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EXISTENCE OF PERIODIC SOLUTIONS OF STAGE-STRUCTURED NONAUTONOMOUS COOPERATIVE SYSTEM WITH DELAY

XIN MIN WU*

Department of Mathematics
Shoayang University
Shaoyang, Hunan 422000
P. R. China
e-mail: wxm6138@163.com

Abstract

By using Mawhin continuation theory of coincidence degree theory, we derive the existence of periodic solutions of stage-structured nonautonomous cooperative system with delay.

1. Introduction

In a natural world, there exist many individuals of species which experience two stages in the lifetime, i.e., immature stage and mature stage, for example, animal and amphibian. Therefore, to make the models more practical, species are usually considered by dividing the individuals into two stages. Recently, there exist many papers [1, 4] in the literature which investigate some stage-structured predator-prey systems, however, the papers which investigate stage-structured cooperative systems are scarce. In this paper, we study stage-structured nonautonomous

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cooperative system of two species. Consider the following model:

$$\begin{cases} \dot{X}_1(t) = -a_1(t)x_1(t) + b_1(t)x_2(t) - \beta_1(t)x_1(t)x_3(t), \\ \dot{X}_2(t) = a_2(t)x_1(t) - b_2(t)x_2(t) - C(t)x_2^2(t), \\ \dot{X}_3(t) = x_3(t)(-d(t) - e(t)x_3(t)) + \beta_2(t)x_1(t-T)x_3(t-T). \end{cases} \tag{1.1}$$

Here $x_1(t)$ and $x_2(t)$ are immature and mature population densities of prey species respectively, and $x_3(t)$ represents the population density of predator species. In our this model, the predator species can just prey on immature individuals of prey species, and this is natural because the mature individuals have developed some protective instinct. All coefficients $a_i(t)$, $b_i(t)$, $\beta_i(t)$; $(i = 1, 2)$, $c(t)$, $d(t)$, $e(t)$ are continuous functions. $T > 0$ is digest delay time.

2. The Existence of a Positive Periodic Solution

In this section, based on Mawhin's continuation theorem, we shall show the existence of at least one positive periodic solution of system (1.1) to do so, we need to make some preparations.

Let X and Y be real Banach spaces, let $L : DomL \subset Z \to Y$ be a Fredholm mapping of index zero, and let $P : X \to X$, $Q : y \to y$ be continuous projectors such that $\operatorname{Im} p = KerL$, $KerQ = \operatorname{Im} L$, and $X = KerL \oplus KerP$, $y = \operatorname{Im} L \oplus \operatorname{Im} Q$. Denote by Lp the restriction of L to $DomL \cap KerP$, $Kp : \operatorname{Im} L \to KerP \cap DomL$ the inverse (to Lp), and $T : \operatorname{Im} Q \to KerL$ an isomorphism of $\operatorname{Im} Q$ onto $KerL$.

For convenience, we introduce Mawhin's continuation theorem [2, p. 40] as follows.

Lemma 1. *Let $\Omega \subset X$ be an open bounded set and let $N : X \to y$ be a continuous operator which is L-compact on $\overline{\Omega}$ (i.e., $QN : \overline{\Omega} \to y$ and $Kp(I - Q)N : \overline{\Omega} \to y$ are compact). Assume the following*:

(i) *For each $\lambda \in (0, 1)$, $x \in \partial\Omega \cap DomL$, $Lx \neq \lambda Nx$.*

(ii) *For each* $x \in 2\Omega \cap KerL,\ QNx \neq 0.$

(iii) $\deg\{JQN, \Omega \cap KerL, 0\} \neq 0.$

Then $Lx = Nx$ *has at least one solution in* $\overline{\Omega} \cap DomL$. *In what follows, we use the following notation*:

$$\bar{f} = \frac{1}{w}\int_0^w f(t)dt, \quad f^l = \min_{t\in[0,w]} |f(t)|, \quad f^M = \max_{t\in[0,w]} |f(t)|,$$

where f is a periodic continuous function with period $w > 0$.

(H_1) All the coefficients in system (1.1) are positive continuous w-periodic functions.

Now we state our fundamental theorem about the existence of a positive w-periodic solution of system (1.1).

Theorem. *In addition to assumption* (H_1), *we assume the following*:

$$(H_2)\ b_2^l \beta_2^l \left[\frac{a_2^l b_2^l}{a_1^M + \beta_1^M \left(\frac{\beta_2^M}{e^l} \bullet \frac{a_2^M}{c^l} \right) \left(\frac{b_2^M}{a_1^l} \right)^2} - b_2^M \right]$$

$$> C^M d^M \left[a_1^M + \beta_1^M \left(\frac{\beta_2^M a_2^M}{e^l c^l} \right) \left(\frac{b_2^M}{a_1^l} \right)^2 \right].$$

Then system (1.1) *has at least one positive w-periodic solution.*

Proof. Consider the system

$$\begin{cases} u_1'(t) = -a_1(t) + b_1(t)e^{u_2(t)-u_1(t)} - \beta_1(t)e^{u_3(t)}, \\ u_2'(t) = a_2(t)e^{u_1(t)-u_2(t)} - b_2(t) - c(t)e^{u_2(t)}, \\ u_3'(t) = -d(t) - e(t)e^{u_3(t)} + \beta_2(t)e^{u_1(t-T)+u_3(t-T)-u_3(t)}, \end{cases} \tag{2.1}$$

where all parameters are the same as those in system (1.1). It is easy to see that if system (2.1) has an w-periodic solution $(u_1^*(t), u_2^*(t), u_3^*(t))^T$, then $(e^{u_1^*}, e^{u_2^*}, e^{u_3^*})^T$ is a positive w-periodic solution of system (1.1). Therefore, for system (1.1) to have at least one positive w-periodic solution, it is sufficient that (2.1) has at least one w-periodic solution. To apply lemma to system (2.1), we first define

$$X = Z = \{u(t) = (u_1(t), u_2(t), u_3(t))^T \in (R, R^3), u(t+w) = u(t)\}$$

and

$$\| u \| = \| (u_1(t), u_2(t), u_3(t))^T \| = \sum_{i=1}^{3} \max_{t \in [0, w]} | u_i(t) |$$

for any $u \in X$ (or z). Then X and Z are Banach spaces with the norm $\| \cdot \|$. Let

$$Nu = \begin{bmatrix} -a_1(t) + b_1(t)e^{u_2(t)-u_1(t)} - \beta_1(t)e^{u_3(t)} \\ a_2(t)e^{u_1(t)-u_2(t)} - b_2(t) - c(t)e^{u_2(t)} \\ -d(t) - e(t)e^{u_3(t)} + \beta_2(t)e^{u_1(t-T)+u_3(t-T)-u_3(t)} \end{bmatrix},$$

$$Lu = u' = \frac{du(t)}{dt}, \quad pu = \frac{1}{w}\int_0^w u(t)dt, \quad u \in X,$$

$$Q_Z = \frac{1}{w}\int_0^w Z(t)dt, \quad z \in Z.$$

Then it follows that $KerL = R^3$, $\operatorname{Im} L = \left\{z \in Z : \int_0^w Z(t)dt = 0\right\}$ is closed in Z, $\dim KerL = 3 = \operatorname{codim} \operatorname{Im} L$ and P, Q are continuous projectors such that

$$\operatorname{Im} P = KerL, \quad KerQ = \operatorname{Im} L = \operatorname{Im}(I - Q).$$

Therefore, L is a Fredholm mapping of index zero. Furthermore, the generalized inverse (to L) $Kp : \operatorname{Im} L \to KerP \cap DomL$ is given by

$$Kp(z) = \int_0^t Z(t)ds - \frac{1}{w}\int_0^w \int_0^t z(s)dsdt.$$

Thus

$$QNu = \begin{bmatrix} \frac{1}{w}\int_0^w F_1(s)ds \\ \frac{1}{w}\int_0^w F_2(s)ds \\ \frac{1}{w}\int_0^w F_3(s)ds \end{bmatrix}$$

and

$$Kp(I-Q)Nu = \begin{bmatrix} \int_0^t F_1(s)ds - \frac{1}{w}\int_0^w \int_0^t F_1(s)dsdt + \left(\frac{1}{2} - \frac{t}{w}\right)\int_0^w F_1(s)ds \\ \int_0^t F_2(s)ds - \frac{1}{w}\int_0^w \int_0^t F_2(s)dsdt + \left(\frac{1}{2} - \frac{t}{w}\right)\int_0^w F_2(s)ds \\ \int_0^t F_3(s)ds - \frac{1}{w}\int_0^w \int_0^t F_3(s)dsdt + \left(\frac{1}{2} - \frac{t}{w}\right)\int_0^w F_3(s)ds \end{bmatrix},$$

where

$$F_1(s) = -a_1(s) + b_1(s)e^{u_2(s)-u_1(s)} - \beta_2(s)e^{u_3(s)},$$

$$F_2(s) = a_2(s)e^{u_1(s)-u_2(s)} - b_2(s) - c(s)e^{u_2(s)}$$

and

$$F_3(s) = -d(s) - e(s)e^{u_3(s)} + \beta_2(s)e^{u_1(s-T)+u_3(s-T)-u_3(s)}.$$

Obviously, QN and $Kp(I-Q)N$ are continuous. It is not difficult to show that $Kp(I-Q)N(\overline{\Omega})$ is compact for any open bounded $\Omega \subset X$ by using the Arzela-Ascoli theorem. Moreover, $QN(\overline{\Omega})$ is clearly bounded. Thus, N is L-compact on $\overline{\Omega}$ with any open bounded set $\Omega \subset X$.

Now we reach the point where we search for an appropriate open bounded subset Ω for the application of the theorem. Corresponding to the operator equation $Lx = \lambda Nx$, $\lambda \in (0, 1)$, we have

$$\begin{cases} u_1'(t) = \lambda\lfloor -a_1(t) + b_1(t)e^{u_2(t)-u_1(t)} - \beta_1(t)e^{u_3(t)} \rfloor, \\ u_2'(t) = \lambda\lfloor a_2(t)e^{u_1(t)-u_2(t)} - b_2(t) - c(t)e^{u_2(t)} \rfloor, \\ u_3'(t) = \lambda\lfloor -d(t) - e(t)e^{u_3(t)} + \beta_2(t)e^{u_1(t-T)+u_3(t-T)-u_3(t)} \rfloor. \end{cases} \tag{2.2}$$

Assume that $u = u(t) \in X$ is a solution of system (2.2) for a certain $\lambda \in (0, 1)$. Because of $(u_1(t), u_2(t), u_3(t))^T \in X$, there are $\xi_i, \eta_i \in [0, w]$ such that

$$u_i(\xi_i) = \max_{t \in [0, w]} u_i(t), \quad u_i(\eta_i) = \min_{t \in [0, w]} u_i(t), \quad i = 1, 2, 3.$$

It is clear that

$$u_i'(\xi_i) = 0, \quad u_i'(\eta_i) = 0, \quad i = 1, 2, 3.$$

From this and system (2.2), we obtain

$$\begin{cases} -a_1(\xi_1) + b_2(\xi_1) e^{u_2(\xi_1) - u_1(\xi_1)} - \beta_1(\xi_1) e^{u_3(\xi_1)} = 0 & (2.3) \\ a_2(\xi_2) e^{u_1(\xi_2) - u_2(\xi_2)} - b_2(\xi_2) - c(\xi_2) e^{u_2(\xi_2)} = 0 & (2.4) \\ -d(\xi_3) - e(\xi_3)^{u_3(\xi_3)} + \beta_2(\xi_3) e^{u_1(\xi_3 - T)} e^{u_3(\xi_3 - T)} e^{-u_3(\xi_3)} = 0 & (2.5) \end{cases}$$

and

$$-a_1(\eta_1) + b_2(\eta_1) e^{u_2(\eta_1) - u_1(\eta_1)} - \beta_1(\eta_1) e^{u_3(\eta_1)} = 0 \tag{2.6}$$

$$a_2(\eta_2) e^{u_1(\eta_2) - u_2(\eta_2)} - b_2(\eta_2) - c(\eta_2) e^{u_2(\eta_2)} = 0 \tag{2.7}$$

$$-d(\eta_3) - e(\eta_3) e^{u_3(\eta_3)} + \beta_2(\eta_3) e^{u_1(\eta_3 - T) + u_3(\eta_3 - T) - u_3(\eta_3)} = 0. \tag{2.8}$$

From (2.3), we get

$$b_2(\xi_1) e^{u_2(\xi_1)} - a_1(\xi_1) e^{u_1(\xi_1)} = \beta_1(\xi_1) e^{u_3(\xi_1)} > 0,$$

that is,

$$a_1^l e^{u_1(\xi_1)} \leq b_2(\xi_1) e^{u_2(\xi_1)} \leq b_2^M e^{u_2(\xi_2)}.$$

Hence

$$e^{u_1(\xi_1)} \leq \frac{b_2^M}{a_1^l} e^{u_2(\xi_2)}. \tag{2.9}$$

From (2.3) and (2.8), we have

$$c(\xi_2)e^{2u_2(\xi_2)} = a_2(\xi_2)e^{u_1(\xi_2)} - b_2(\xi_2)e^{u_2(\xi_2)}$$

$$\leq a_2(\xi_2)e^{u_1(\xi_1)} < \frac{a_2^M b_2^M}{a_1^l} e^{u_2(\xi_2)}.$$

Hence

$$e^{u_2(\xi_2)} \leq \frac{a_2^M b_2^M}{c^l a_1^l} \stackrel{def}{=} d_2. \tag{2.10}$$

From (2.8) and (2.9), we have

$$e^{u_1(\xi_1)} \leq \frac{b_2^M}{a_1^l} e^{u_2(\xi_2)} \leq \frac{a_2^M}{c^l}\left(\frac{b_2^M}{a_1^l}\right)^2 \stackrel{def}{=} d_1. \tag{2.11}$$

From (2.4), we get

$$e^l e^{2u_3(\xi_3)} \leq e(\xi_3)e^{2u_3(\xi_3)} = \beta_2(\xi_2)e^{u_1(\xi_3-T)+u_3(\xi_3-T)} - d(\xi_3)e^{u_3(\xi_3)}$$

$$\leq \beta_2^M e^{u_1(\xi_1)+u_3(\xi_3)},$$

which together with (2.9) implies

$$e^{u_3(\xi_3)} \leq \frac{\beta_2^M}{e^l} e^{u_1(\xi_1)} \leq \frac{\beta_2^M}{e^l} \cdot \frac{a_2^M}{c^l}\left(\frac{b_2^M}{a_1^l}\right)^2 \stackrel{def}{=} d_3. \tag{2.12}$$

Multiplying (2.5)-(2.7) by $e^{u_i(t)}$ $(i = 1, 2, 3)$ gives

$$-a_1(\eta_1)e^{u_1(\eta_1)} + b_2(\eta_1)e^{u_2(\eta_1)} - \beta_1(\eta_1)e^{u_3(\eta_1)} = 0 \tag{2.13}$$

$$a_2(\eta_2)e^{u_1(\eta_1)} - b_2(\eta_2)e^{u_2(\eta_2)} - c(\eta_2)e^{2u_2(\eta_2)} = 0 \tag{2.14}$$

$$-d(\eta_3)e^{u_3(\eta_3)} - e(\eta_3)e^{2u_3(\eta_3)} + \beta_2(\eta_3)e^{u_1(\eta_3-T)+u_3(\eta_3-T)} = 0. \tag{2.15}$$

Equation (2.13) implies that

$$c^M e^{2u_2(\eta_2)} + b_2^M e^{u_2(\eta_2)} > a_2^l e^{u_1(\eta_2)} > a_2^l e^{u_1(\eta_1)}.$$

From (2.12) and (2.13), we have

$$\begin{aligned} b_2^l e^{u_2(\eta_2)} < b_2(\eta_1) e^{u_2(\eta_1)} &= a_1(\eta_1) e^{u_1(\eta_1) + \beta_1(\eta_1) e^{u_3(\eta_1)+u_1(\eta_1)}} \\ &< a_1^M e^{u_1(\eta_1)} + \beta_1^M e^{u_3(\xi_3)+u_1(\eta_1)} \\ &\le \left\{ a_1^M + \beta_1^M \left[\frac{\beta_2^M}{e^l} \cdot \frac{a_2^M}{c^l} \left(\frac{b_2^M}{a_1^l} \right)^2 \right] \right\} e^{u_1(\eta_1)}, \end{aligned}$$

$$e^{u_1(\eta_1)} > \frac{b_2^l}{\left\{ a_1^M + \beta_1^M \left[\frac{\beta_2^M}{e^l} \cdot \frac{a_2^M}{c^l} \left(\frac{b_2^M}{a_2^l} \right)^2 \right] \right\}} e^{u_2(\eta_2)}. \tag{2.16}$$

Substituting (2.16) into (2.15) gives

$$c^M e^{2u_2(\eta_2)} + b_2^M e^{u_2(\eta_2)} > \frac{a_2^l b_2^l}{a_1^M + \beta_1^M \left[\frac{\beta_2^M}{e^l} \cdot \frac{a_2^M}{c^l} \left(\frac{b_2^M}{a_1^l} \right)^2 \right]} e^{u_2(\eta_2)}.$$

Hence

$$e^{u_2(\eta_2)} > \frac{1}{c^M} \left\{ \frac{a_2^l b_2^l}{a_1^M + \beta_1^M \left[\frac{\beta_2^M}{e^l} \cdot \frac{a_2^M}{c^l} \left(\frac{b_2^M}{a_1^l} \right)^2 \right]} - b_2^M \right\} \overset{def}{=} p_2 > 0. \tag{2.17}$$

From (2.14) and (2.16), we obtain

$$e^{u_1(\eta_1)} > \frac{b_2^l}{a_1^M + \beta_1^M \left[\frac{\beta_2^M}{e^l} \cdot \frac{a_2^M}{c^l} \left(\frac{b_2^M}{a_1^l} \right)^2 \right]} e^{u_2(\eta_2)}$$

$$> \frac{b_2^l}{a_1^M + \beta_1^M\left[\frac{\beta_2^M}{e^l}\cdot\frac{a_2^M}{c^l}\left(\frac{b_2^M}{a_1^l}\right)^2\right]}$$

$$\cdot\frac{1}{c^M}\left\{\frac{a_2^l b_2^l}{a_1^M + \beta_1^M\left[\frac{\beta_2^M}{e^l}\cdot\frac{a_2^M}{c^l}\left(\frac{b_2^M}{a_1^l}\right)^2\right]} - b_2^M\right\} \stackrel{def}{=} p_1 > 0. \quad (2.18)$$

From (2.14), we have

$$\beta_2^l e^{u_1(\eta_1)+u_3(\eta_3)} \le \beta_2(\eta_3)e^{u_1(\eta_3-T)+u_3(\eta_3-T)} = d(\eta_3)e^{u_3(\eta_3)} + e(\eta_3)e^{2u_3(\eta_3)}$$

$$\le d^M e^{u_3(\eta_3)} + e^M e^{2u_3(\eta_3)}.$$

Thus

$$e^{u_3(\eta_3)} > \frac{\beta_2^l}{e^M}e^{u_1(\eta_1)} - \frac{d^M}{e^M}$$

$$\ge \frac{b_2^l}{a_1^M + \beta_1^M\left[\frac{\beta_2^M}{e^l}\cdot\frac{a_2^M}{c^l}\left(\frac{b_2^M}{a_1^l}\right)^2\right]}$$

$$\cdot\frac{\beta_2^l}{c^M\cdot e^M}\left\{\frac{a_2^l b_2^l}{a_1^M + \beta_1^M\left[\frac{\beta_2^M}{e^l}\cdot\frac{a_2^M}{c^l}\left(\frac{b_2^M}{a_1^l}\right)^2\right]} - b_2^M\right\} - \frac{d^M}{e^M}$$

$$\stackrel{def}{=} p_3 > 0. \quad (2.19)$$

From (2.9)-(2.11) and (2.17)-(2.19), we get

$$|u_1(t)| \le \max\{|\ln d_1|, |\ln p_1|\} \stackrel{def}{=} R_1$$

$$|u_2(t)| \le \max\{|\ln d_2|, |\ln p_2|\} \overset{def}{=} R_2$$

$$|u_3(t)| \le \max\{|\ln d_3|, |\ln p_3|\} \overset{def}{=} R_3.$$

Clearly, R_i $(i = 1, 2, 3)$ are independent of λ. Denote $M = R_1 + R_2 + R_3 + R_o$; here, R_o is sufficiently large such that each solution $(\alpha^*, \beta^*, \gamma^*)^T$ meets the following system:

$$\begin{cases} -\overline{a}_1 + \overline{b}_1 e^{\beta-\alpha} - \overline{\beta}_1 e^{\gamma} = 0 \\ \overline{a}_2 e^{\alpha-\beta} - \overline{b}_2 - \overline{c} e^{\beta} = 0 \\ -\overline{d} - \overline{e} e^{\gamma} + \overline{\beta}_2^{\alpha} = 0 \end{cases} \tag{2.20}$$

satisfies $\|(\alpha^*, \beta^*, \gamma^*)^T\| = |\alpha^*| + |\beta^*| + |\gamma^*| < M$, provided that system (2.2) has a solution or a number of solutions. Now we take $\Omega = \{(u_1(t), u_2(t), u_3(t))^T \in X : \|(u_1, u_2, u_3)^T\| < M\}$. This satisfies condition (i) of Lemma 1 when $(u_1, u_2, u_3)^T \in \partial\Omega \cap KerL = \partial\Omega \cap R^3$, $(u_1, u_1, u_3)^T$ is a constant vector in R^3 with $|u_1| + |u_2| + |u_3| = M$. If system (2.20) has a solution or a number of solutions, then

$$QN\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} -\overline{a}_1 + \overline{b}_1 e^{u_2-u_1} - \overline{\beta}_1 e^{u_3} \\ \overline{a}_2 e^{u_1-u_2} - \overline{b}_2 - \overline{c} e^{u_2} \\ -\overline{d} - \overline{e} e^{u_3} + \overline{\beta}_2 e^{u_1} \end{bmatrix} \neq 0.$$

This proves that condition (ii) of Lemma 1 is satisfied. Finally, we will prove that condition (iii) of Lemma 1 is satisfied. To this end, we define $\phi : DomL \times [0, 1] \to X$ by

$$\phi(u_1, u_2, u_3, \mu) = \begin{bmatrix} -\overline{a}_1 + \overline{b}_2 e^{u_2-u_1} \\ \overline{a}_2 e^{u_1-u_2} - \overline{c} e^{u_2} \\ -\overline{e} e^{u_3} + \overline{\beta}_2 e^{u_1} \end{bmatrix} + \mu \begin{bmatrix} -\overline{\beta}_1 e^{u_3} \\ -\overline{b}_2 \\ -\overline{d} \end{bmatrix},$$

where $\mu \in [0, 1]$ is a parameter. When $(u_1, u_2, u_3)^T \in \partial\Omega \cap KerL = \partial\Omega \cap R^3$, $(u_1, u_2, u_3)^T$ is a constant vector in R^3 with $|u_1| + |u_2| + |u_3| = M$. We will show that when $(u_1, u_2, u_3)^T \in \partial\Omega \cap KerL$, $\phi(u_1, u_2, u_3, \mu) \neq 0$, if the conclusion is not true, then constant vector $(u_1, u_2, u_3)^T$ with $|u_1| + |u_2| + |u_3| = M$ satisfies $\phi(u_1, u_2, u_3, \mu) = 0$. From

$$-\bar{a}_1 + \bar{b}_2 e^{u_2 - u_1} + \mu(-\bar{\beta}_1 e^{u_3}) = 0$$

$$\bar{a}_2 e^{u_1 - u_2} - \bar{c} e^{u_2} + \mu(-\bar{b}_{21}) = 0$$

$$-\bar{e}_1 e^{u_3} + \bar{\beta}_2 e^{u_1} + \mu(-\bar{d}) = 0$$

and following the argument of (2.9)-(2.11), we obtain

$$|u_i| < \max\{|\ln d_i|, |\ln p_i|\}, \quad i = 1, 2, 3.$$

Thus

$$\sum_{i=1}^{3} |u_i| < \sum_{i=1}^{3} \max\{|\ln d_i|, |\ln p_i|\} < M,$$

which contradicts the fact that $|u_1| + |u_2| + |u_3| = M$. Therefore, according to topological degree theory, we have

$$\deg(JQN(u_1, u_2, u_3)^T, \Omega \cap KerL, (0, 0, 0)^T)$$

$$= \deg(\phi(u_1, u_2, u_3, 1), \Omega \cap KerL, (0, 0, 0)^T)$$

$$= \deg(\phi(u_1, u_2, u_3, 0), \Omega \cap KerL, (0, 0, 0)^T)$$

$$= \deg((-\bar{a}_1 + \bar{b}_2 e^{u_2 - u_1}, \bar{a}_2 e^{u_1 - u_2} - \bar{c} e^{u_2},$$

$$-\bar{e} e^{u_3} + \bar{\beta}_2 e^{u_1})^T, \Omega \cap KerL, (0, 0, 0)^T).$$

Because of condition (i), the system of algebraic equations

$$\begin{cases} -\bar{a}_1 + \dfrac{\bar{b}_2 y}{x} = 0 \\ \dfrac{\bar{a}_2 x}{y} - \bar{c} y = 0 \\ -\bar{e} z + \bar{\beta}_2 x = 0 \end{cases}$$

has a unique solution $(x^*, y^*, z^*)^T$ which satisfies $x^* > 0$, $y^* > 0$, and $z^* > 0$, and thus,

$$\deg((-\bar{a}_1 + \bar{b}_2 e^{u_2 - u_1}, \bar{a}_2 e^{u_1 - u_2} - \bar{c} e^{u_2},$$

$$- \bar{e} e^{u_3} + \bar{\beta}_2 e^{u_1})^T, \Omega \cap KerL, (0, 0, 0)^T)$$

$$sign \begin{vmatrix} \dfrac{-\bar{b}_2 y^*}{(x^*)^2}, & \dfrac{\bar{b}_2}{x^*}, & 0 \\ \dfrac{\bar{a}_2}{y^*}, & \dfrac{\bar{a}_2 x^*}{-(y^*)^2} - \bar{c}, & 0 \\ \bar{\beta}_2, & 0, & \bar{e} z^* \end{vmatrix} = sign\left[-\bar{e}\bar{c}\bar{b}_2 \frac{z^* y^*}{(x^*)^2}\right] = -1$$

consequently,

$$\deg(JQN(u_1, u_2, u_3)^T, \Omega \cap KerL, (0, 0, 0)^T) \neq 0.$$

This shows that condition (iii) of Lemma 1 is satisfied. By now Ω verifies all the requirements of Lemma 1 and then system (2.2) has at least one w-periodic solution. This completes the proof.

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