continuous maize treatment. For a better and more informative analysis slope = 2 may be omitted from the analysis in order to come up with more realistic estimates.

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