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NUMERICAL COMPUTATION OF SOME ASPECTS OF 26 DECEMBER 2004 TSUNAMI ALONG THE WEST COAST OF THAILAND AND PENINSULAR MALAYSIA USING A CARTESIAN COORDINATE SHALLOW WATER MODEL

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Abstract

The Indonesian tsunami of 26 December 2004 was devastating along some parts of west coast of Thailand and Peninsular Malaysia. From this event it is evident that the west coast of Thailand and Malaysia is in a vulnerable position for tsunami surge due to an active seismic zone at Sumatra. A numerical model based on the non-linear shallow water equations in Cartesian coordinates was discussed and the results were presented in a previous study (Roy and Izani [8]). In the present study, we investigate some aspects of the tsunami along the west coast of Thailand and Malaysia using this model and for that purpose the source of Indonesian tsunami of 2004 is used. We compute wave propagation, maximum surge level and arrival time of maximum surge at the west coast of Peninsular Malaysia and Thailand. We also study the effect of different orientation of the source along the coastal belt. Our studies show that Cartesian coordinate shallow water model is capable of simulating different aspects of tsunami with reasonable accuracy. It is also found that orientation of the source has a significant effect on the

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maximum surge amplitude along the coastal belt. Further study is carried out to investigate the effect of convective terms and it is found that in the framework of shallow water equations, their effect is insignificant in tsunami computation.

1. Introduction

The Indonesian tsunami of 26 December 2004, having its source near Sumatra, was very severe and caused tremendous loss of life and property along the coastal belts surrounding the Indian Ocean including some parts of the west coast of Peninsular Malaysia and Thailand. From this event it is evident that some parts of the west coast of Peninsular Malaysia and Thailand are in vulnerable positions due to an active seismic zone near Sumatra. Accurate computation of the propagation, wave amplitude and run-up along the coastal belts of Indian Ocean is necessary for mitigating the sufferings of the people. So, it is essential that tsunamis are studied in detail and prediction models be developed to simulate different aspects of tsunami along the coastal belts.

Many analyses have been carried out on tsunami computation throughout the world. Some of these are Imamura and Gica [1], Kienle et al. [2], Kowalik and Whitmore [3], Kowalik et al. [4], Titov and Gonzalez [11] and Zahibo et al. [12]. Very recently some works on tsunami computation in the Indian Ocean have been carried out by Roy and Izani [8], Roy and Ismail [9], Roy et al. [10] and Rahman [6].

The west coast of southern Thailand and Peninsular Malaysia is facing towards the Sumatra Island where there is an active seismic zone (Figure 1). Penang Island in Malaysia and the Phuket in Thailand were lashed by high tsunami surges due to the Indonesian tsunami of 2004. From a literature survey, it is found that not much work has been done on tsunami modeling for this region. In the present study, a Cartesian coordinate shallow water model of Roy and Izani [8] has been applied to compute some aspects of tsunami that originated at Sumatra on 26 December 2004 along the west coasts of Peninsular Malaysia and Thailand (Figure 1). The analysis area is a rectangular region approximately between 2°N to 14°N and 91°E to 101.5°E. The model area includes the source region of Indonesian tsunami of 2004.

2. Shallow Water Model

2.1. Depth averaged shallow water equations

A system of rectangular Cartesian coordinates is used in which the origin, O, is in the undisturbed sea surface (MSL) and Oz is directed vertically upwards. We consider the displaced position of the free surface as $z = \zeta(x, y, t)$ and the sea floor as $z = -h(x, y)$ so that, the total depth of the fluid layer is $\zeta + h$. The vertically integrated shallow water equations are

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x}[(\zeta + h)u] + \frac{\partial}{\partial y}[(\zeta + h)v] = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv = -g\frac{\partial \zeta}{\partial x} + \frac{T_x - F_x}{\rho(\zeta + h)}, \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + fu = -g\frac{\partial \zeta}{\partial y} + \frac{T_y - F_y}{\rho(\zeta + h)}, \tag{3}$$

where

$u, v =$ velocity components of sea water in x and y directions, $\mathrm{m\,s^{-1}}$

$f =$ Coriolis parameter, $\mathrm{s^{-1}} = 2\Omega \sin\varphi$

$\Omega =$ angular speed of earth, $\mathrm{rad\,s^{-1}}$

$\varphi =$ latitude of a location in the analysis area, rad

$g =$ gravity acceleration, $\mathrm{m\,s^{-2}}$

$T_x, T_y =$ components of wind stress, $\mathrm{N\,m^{-2}}$

$F_x, F_y =$ components of bottom friction, $\mathrm{N\,m^{-2}}$

$h =$ ocean depth from the mean sea level, m.

Since our study is on simulation of tsunami only, we neglect the wind stress terms T_x and T_y. On the other hand, the parameterization of the bottom stress is done by the depth averaged velocity components:

$$F_x = \rho C_f u(u^2 + v^2)^{1/2} \quad \text{and} \quad F_y = \rho C_f v(u^2 + v^2)^{1/2}, \tag{4}$$

where

C_f = the friction coefficient

ρ = water density, kg m^{-3}.

For numerical treatment it is convenient to express the equations (2) and (3) in the flux form by using the equation (1). Using the parameterization formulas (4), the equations (1)-(3) may be expressed as

$$\frac{\partial \zeta}{\partial t} + \frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} = 0, \tag{5}$$

$$\frac{\partial \tilde{u}}{\partial t} + \frac{\partial (u\tilde{u})}{\partial x} + \frac{\partial (v\tilde{u})}{\partial y} - f\tilde{v} = -g(\zeta + h)\frac{\partial \zeta}{\partial x} - \frac{C_f \tilde{u}(u^2 + v^2)^{1/2}}{\zeta + h}, \tag{6}$$

$$\frac{\partial \tilde{v}}{\partial t} + \frac{\partial (u\tilde{v})}{\partial x} + \frac{\partial (v\tilde{v})}{\partial y} + f\tilde{u} = -g(\zeta + h)\frac{\partial \zeta}{\partial y} - \frac{C_f \tilde{v}(u^2 + v^2)^{1/2}}{\zeta + h}, \tag{7}$$

where $(\tilde{u}, \tilde{v}) = (\zeta + h)(u, v)$.

In the bottom stress terms of (6) and (7), u and v have been replaced by $\tilde{u}$ and $\tilde{v}$ in order to solve the equations in a semi-implicit manner.

2.2. Boundary conditions

Other than the west coast of Peninsular Malaysia and Thailand, the boundaries are considered as straight lines in the sea. The southern and northern open sea boundaries lie parallel to x-axis and the western open sea boundary lies parallel to y-axis. The radiation boundary conditions for the southern, northern and western open sea boundaries, due to Roy [7], are

$$u - (g/h)^{1/2}\zeta = 0 \text{ at the west open boundary; parallel to } y\text{-axis} \tag{8}$$

$$v + (g/h)^{1/2}\zeta = 0 \text{ at the south open boundary; along } x\text{-axis} \tag{9}$$

$$v - (g/h)^{1/2}\zeta = 0 \text{ at the north open boundary; parallel to } x\text{-axis.} \tag{10}$$

The coastal belts of the main land and islands are the closed boundaries where the normal components of the flow are taken as zero.

2.3. Initial condition (tsunami source generation)

The generation mechanism of the 26 December 2004 tsunami was mainly due to a static sea floor uplift caused by an abrupt slip at the India/Burma plate interface. A detailed description of the estimation of the extent of the earthquake rapture as well as the maximum uplift and subsidence of the seabed is given in Kowalik et al. [4] and this estimation is based on Okada [5]. From the deformation contour, it is seen that the estimated source zone is between 92°E to 97°E and 3°N to 10°N, elongated along the fault which is aligned from south-east to north-west, with a maximum uplift of 507 cm at the west and maximum subsidence of 474 cm at the east (Figure 4 of Kowalik et al. [4]). The uplift to subsidence is approximately from west to east. Following Kowalik et al. [4], the disturbance in the form of rise and fall of sea surface is assigned as the initial condition in the model with a maximum rise of 5 m to maximum fall of 4.75 m. The vertical cross-section of the source along the 155th gridline is shown in Figure 2. This gridline is parallel to x-axis and passing through the Phuket region. In all other regions the initial sea surface elevations are taken as zero. The initial x and y components of velocity are also taken as zero throughout the model area.

3. Grid Generation

A rectangular grid system is generated in the analysis area using a set of equidistant straight lines parallel to x-axis and a set of equidistant straight lines parallel to y-axis. The space between any two consecutive gridlines parallel to y-axis is Δx and that between any two consecutive gridlines parallel to x-axis is Δy. Let there be M gridlines parallel to y-axis and N gridlines parallel to x-axis so that the total number of grid points are $M \times N$. We define the grid points $(x_i,\ y_j)$ in the domain by

$$x_i = (i - 1)\Delta x, \quad i = 1, 2, 3, \ldots, M, \tag{11}$$

$$y_j = (j - 1)\Delta y, \quad j = 1, 2, 3, \ldots, N. \tag{12}$$

The sequence of discrete time instants is given by

$$t_k = k\Delta t, \quad k = 1, 2, 3, \ldots. \tag{13}$$

4. Numerical Scheme and Model Data Set-up

The governing equations (5)-(7) together with the boundary conditions (8)-(10) are discretized by finite difference (forward in time and central in space) and are solved by a conditionally stable semi-implicit method using a staggered grid system which is similar to Arakawa C system.

The origin of the Cartesian coordinate system is at O (3.125°N, 101.5°E) x-axis is directed towards west at an angle 15° with the latitude line through O and the y-axis is directed towards north inclined at an angle 15° with the longitude line through O. The grid size of the rectangular mesh is given by $\Delta x = \Delta y = 4\text{ km}$ and number of grids in x-direction and y-direction are respectively $M = 230$ and $N = 319$ so that there are 73370 grid points in the computational domain. The time step Δt is taken as 10 seconds and this satisfies the CFL criterion and thus ensures the stability of the numerical scheme. The value of the friction coefficient C_f is taken as 0.0033 throughout the model area. The depth data for the model area are collected from the Admiralty bathymetric charts.

5. Results and Discussions

In computing different aspects of tsunami, we solve the set of equations and boundary conditions using finite difference method and the results are presented below.

5.1. Propagation of tsunami towards Phuket

The propagation of the tsunami wave, which is originated at the source in the form sea level rise and fall (Figure 2), has been studied. The propagation of the disturbance along the 155th gridline, parallel to x-axis, starting from the western open boundary (725 km from Phuket) to the coast of Phuket is shown in Figure 3. The figure shows the disturbance pattern at 30 min, 60 min, 90 min and 103 min (time of 1st crest at Phuket) along the 155th gridline. Considering the time of first sea surface rise as time of arrival of tsunami, we see that the wave reaches the locations at distances 375 km, 175 km, 25 km from Phuket at 30 min, 60 min, 90 min, respectively. Finally, the first crest reaches the Phuket coast at 103 min. Thus the tsunami gradually propagates towards Phuket with time and attains a maximum of 3.5 m at the coast at 103 min (Figure 3b).

5.2. Estimation of maximum water level

Figure 4 depicts the maximum water level contours along the coast from Penang Island to Phuket; the surge amplitude is increasing along the coast from south to north. The maximum water level at Penang Island is from 2 m to 3.5 m, where as the same at Phuket region is 3.5 m to 7 m. The surge amplitude is increasing very fast near the shoreline everywhere. The computed water levels indicate that the north and west coasts of Penang Island is vulnerable for stronger surges. Similarly the north-west part of Phuket is at risk of highest surge due to the source at Sumatra.

5.3. Surge sensitivity due to source orientation

The fault line of the source of Indonesian tsunami 2004 is inclined to a longitude line at an angle approximately 15°. Thus the source is elongated from south-east to north-west. In order to investigate the effect of the sources with same intensity but different orientations, simulations have been carried out for two different orientations which are as follows: the source rotated at an angle 22.5° clockwise (source oriented to right) and the source rotated at an angle 22.5° anti-clockwise (source oriented to left); henceforth will be abbreviated as SOR and SOL, respectively. The maximum water levels along coastal belts associated with the SOR and SOL are presented in Figure 5. There is over all increase of the water level along the entire coast with the maximum level (15 m) at the middle due to SOR (Figure 5a). The highest intensity attained along the north-west coast of Phuket is 6.5 m and at Penang the same is 4.5 m due to SOR. On the other hand, lowest intensity is attained along the entire coast due to SOL (Figure 5b). For this orientation the maximum water levels along the coasts of Phuket and Penang are 5 m and 2 m, respectively. Thus, the tsunami response along the west coast of Malaysia and Thailand is sensitive to the orientations of the source and the coast will be more affected due to SOR.

5.4. Arrival time of maximum surge

The arrival time of the maximum surge is an important parameter in the tsunami prediction and warning. Figure 6 shows the contour plot of time, in minutes, for attaining +0.1 m sea level rise at each grid point in the model domain. Thus considering the 0.1 m sea level rise as the arrival of tsunami, it is seen that after initiating the source the disturbance propagates gradually towards the coast (Figure 6). The arrival times of tsunami at Phuket and Penang are approximately 90 min to 100 min and 230 min, respectively. The times of attaining maximum elevations along these regions are also computed and it is found that this time at each location is approximately 10 to 15 min later than the time of attaining +0.1 m. Thus, the times of attaining maximum surges at Phuket and Penang are 100 min to 110 min and 240 min, respectively.

6. Investigation on the Effect of Non-linear Terms

Studies are carried out to investigate the effect of non-linear terms of the momentum equations. Simulations are performed with the corresponding linear model by excluding the convective terms and the results are compared with those of the non-linear model.

Figure 7 shows the propagation of the disturbance along the 155th gridline, parallel to x-axis, starting from the western open boundary (725 km from Phuket) towards Phuket for both the models. Solid line and dash-dot line represent the disturbance patterns computed through non-linear and linear models respectively at 60 min and 103 min (time of attaining maximum surge at Phuket) along the 155th gridline. Considering the first sea surface rise as the time of arrival of tsunami, it is seen that the tsunami reaches the location at distance 175 km from Phuket at 60 min both the cases. At 103 min the first crest associated with both models hits the coast. Thus, the computed propagation patterns associated with linear and non-linear models are almost identical. This indicates the insignificance of convective terms and confirms that their effect is negligible in computing tsunami travel time. However, the maximum water levels at the time (103 min) of hitting the coast are 3.5 m and 3.2 m associated with the non-linear and linear models, respectively (Figure 7b). Thus, non-linear terms are found to be weakly significant for the wave height near the coast.

7. Comparison Between Model Results and Observed Data

The computed arrival time of high surge is compared with the data available in U.S.G.S. website. According to U.S.G.S. report the tsunami waves reached at Phuket in two hours time after the earthquake [(http://staff.aist.go.jp/kenji.satake/Sumatra-E.html) (Tsunami travel time in hours for the entire Indian Ocean)] and the computed time of attaining maximum surge is 100 min to 110 min. Thus the computed time is almost identical with the website data.

On the other hand, the authors of this paper made a survey to estimate the water levels at different locations of Penang Island and time of arrival of high waves along the coastal belts. On the basis of the authors' survey and the reports in newspapers, the arrival time of high surge at different locations of north coast was approximately between 1310 and 1345 (Malaysian time). The rapture time was 0859 (Malaysian time) and so the propagation time was between 4 hr 11 min and 4 hr 46 min. The computed arrival time of high surge of tsunami at the north coast of Penang Island is 245 min, that is, 4 hr 5 min. The above website data also confirms the fact that the arrival time of maximum surge at Penang is between 3 hr 30 min and 4 hours. So the computed time of arrival of high tsunami surge at the north coast of Penang is in reasonable agreement with the observation.

Conclusion

Numerical simulation of some aspects of tsunami is performed in the framework of the non-linear shallow water equations in Cartesian coordinates along the west coast of Thailand and Malaysia. Simulation of the maximum surge along the coast associated with several orientations of the source has been performed. This analysis demonstrates that the maximum surge along the coastal belt is sensitive to the orientation of the source.

The computed arrival times of tsunami at different locations are in good agreement with the observed arrival times around the Phuket and Penang.

This numerical model can be applied to any localized coastal region for simulating tsunami hazards and to provide a tool for developing effective warning system.

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Figure 1. Model domain including west coast of Thailand, Peninsular Malaysia and source zone west of North Sumatra.

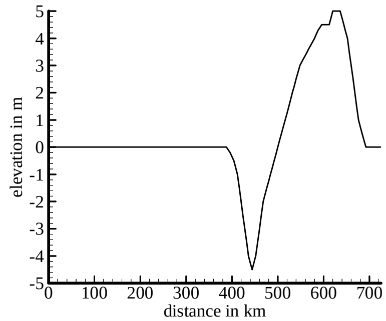


Figure 2. Vertical cross-section of the source associated with the Indonesian tsunami of 26 December 2004 along 155th gridline; the distance is measured from Phuket.

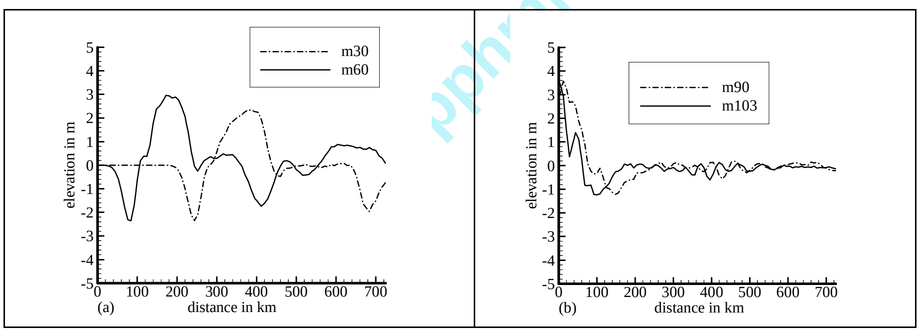


Figure 3. Disturbance pattern at different instants of time along 155th gridline: (a) 30 min, 60 min; (b) 90 min, 103 min.

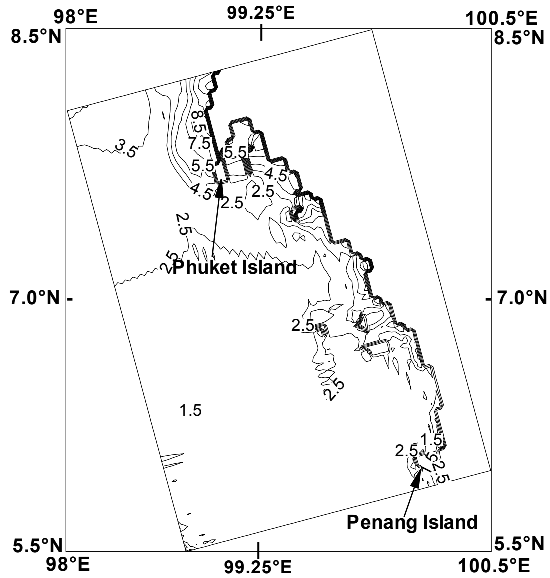
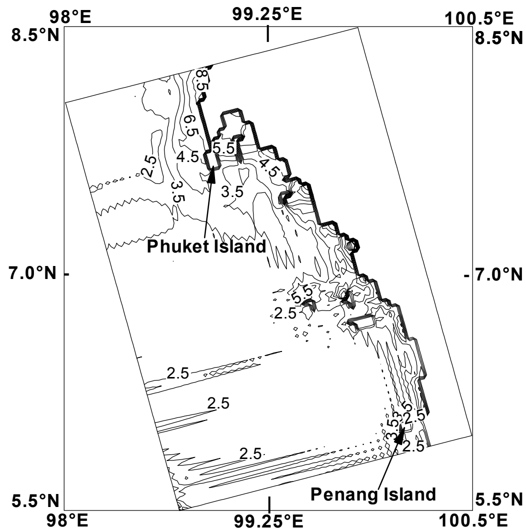
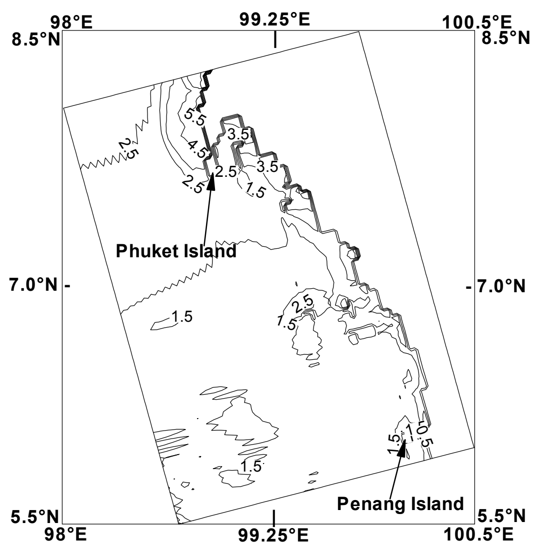


Figure 4. Contour of maximum water levels along the west coast of Malaysia and Thailand (from Penang Island to Phuket) due to the source.



(a)



(b)

Figure 5. Contour of maximum water levels along the west coast of Malaysia and Thailand (from Penang Island to Phuket) due to the source orientations: (a) oriented to right (SOR), (b) oriented to left (SOL).

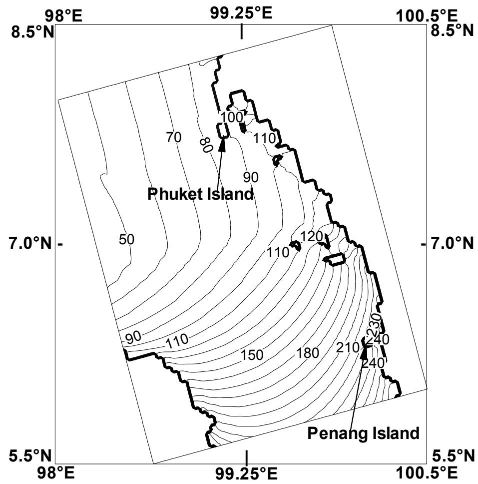


Figure 6. Contour showing tsunami propagation time in minutes, sea level rise of +0.1 m is considered as the arrival of tsunami.

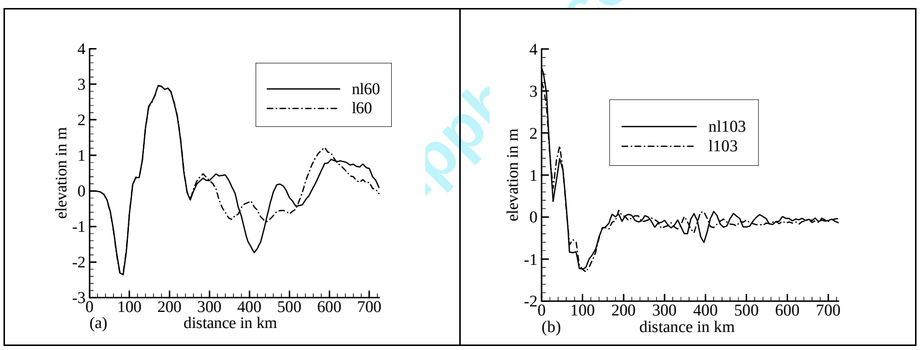


Figure 7. Computed disturbance patterns associated with linear and non-linear models along 155th gridline (the distance along the gridline is measured from Phuket): (a) at 60 min, (b) 103 min (time of hitting the coast).

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