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PROPAGATION OF THERMAL WAVES IN A THIN FILM SUBJECTED TO HEATING AND COOLING ON EITHER SIDE

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Abstract

Wave nature of heat propagation in a thin film subjected to heating and cooling on either side is investigated by solving the hyperbolic heat conduction equation. Analysis expressions for the temperature and heat flux distributions and computation results for the time history of heat transfer behavior are obtained. The results show that in transient heat conduction, a heat pulse is transported as wave, which is attenuated in the film, and non-Fourier heat conduction is extremely significant within a certain range of film thickness and time.

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Nomenclature

a	separation constant, $a_i = i\pi/[x_0/(\tau C)]$ and $i = 1, 2, \ldots$
A	a function of dimensionless time β
B	a function of dimensionless distance δ
C	thermal wave speed
Cp	specific heat
K	thermal conductivity
q	heat flux
q_m	heat flux defined as $q_m = KT_0/x_0$
T	temperature
T_0	internal temperature
T_1	imposed wall temperature
t	time
x	distance
x_0	half thickness of the film

Greek letters

α	thermal diffusivity
β	dimensionless time defined as $t/(2\tau)$
δ	dimensionless distance defined as $x/(2\tau C)$
ρ	density
τ	thermal relaxation time
θ	dimensionless temperature defined as $T_1(x - x_0)/(T_0 x_0)$

1. Introduction

In the classical theory of diffusion, Fourier's law serves as the constitutive equation relating heat flux to the temperature gradient. In one-dimensional heat conduction, it can be expressed as

$$q(x, t) = -K \frac{\partial T(x, t)}{\partial x}. \tag{1}$$

Here, q denotes the heat flux; T is temperature; x is distance from the left side wall of the film; t is time and K is thermal conductivity of the medium. According to this law, if the material conducting heat is subjected to a thermal disturbance, then the effect will be felt instantaneously in all parts of the conducting medium. In other words, heat propagates at an infinite speed. This phenomenon is physically anomalous and can be remedied through the introduction of a hyperbolic equation based on a relaxation model for heat conduction that accounts for a finite thermal propagation speed.

In most practical heat transfer applications, the effect of a finite speed of propagation is negligible since materials in which heat propagates are macroscopic in dimension such that Fourier's law is accurate and appropriate. However, this law noticeably breaks down in situations involving extremely short time response, extremely high-rate change of temperature of heat flux, temperature approaching absolute zero and initial conditions involving the time rate of change of temperature, because the wave nature of heat propagation becomes dominant [5, 6, 12, 16]. Several issues of basic scientific interest arise in cases such as laser penetration and welding, explosive bonding, electrical discharge machining, and heating and cooling of micro-electronic elements involving a duration time of a nanosecond or ever picosecond in which energy is absorbed within a distance of microns from the surface. For example, the issue of energy transfer into a lattice and the resulting temperature in the lattice during such a short period of time and over such a tiny region is of fundamental importance but remains a matter of controversy [2]. It is apparent that a more accurate constitutive law describing the nature of heat conduction needs to be introduced.

Recently, considerable interest has been generated toward the hyperbolic heat conduction (HHC) equation and its potential applications in engineering and technology. A comprehensive survey of the pertinent literature is available in [19]. Theoretical predictions are available in the literature for some specific cases. Some researchers dealt with wave characteristics and finite propagation speed in transient heat transfer conduction [3, 8, 10, 12, 14, 16, 26-28]. Several analytical and numerical solutions of the HHC equation have been presented in the literature. Carey and Tsai [4] analyzed a propagating heat wave reflected at a boundary, in which the numerical methods based on a variation formulation of the problem and the Galerkin finite-element method are employed. Glass et al. [9] used a numerical technique based on MacCormack's predictor-corrector scheme to solve the HHC equation. As the other method, Frankel et al. [7] developed a general three-dimensional constant property heat flux formulation based on the HHC approximation. They reported that the flux-formulation is more convenient to solve problems involving flux-specified boundary conditions. The hyperbolic heat transfer was used by Baumeister and Hamill [1] to study the propagation of a temperature pulse in a semi-infinite medium, and by Vick and Ozisik [26] and Ozisik and Vick [18] to study the propagation of a heat pulse. Using the parabolic and hyperbolic models of heat conduction, Kar et al. [11] studied heat conduction due to short-pulse heating for various boundary conditions. They reported that the predicted temperature distribution is substantially affected by the temperature dependent thermal properties. Lewandowska [13] also dealt with the parabolic and hyperbolic heat conduction in the one-dimensional, semi-infinite body with the insulated boundary and discussed different time characteristics of the heat source capacity. Size effects on nonequilibrium laser heating of metal films were investigated by Qin and Tien [20]. Tan and Yang [22, 23] presented theoretical predictions for the propagation of heat from a step change in temperature on both side walls of a thin film and heat transfer during asymmetrical collision of thermal waves in a thin film. Results were obtained for the time history of propagation process, magnitude and shape of thermal waves and the range of film thickness and duration time. The similar study was carried out by Torii and Yang [24, 25], who employed a numerical technique based on MacCormack's predictor-corrector scheme to solve the non-Fourier, hyperbolic heat conduction equation.

This paper deals with the wave behavior during transient heat conduction in a very thin film (solid plate) subject to heating and cooling on either side. Analytical solutions are obtained by means of the method of separation of variables to solve the non-Fourier, hyperbolic-type heat conduction equation.

2. Formulation of Problem and Solutions

The thermal wave model allows a time lag between the heat flux and the temperature gradient. In one-dimensional heat conduction, this special feature can be illustrated mathematically by the following equation:

$$q(x,\, t + \tau) = -K \frac{\partial T(x,\, t)}{\partial x}. \tag{2}$$

Here, τ denotes the relaxation time, an intrinsic thermal property of the medium, $\tau = \alpha/C^2$, where C is the speed of "second sound" (thermal shock wave) and α represents the thermal diffusivity of the medium. The thermal wave speed C becomes finite for $\tau > 0$. As τ approaches zero, the thermal wave speed C approaches infinity and equation (2) reduces to the classical parabolic heat conduction equation.

Assuming that the relaxation time τ is small, the second and the higher order terms can be neglected, hence equation (2) takes the form

$$\tau \frac{\partial q}{\partial t} + q + K \frac{\partial T}{\partial x} = 0. \tag{3}$$

In one-dimensional heat conduction, the conservation of energy is given by

$$\rho c_p \frac{\partial T}{\partial t} + \frac{\partial q}{\partial x} = 0. \tag{4}$$

Here, ρ and c_p denote the density and specific heat of the medium, respectively.

A combination of equations (3) and (4) yields the hyperbolic conduction equation as

$$\tau \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}. \tag{5}$$

Consider a very thin film with a thickness of $2x_0$ maintained at a uniform, initial temperature T_0. A heating temperature T_1 and a cooling temperature $-T_1$ are suddenly imposed on the walls at $x = 0$ and $x = 2x_0$, respectively. In other words, one-side wall is raised from T_0 by $T_0 + T_1$ and the other wall is cooled from T_0 by $T_0 - T_1$. Thus the initial and boundary conditions are

$$T = T_0 \quad \text{at } t = 0, \ 0 < x < 2x_0, \tag{6}$$

$$\partial T/\partial x = 0 \quad \text{at } t = 0, \ 0 < x < 2x_0, \tag{7}$$

$$T = T_0 + T_1 \quad \text{at } t > 0, \ x = 0, \tag{8}$$

$$T = T_0 - T_1 \quad \text{at } t > 0, \ x = 2x_0. \tag{9}$$

With the introduction of the following dimensionless quantities

$$\theta(\beta, \delta) = \frac{(T - T_0)x_0 + T_1(x - x_0)}{T_0 x_0},$$

$$\beta = \frac{t}{2\tau},$$

$$\delta = \frac{x}{2\tau C},$$

equations (5) and (9) are reduced to

$$\frac{\partial^2\theta}{\partial\beta^2} + 2\frac{\partial\theta}{\partial\beta} = \frac{\partial^2\beta}{\partial\delta^2}, \tag{10}$$

$$\theta = T_1(x - x_0)/(T_0 x_0) \quad \text{at } \beta = 0, \ 0 < \delta < x_0/(\tau C), \tag{11}$$

$$\partial\theta/\partial\beta = 0 \quad \text{at } \beta = 0, \ 0 < \delta < x_0/(\tau C), \tag{12}$$

$$\theta = 0 \quad \text{at } \beta > 0, \ \delta = 0, \tag{13}$$

$$\theta = 0 \quad \text{at } \beta > 0, \ 0 = x_0/(\tau C). \tag{14}$$

The method of separation of variables is applied to the heat conduction (10) using

$$\theta(\beta, \delta) = A(\beta)B(\delta). \tag{15}$$

One obtains the general solutions as

$$\theta = [D\cos(\beta\sqrt{a^2-1}) + E\sin(\beta\sqrt{a^2-1})][H\cos(a\delta) + J\sin(a\delta)]e^{-\beta}$$
$$\text{at } a > 1, \tag{16}$$

$$\theta = (Fe^{\beta\sqrt{1-a^2}} + Ge^{-\beta\sqrt{1-a^2}})[H\cos(a\delta) + J\sin(a\delta)]e^{-\beta}$$
$$\text{at } a < 1. \tag{17}$$

Here, a is a constant and D, E, F, G, H and J are the coefficients to be determined by the initial and boundary conditions. For the sake of brevity, the intermediate steps of derivation are omitted here. The general solutions thus obtained are recast in the dimensional expression for the temperature time history as

$$\theta = -\sum_{i=1}^{\infty} \frac{2T_1[(-1)^i+1]}{i\pi T_0}\left[\cos(\beta\sqrt{a_i^2-1}) + \frac{1}{\sqrt{a_i^2-1}}\sin(\beta\sqrt{a_i^2-1})\right]e^{-\beta}\sin(a_i\delta) \text{ at } x_0/(\tau C) < \pi, \tag{18}$$

$$\theta = -\sum_{i=1}^{\mathrm{int}\left(\frac{x_0}{\pi\tau C}-\frac{1}{2}\right)} \frac{T_1[(-1)^i+1]}{i\pi T_0}\left[\frac{\sqrt{1-a_i^2}+1}{\sqrt{1-a_i^2}}e^{\beta\sqrt{1-a_i^2}} + \frac{\sqrt{1-a_i^2}-1}{\sqrt{1-a_i^2}}e^{-\beta\sqrt{1-a_i^2}}\right]e^{-\beta}\sin(a_i\delta)$$
$$- \sum_{i=\mathrm{int}\left(\frac{x_0}{\pi\tau C}+\frac{1}{2}\right)}^{\infty} \frac{2T_1[(-1)^i+1]}{i\pi T_0}\left[\cos(\beta\sqrt{a_i^2-1}) + \frac{1}{\sqrt{a_i^2-1}}\sin(\beta\sqrt{a_i^2-1})\right]e^{-\beta}\sin(a_i\delta) \text{ at } x_0/(\tau C) > (i-1)\pi. \tag{19}$$

Here, $a_i = i\pi/[x_0/(\tau C)]$ and i is an integer (1, 2, ...).

Equation (3) is solved for the time history of heat flux as

$$q(x,\ t) = -\frac{K}{\tau}\cdot e^{-\frac{t}{\tau}}\cdot\int_0^t\left(e^{\frac{t}{\tau}}\cdot\frac{\partial T(x,\ t)}{\partial x}\right)dt. \tag{20}$$

According to this equation, the heat flux q at time t depends on the entire time history $t = 0$ to t during which temperature gradient is established, rather than on the point value of temperature gradient at time t as in the case of simple diffusion given by equation (1).

A substitution of equations (18) and (19) into equation (20) yields

$$\frac{q}{q_m} = \frac{T_1}{T_0} - \frac{T_1}{T_0}e^{-2\beta} + \sum_{i=1}^{\infty}\frac{2T_1[(-1)^i+1]}{T_0\sqrt{a_i^2-1}}e^{-\beta}\sin(\beta\sqrt{a_i^2-1})\cos(a_i\delta) \text{ at } x_0/(\tau C) < \pi, \tag{21}$$

$$\frac{q}{q_m} = \frac{T_1}{T_0} - \frac{T_1}{T_0}e^{-2\beta} + \sum_{i=1}^{\mathrm{int}\left(\frac{x_0}{\pi\tau C}-\frac{1}{2}\right)}\frac{T_1[(-1)^i+1]}{T_0\sqrt{1-a_i^2}}\left(e^{\beta\sqrt{1-a_i^2}} - e^{-\beta\sqrt{1-a_i^2}}\right)e^{-\beta}\cos(a_i\delta) + \sum_{i=\mathrm{int}\left(\frac{x_0}{\pi\tau C}+\frac{1}{2}\right)}^{\infty}\frac{2T_1[(-1)^i+1]}{T_0\sqrt{a_i^2-1}}e^{-\beta}\sin(\beta\sqrt{a_i^2-1})\cos(a_i\delta)$$

$$\text{at } x_0/(\tau C) > (i-1)\pi. \tag{22}$$

3. Results and Discussion

Analytical expressions and computation results are obtained which display the unusual nature of hyperbolic heat conduction in the films with different values of $x_0/(\tau C)$. For convenience in analysis and computation, a set of $T_1 = T_0$ is selected to demonstrate heat transfer characteristics resulting from temperature changes of heating and cooling on either side of a thin film.

Figures 1 to 3 show the time history of temperature $[(T - T_0)/T_1]$ in the film having $x_0/(\tau C)$ of 1, 3, and 10, respectively. One can see that the relaxation behavior in thermal wave induces many phenomena which cannot be described by the classical diffusion model. Figure 1 is prepared to illustrate in detail the propagation process of thermal waves in a film with the value of $x_0/(\tau C)$ being 1. Clearly, the same as other wave phenomena, sharp wave-fronts exist in the thermal wave propagation while the temperature levels decrease when the thermal waves penetrate into the medium. At $\beta = 0$, the temperature in the film is uniform and equals to its initial temperature T_0 [namely $(T - T_0)/T_1 = 0$], and temperature of T_1 and $-T_1$ are suddenly imposed on either side of the film. The temperature distribution in the film is expressed by curve A1'1B at this moment. Then, two wave-fronts appear and advance towards the center in the physical domain which separates the heat affected zone from the thermally undisturbed zone. Across the wave-fronts, the temperature presents a finite jump. At $\beta = 0.5$, thermal wave-fronts meet at the center of the film with a temperature distribution curve A4'4B in Figure 1(a). After the wave-fronts from two sides meet at the center of the film, reverse thermal wave-fronts occur and travel towards side walls of the film (see Figure 1b). The temperature undergoes a significant decrease near the left side of the center and a significant increase near the right of the center, resulting in a constant temperature of $T = T_0$ at the center. After thermal wave-fronts are reflected from the boundaries, the pattern is continued in Figure 1(c) with smaller wave-fronts. By several times of collision, reflection and continuous attenuation of the thermal waves as they propagate back and forth between the two boundaries, the wave-fronts become weak and the results predicted by the wave theory collapse onto those predicted by the diffusion model at $\beta = 5.0$ and thereafter.

Similarly, one can see in Figures 2 and 3 that heating and cooling on either side of a film gives rise to the propagation of two severe thermal wave-fronts in the film at a finite velocity. Each of these wave-fronts decays exponentially with time and dissipates energy along its path by diffusion. One notices an undisturbed region $t/(2\tau) < \delta < (2x_0 - Ct)/(2\tau C)$

of no temperature change when the dimensionless time β is less than $x_0/(2\tau C)$. The local temperatures at locations $\delta = t/(2\tau)$ and $\delta = (2x_0 - Ct)/(2\tau C)$ exhibit a step discontinuity similar to that of semi-infinite body problem [1, 21].

The present analytical solution predicts the existence of thermal waves in a very thin film and exhibits the propagation process of thermal waves, the magnitude and shape of thermal waves, and the regularity of the thermal wave decaying process in the films with different values of $x_0/(2\tau C)$. Such behavior is characteristics of a thermal system with a relaxation or start up time unseen in the classical linear or nonlinear diffusion theory. It is seen in each figure that the peak of thermal wave-fronts from two sides of the film decay exponentially with time up to $\beta = 5$ (namely $t = 10\tau$). And at the same time, it is also found that the non-Fourier effect is far more significant in a system with a larger relaxation time τ. For example, larger temperature waves can be seen in a film with $x_0/(\tau C)$ of unity in Figure 1. However, in a film with $x_0/(\tau C)$ $= 10$, wave-fronts are too weak as they approach the symmetrical center to produce temperature waves and reverse thermal waves. It behaves like diffusion domination. For $\tau = 0$, equation (19) is reduced to

$$T = \frac{T_1(x_0 - x)}{x_0} + T_0 - \sum_{i=1}^{\infty} \frac{2T_1[(-1)^i + 1]}{i\pi} e^{-\left(\frac{i\pi}{2}\right)^2 \frac{\alpha t}{x_0^2}} \sin\left(\frac{i\pi}{2}\frac{x}{x_0}\right) \tag{23}$$

which is the solution of the diffusion mechanism.

Figures 1 to 3 reveal that the thermal relaxation time τ plays a primary role in deciding a domain to be wave dominating or diffusion dominating. Several investigations estimated the magnitude of thermal relaxation time τ to range from 10^{-10}sec for gases at standard conditions to 10^{-14}sec for metals [15] with that for liquids [17] and insulators [6] falling within this range. With τ known, one can estimate the range of film thickness within which heat propagates as a wave.

Figures 4 to 6 show the time history of heat flux q/q_m at different positions in the films with $x_0/(\tau C) = 1, 3,$ and 10, respectively. It is seen that the absolute values of heat flux at the side wall $(\delta = 0)$ increase instantly from zero to a maximum upon an introduction of the transient followed by a decrease with time. The maximum value of q/q_m at $\beta = 0+$ is inversely proportional to the relaxation time, giving the values of 0.96, 2.88 and 9.6 at the left side wall of the films with $x_0/(\tau C) = 1$, 3, and 10, respectively. As the relaxation time τ approaches zero, the wall heat flux approaches infinite, in accordance with the parabolic heat conduction theory. Heat flux at the center of the film $(\delta = 0.5)$ does not occur until thermal wave-fronts from both sides of the film meet at the center. A thermal shock wave is induced at a certain position in the film when the wave-front arrives at and therefore results in a significant increase in temperature at this position.

It is of interest to note in Figure 4 that the first thermal shock wave in curve 1 occurs at an instant of $\beta = t/(2\tau) = 1$, corresponding to

$$\frac{x_0}{\tau C} = \frac{t}{2\tau} \text{ or } C = \frac{2x_0}{t}. \tag{24}$$

Since $2x_0$ represents the film thickness, one can use the equation to determine the speed of "second sound" C in the film, by measuring the time interval t followed by evaluating the relaxation time of medium using the expression

$$\tau = \frac{\alpha}{C^2} = \frac{\alpha t^2}{4x_0^2}. \tag{25}$$

4. Conclusion

Heat wave and hyperbolic heat transfer phenomena have been theoretically studied in a very thin film subjected to heating and cooling on either side, using the method of separation of variables. Results have been obtained for the propagation process, magnitude and shape of thermal waves and the range of film thickness and duration time of thermal wave propagation.

It is revealed that thermal waves appear only when τC is of the same order as or larger than one half the film thickness, namely $x_0/(\tau C) < 10$. The smaller the value of $x_0/(\tau C)$, the more pronounced the temperature waves. The criterion for the occurrence of thermal shock waves in a thin film is for the film thickness to be $2x_0 < 20\tau C = 20(\tau\alpha)^{0.5}$.

Wall heat flux undergoes a sharp change at an introduction of transient and the heat flux at a certain position has a step change when the wave-front arrives. After a very short period of time ($\beta > 5$), temperature waves disappear and a uniform heat flux is established throughout the film. The shorter the thermal relaxation time, the larger the maximum heat flux at zero time. As the thermal relaxation time approaches zero, the maximum heat flux approaches infinite in accordance with the parabolic heat conduction theory.

References

[1] K. J. Baumeister and T. D. Hamill, Hyperbolic heat-conduction equation - a solution for the semi-infinite body problem, J. Heat Trans. 91 (1969), 543-548.

[2] N. Bloembergen, H. Kurz, J. M. Kiu and R. Yen, Fundamentals of energy transfer during picosecond irradiation of silicon, Laser and Electron-beam Interactions with Solids, pp. 3-10, Elsevier Science Publ. Co., Inc., New York, 1982.

[3] J. P. Brazel and E. J. Nolan, Non-Fourier effects in the transmission of heat, Proceedings of 6th Conference on Thermal Conductivity, pp. 237-254, Dayton, OH, 1966.

[4] G. F. Carey and M. Tsai, Hyperbolic heat transfer with reflection, Numer. Heat Trans. 5 (1982), 309-327.

[5] S. H. Chan, D. Low and K. Mueller, Parabolic heat conduction in catalytic supported crystallites, AIChE J. 17 (1971), 1499-1506.

[6] M. Chester, Second sound in solids, Phys. Rev. 131 (1963), 2013-2015.

[7] J. I. Frankel, B. Vick and M. N. Ozisik, Flux formulation of hyperbolic heat conduction, J. Appl. Phys. 58 (1985), 3340-3345.

[8] D. E. Glass, M. N. Ozisik and B. Vick, Hyperbolic heat conduction with surface radiation, Int. J. Heat and Mass Trans. 28 (1985), 1823-1830.

[9] D. E. Glass, M. N. Ozisik, D. S. McRae and B. Vick, On the numerical solution of hyperbolic heat conduction, Numer. Heat Trans. 8 (1985), 497-504.

[10] T. T. Kao, Non-Fourier heat conduction in thin surface layers, J. Heat Trans. 99 (1977), 343-345.

[11] A. Kar, C. L. Chan and J. Mazumder, Comparative studies on nonlinear hyperbolic heat conduction for various boundary conditions: analytical and numerical solutions, J. Heat Trans. 114 (1992), 14-20.

[12] M. S. Kazimi and C. A. Erdman, On the interface temperature of two suddenly contacting materials, J. Heat Trans. 97 (1975), 615-617.

[13] M. Lewandowska, Hyperbolic heat conduction in the semi-infinite body with a time-dependent laser heat source, Int. J. Heat and Mass Trans. 37 (2001), 333-342.

[14] A. V. Luikov, Analytical Heat Diffusion Theory, pp. 245-248, Academic Press, New York, 1968.

[15] M. J. Maurer, Relaxation model for heat conduction in metals, J. Appl. Phys. 40 (1969), 5123-5130.

[16] M. J. Mourer and H. A. Thompson, Non-Fourier effects at high heat flux, J. Heat Trans. 95 (1973), 284-286.

[17] R. E. Nettleton, Relaxation theory of thermal conduction in liquids, Phys. Fluids 3 (1960), 216-225.

[18] M. N. Ozisik and S. Vick, Propagation and reflection of thermal waves in a finite medium, Int. J. Heat and Mass Trans. 27 (1984), 1845-1854.

[19] M. N. Ozisik and D. Y. Tzou, On the wave theory in heat conduction, J. Heat Trans. 116 (1994), 526-535.

[20] T. Q. Qin and C. L. Tien, Size effect on non-equilibrium laser heating of metals, J. Heat Trans. 115 (1993), 842-847.

[21] Y. Taitel, On the parabolic, hyperbolic and discrete formulation of the heat conduction equation, Int. J. Heat and Mass Trans. 15 (1972), 367-371.

[22] Z. M. Tan and W.-J. Yang, Non-Fourier heat conduction in a thin film subjected to a sudden temperature change on two sides, J. Non-equilibrium Thermodyn. 22 (1997), 75-87.

[23] Z. M. Tan and W.-J. Yang, Heat transfer during asymmetrical collision of thermal waves in a thin film, Int. J. Heat and Mass Trans. 40 (1997), 3999-4006.

[24] S. Torii and W.-J. Yang, Thermal wave propagation phenomena in thin solid films with temperature-dependent thermal conductivity, Int. J. Trans. Phenom. 4 (2002), 133-140.

[25] S. Torii and W.-J. Yang, Propagation phenomena during asymmetrical collision of thermal waves in a very thin film, Proceedings of the 1st International Conference on Heat Transfer, Fluid Mech. Thermodyn., pp. 409-414, South Africa, 2002.

[26] B. Vick and M. N. Ozisik, Growth and decay of a thermal pulse predicted by the hyperbolic heat conduction equation, J. Heat Trans. 105 (1983), 902-907.

[27] D. C. Wiggert, Analysis of early-time transient heat conduction by method of characteristics, J. Heat Trans. 99 (1977), 35-40.

[28] W. W. Yuen and S. C. Lee, Non-Fourier heat conduction in a semi-infinite solid subjected to oscillatory surface thermal disturbances, J. Heat Trans. 111 (1989), 178-181.

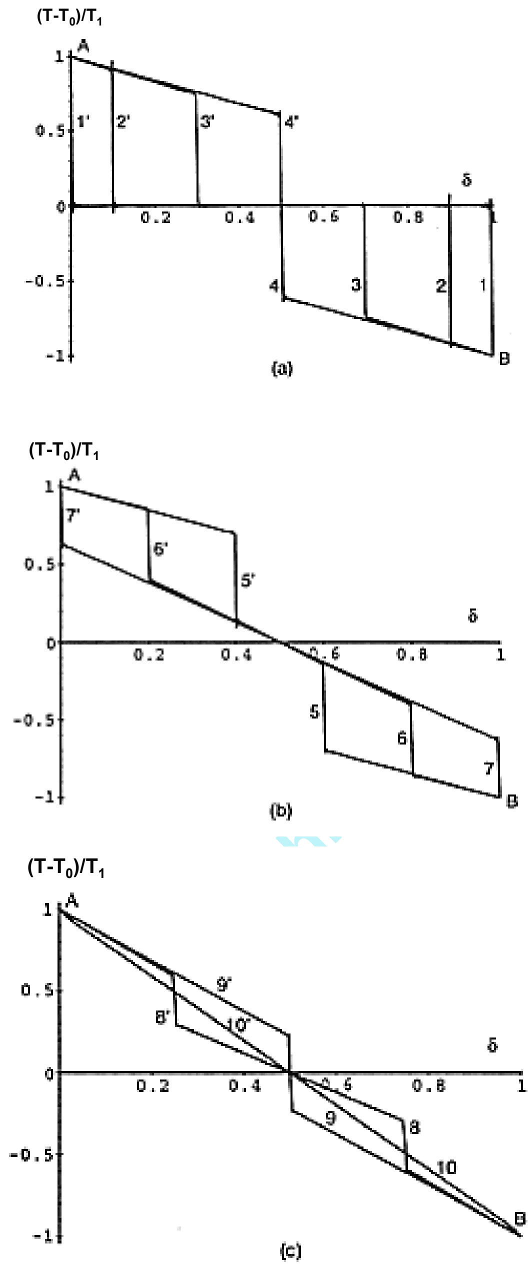

Figure 1. Instantaneous temperature distributions in the film at $x_0/\tau C = 1$. 1. $\beta = 0$, 2. $\beta = 0.1$, 3. $\beta = 0.3$, 4. $\beta = 0.5$, 5. $\beta = 0.6$, 6. $\beta = 0.8$, 7. $\beta = 1.0$, 8. $\beta = 1.25$, 9. $\beta = 1.5$, 10. $\beta = 5.0$.

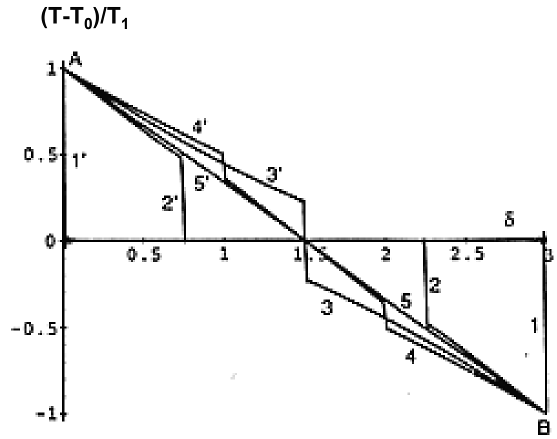

Figure 2. Instantaneous temperature distributions in the film at $x_0/\tau C = 3$. 1. $\beta = 0.00$, 2. $\beta = 0.75$, 3. $\beta = 1.50$, 4. $\beta = 2.00$, 5. $\beta = 5.00$.

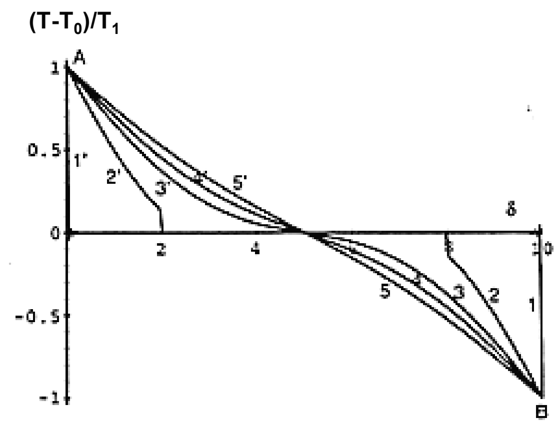

Figure 3. Instantaneous temperature distributions in the film at $x_0/\tau C = 10$. 1. $\beta = 0.0$, 2. $\beta = 2.0$, 3. $\beta = 5.0$, 4. $\beta = 7.0$, 5. $\beta = 10.0$.

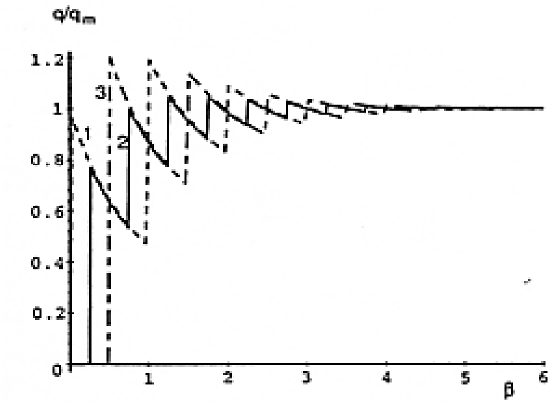

Figure 4. Time history of heat flux in the films at $x_0/\tau C = 1$.
1. $\delta = 0.00$, 2. $\delta = 0.25$, 3. $\delta = 0.50$.

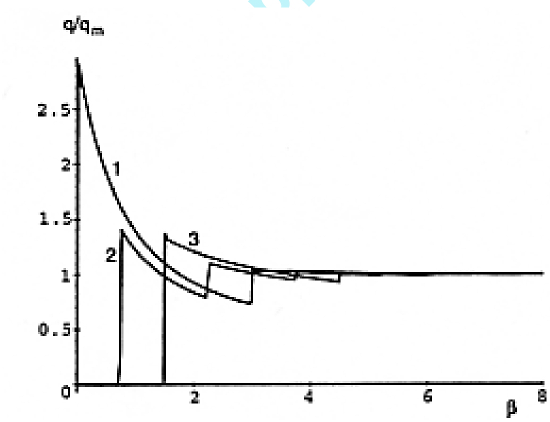

Figure 5. Time history of heat flux in the films at $x_0/\tau C = 3$.
1. $\delta = 0.00$, 2. $\delta = 0.75$, 3. $\delta = 1.50$.

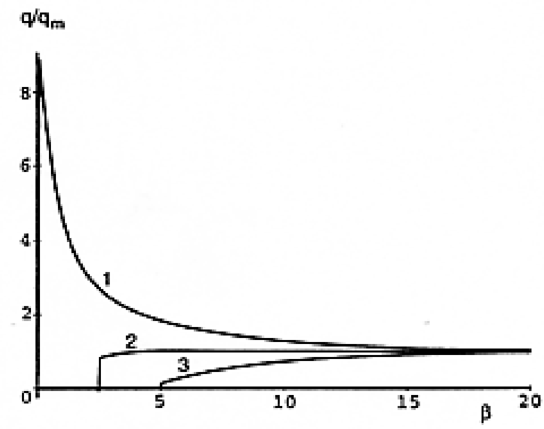

Figure 6. Time history of heat flux in the films at $x_0/\tau C = 10$.

1. $\delta = 0.0$, 2. $\delta = 2.5$, 3. $\delta = 5.0$.

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