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RANDOM COEFFICIENT REGRESSION IN SIMULTANEOUS LINEAR EQUATIONS MODEL

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Abstract

In this paper, we will consider a random coefficients regression in simultaneous equations model (SEM). Some facts about SEM in econometrics theory and practice are introduced. Estimation problems concerning double k-class in SEM which have random coefficients are considered. Bias and mean squared error (MSE) of double k-class estimators are derived for random coefficients in a single equation SEM. For given k_1, we suggested four alternative values of k_2. Three of them are better, in the sense of MSE, than the other values of k_2.

1. Introduction

The econometricians distinguish three approaches to estimating the simultaneous linear equation model: the naive approach, the limited-information approach, and the full information approach.

The naive approach consists of estimating a single equation using the technique of the ordinary least squares method (OLS). This approach ignores the information as to which of the predetermined variables in question are endogenous and which are exogenous, and the estimators

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are biased and inconsistent because of the inclusion of the endogenous variables into the set of the predetermined variables.

The limited-information approach considers one equation at a time, estimating the structural form as does OLS. It uses the information as to which variables, both endogenous and exogenous, are included in the other equations of the model but excluded from the equation being estimated. In this group there are, for example, the following methods: the indirect least squares method (ILS), the two-stage least squares method (2-SLS) and double k-class estimators as the generalization of the 2-SLS and k-class estimators.

The full-information approach estimates the entire model of the simultaneous linear equations simultaneously using all information available on each of the equations of the system. This approach includes two methods: the three-stage least squares method (3-SLS) and full-information maximum likelihood method (FIML).

The classical simultaneous equation model of econometrics is a system of M structural equations which may be compiled to give the following equation:

$$Y_1\gamma_{11} + Y_2\gamma_{21} + \cdots + Y_M\gamma_{M1} + X_1\beta_{11} + X_2\beta_{21} + \cdots + X_K\beta_{K1} = \varepsilon_1$$

$$Y_1\gamma_{12} + Y_2\gamma_{22} + \cdots + Y_M\gamma_{M2} + X_1\beta_{12} + X_2\beta_{22} + \cdots + X_K\beta_{K2} = \varepsilon_2$$

$$\vdots$$

$$Y_1\gamma_{1M} + Y_2\gamma_{2M} + \cdots + Y_M\gamma_{MM} + X_1\beta_{1M} + X_2\beta_{2M} + \cdots + X_K\beta_{KM} = \varepsilon_M, \quad (1)$$

where $Y_1, Y_2, \ldots, Y_M$ are $T \times 1$ endogenous variables, $X_1, X_2, \ldots, X_K$ are $T \times 1$ exogenous (predetermined) variables and $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_M$ denote $T \times 1$ stochastic disturbance terms or random variables. The γ's are coefficients of endogenous variables, and the β's are coefficients of predetermined variables. The model under consideration is complete, i.e., the number of equations equals to the number of endogenous variables. Each equation of the model (1) represents one aspect of the structure of the model and it is called a *structural equation* and the model is called the *structural form*.

In general, we can distinguish three types of equation. First, there are behavioral equations. They describe the behavior of economic subjects, for instance, the marginal productivity conditions. Second, we have technical or institutional equations, for example, production function or other technology-induced relationships. Such equations are stochastically. Third, there are identities or definitional equations, strictly non stochastic equations. A typical example would be, say, the usual national income identity. This kind of equations can be eliminated from the system and we shall always assume that the system (1) contains no identities or that, if does, they have been substituted out.

The structural form (1) in a matrix notation is the most convenient and the most easily manipulated form of expressing the structural equations. In matrix notation, the system (1) is written as:

$$Y\Gamma + XB = E, \tag{2}$$

where Y is a $T \times M$ matrix of observations on M endogenous variables, X is a $T \times K$ matrix of observations on K predetermined variables, and E is a $T \times M$ matrix consisting of M additive stochastic disturbance terms. Rows of a disturbance matrix are assumed to be stochastically independent normal distribution with zero mean and an unknown but finite covariance matrix $\sum \otimes I_T$. The matrices Γ and B are the matrices of M^2 and MK structural coefficients, respectively.

The matrix Γ is assumed to be square and nonsingular. Post multiplying the matrix equation (2) by the inverse of Γ and solving for Y yields:

$$Y = X\prod + U. \tag{3}$$

This is the matrix notation of the reduced form for the structural model (1). Where $\prod = -B\Gamma^{-1}$ is a matrix of reduced form coefficients, and $U = E\Gamma^{-1}$ is the matrix of reduced form stochastic disturbance terms.

The simultaneous linear equation model (1), we wish to express the structural coefficients as explicit functions of the reduced-form coefficients, but it is sometimes difficult or even impossible. Determination of whether there is a one-to-one correspondence between the structural coefficients and the reduced-form coefficients is called the *identification* problem. The identification is prior to choosing the examination method and the estimation. If the identifiability conditions are satisfied, the econometrician may then proceed to estimate the parameters in the model under consideration.

The *order condition for identifiability*: In order to be identified, an equation must exclude at least $M-1$ of the variables appearing in the model. If exactly $M-1$ variables are excluded from an equation, it is called "*justidentified*" equation and if more than $M-1$ variables are excluded from an equation, it is called "*overidentified*" equation. But if less than $M-1$ variables are excluded from an equation, it is called "*underidentified*" equation and it is never possible to compute the structural parameters.

A more rigorous rule is called the *rank condition for identifiability*: An equation is identified if and only if there is at least one non-zero determinant of order $M-1$ in the array of coefficients which those variables excluded from the equation in question appears in the other equations.

A simultaneous linear equation model is identified if all the equations are identified. The order condition for identifiability is necessary but not sufficient, but the rank condition for identifiability is both necessary and sufficient. If the rank condition for identifiability is satisfied, the order condition for identifiability has also been satisfied, but not vice versa.

An econometrician makes the choice among the different econometrical methods and techniques on the basis of the nature of the matrix of coefficients of the endogenous variables Γ and the covariance matrix Σ.

First, we examine the matrix Γ. If it is diagonal, we check the matrix Σ. If the matrix Σ is diagonal, too, we have to check whether $\mathrm{Cov}(E) = \sigma^2 I$ or not. If it does, we apply the ordinary least squares method (OLS); if not, we use the generalized least squares method (GLS). If the matrix Σ is not diagonal, an econometrician uses seemingly unrelated equation estimation methods. If the matrix Γ is triangular and the matrix $\sum$ is diagonal, we shall use the ordinary least squares method to the recursive model. If the matrix $\sum$ is not diagonal and some equations are "*overidentified*" while none is not identified, we shall apply the three-stage least squares method (3-SLS). If these conditions are not satisfied, we apply the indirect least squares method (ILS) or the two-stage least squares method (2-SLS), depending on whether an equation is "*justidentified*" or "*overidentified*". The 2-SLS estimator is both reasonable and appropriate as an estimation method.

Dwivedi and Srivastava [4] studied the exact finite sample properties of Nagar's [10] double k-class estimators. They also analyzed a result originally derived by Srivastava et al. [11] that it is always possible to choose k such that double k-class estimators have smaller mean squared error (MSE) than that of k-class estimators. Chaturvedi and Shalabh [3] considered a family of feasible generalized double k-class estimators in a linear regression model. Chao and Phillips [2], Geweke [7], Kleibergen and van Dijk [9], Zellner [16] and Gao and Lahiri [6] developed the Bayesian approaches in SEM. Hsiao and Pesaran [8] provided a review of linear panel data with slop heterogeneity. Anderson [1] provided a reduced rank regression analysis for maximum likelihood estimators of a matrix of regression coefficients of specified rank and of corresponding linear restrictions on such matrices.

2. Random Coefficient in Simultaneous Equations Model

In this section, we will study the problem of SEM when we have a random coefficients regression in the simultaneous equations system. Suppose that

$$B = \overline{B} + e, \tag{4}$$

where e is a $K \times M$ matrix of random variables and independently distributed with mean zero and covariance Δ. To obtain random coefficients reduced form, we combine equations (2) and (4) together, and get

$$Y = X \prod^{*} + V, \tag{5}$$

where

$$\prod^{*} = -\overline{B}\Gamma^{-1},$$

$$V = U - Xe\Gamma^{-1},$$

$$E(V) = 0,$$

and

$$\mathrm{Cov}(V) = \Gamma'^{-1}(\Sigma \otimes I)\Gamma^{-1} + (\Gamma'^{-1}\Delta\Gamma^{-1} \otimes X'X). \tag{6}$$

The GLS estimator of $\prod^{*}$ is

$$\hat{\prod}^{*} = (X'[\Gamma'^{-1}(\Sigma \otimes I)\Gamma^{-1} + (\Gamma'^{-1}\Delta\Gamma^{-1} \otimes X'X)]^{-1}X)^{-1}X'[\Gamma'^{-1}(\Sigma \otimes I)\Gamma^{-1} + (\Gamma'^{-1}\Delta\Gamma^{-1} \otimes X'X)]^{-1}Y. \tag{7}$$

Since Σ and Δ are unknown, two step OLS procedure can be applied. In the first step, we estimate Σ and Δ by

$$\hat{\Sigma} = \frac{1}{T-K}\hat{V}'\hat{V} = \frac{1}{T-K}(Y - X\hat{\prod})'(Y - X\hat{\prod}) \tag{8}$$

and

$$\hat{\Delta} = \frac{1}{M-1}\sum_{i=1}^{M}(\hat{\prod}_i - \hat{\overline{\prod}})(\hat{\prod}_i - \hat{\overline{\prod}})', \tag{9}$$

where $\hat{\prod}_i = (X'X)^{-1}X'Y$ and $\hat{\overline{\prod}} = \frac{1}{M}\sum_{i=1}^{M}\hat{\prod}_i$. In the second step, we estimate $\hat{\prod}^{*}$ by substituting $\hat{\Sigma}$ and $\hat{\Delta}$ into equation (7), we get the feasible generalized estimators as

$$\hat{\prod}^{*} = (X'[\Gamma'^{-1}(\hat{\Sigma} \otimes I)\Gamma^{-1} + (\Gamma'^{-1}\hat{\Delta}\Gamma^{-1} \otimes X'X)]^{-1}X)^{-1}X'[\Gamma'^{-1}(\hat{\Sigma} \otimes I)\Gamma^{-1} + (\Gamma'^{-1}\hat{\Delta}\Gamma^{-1} \otimes X'X)]^{-1}Y. \tag{10}$$

We can estimate the structural form parameters Γ and $\overline{B}$ from the reduced form estimate $\hat{\prod}^{*}$. Note that, the classical two-stage least squared estimator of SEM can be obtained from (10) by assuming that the covariance matrix of the random coefficient is zero.

The feasible generalized double k-class estimators of (10), when $M = 2$, which are characterized by two non-stochastic scalars k_1 and k_2. Terming it as feasible random coefficients double k-class estimators defined by

$$\begin{aligned}\hat{\Pi}^*_{kk} = \Bigg\{1 - \Bigg[\left(\frac{k_1}{T-K+2}\right)(Y - X\hat{\Pi}^*)'[\Gamma'^{-1}(\hat{\Sigma}\otimes I)\Gamma^{-1} \\ + (\Gamma'^{-1}\hat{\Delta}\Gamma^{-1}\otimes X'X)]^{-1}(Y - X\hat{\Pi}^*)\Bigg]\Bigg/\Bigg[Y'[\Gamma'^{-1}(\hat{\Sigma}\otimes I)\Gamma^{-1} \\ + (\Gamma'^{-1}\hat{\Delta}\Gamma^{-1}\otimes X'X)]^{-1}Y - k_2(Y - X\hat{\Pi}^*)'[\Gamma'^{-1}(\hat{\Sigma}\otimes I)\Gamma^{-1} \\ + (\Gamma'^{-1}\hat{\Delta}\Gamma^{-1}\otimes X'X)]^{-1}(Y - X\hat{\Pi}^*)\Bigg]\hat{\Pi}^*\Bigg\}. \qquad (11)\end{aligned}$$

The feasible generalized double k-class estimators which presented by Ullah and Ullah [13], Wan and Chaturvedi [14], and Chaturvedi and Shalabh [3] can be obtained as special case from (11) when we have $\hat{\Delta} = 0$.

3. Random Coefficient in Single Equation Model

The structural form of a single equation model can be written in the following way:

$$y = Y_2\gamma + X_1\beta + \varepsilon_1, \qquad (12)$$

where y is the $T \times 1$ vector of the first endogenous variables and Y_2 is the $T \times (M-1)$ matrix of included endogenous variables appearing in the first equation, and X_1 is the $T \times \ell_1$ matrix of exogenous variables included in the structural equation (12), $\ell_1 \leq K$. Suppose that the coefficients of exogenous variables are random, that is,

$$\beta = \bar{\beta} + \alpha_1,$$

where $E(\alpha_1) = 0$, and $\mathrm{Cov}(\alpha_1) = \Delta_1$, then

$$y = Y_2\gamma + X_1\overline{\beta} + X_1\alpha_1 + \varepsilon_1$$

$$= Y_2\gamma + X_1\overline{\beta} + \varepsilon_1^*, \qquad (13)$$

where $E(\varepsilon_1^*) = 0$, and $\mathrm{Cov}(\varepsilon_1^*) = X_1'\Delta_1 X_1 + \sigma_{11}I = \Omega_{11}$.

Let us consider the $(M-1)$ endogenous variables Y_2 can be written in the reduced form as follows:

$$Y_2 = X_1\Pi_1 + X_2\Pi_2 + V_2, \qquad (14)$$

where X_2 is a $T \times \ell_2$ observation matrix of exogenous variables excluded form (13), and ε_1^* and V_2 are a $T \times 1$ vector and $T \times (M-1)$ matrix of random disturbances to the system. We assume that $(\varepsilon_1^*, V_2) \sim N(0, \Omega \otimes I_T)$, where $M \times M$ covariance matrix Ω is positive definite symmetric and is partitioned as

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{21}' \\ \Omega_{21} & \Omega_{22} \end{bmatrix}.$$

The structural model (13) and (14) can be written in its reduced form as following:

$$(y \quad Y_2) = (X_1 \quad X_2)\begin{pmatrix} \pi_1 & \Pi_1 \\ \Pi_2\gamma & \Pi_2 \end{pmatrix} + (\xi_1 \quad V_2), \qquad (15)$$

where

$$\pi_1 = \overline{\beta} + \Pi_1\gamma,$$

$$\xi_1 = \varepsilon_1^* + V_2\gamma,$$

$$(\xi_1, V_2) \sim N(0, \Omega^* \otimes I_T),$$

$$\Omega = P'\Omega^* P,$$

$$P = \begin{pmatrix} I & 0 \\ -\gamma & I_{M-1} \end{pmatrix}$$

and

$$\Omega^* = \begin{bmatrix} \Omega_{11}^* & \Omega_{21}'^* \\ \Omega_{21}^* & \Omega_{22}^* \end{bmatrix}.$$

Note that, the equation (13) is fully identified if and only if $Rank(\prod_2) = (M-1) \leq \ell_2$. For simplified, we suppose that $X_1'X_2 = 0$ in two equations model, so the double k-class estimator for the structural coefficients in model (13) and (14) is given by

$$\begin{bmatrix} \hat{\gamma} \\ \hat{\beta} \end{bmatrix} = \begin{bmatrix} Y_2'Y_2 - k_1\hat{V}_2'\hat{V}_2 & Y_2'X_1 \\ X_1'Y_2 & X_1'X_1 \end{bmatrix}^{-1} \begin{bmatrix} (Y_2 - k_2\hat{V}_2)'y \\ X_1'y \end{bmatrix}, \tag{16}$$

where

$$\hat{V}_2 = Q_X Y_2,$$

$$Q_X = I - X(X'X)^{-1}X'$$

$$= I - X_1(X_1'X_1)^{-1}X_1' - X_2(X_2'X_2)^{-1}X_2'.$$

From equation (16), the following estimators have been obtained when $\Delta_1 = 0$.

(i) Ordinary least squares (OLS), when $k_1 = k_2 = 0$,

(ii) Two-stage least squares (2-SLS), when $k_1 = k_2 = 1$,

(iii) Zellner's [15] Bayesian minimum expected loss estimator (MELO), $k_1 = k_2 = 1 - \dfrac{K}{(T-K-M-1)}$.

(iv) Zellner's Bayesian method of moments relative to balanced loss function (BMOM), see Tsurumi [12], when

$$k_1 = 1 - \frac{K}{T-K}$$

$$k_2 = 1 - \frac{(1-\omega)K}{T-K} \text{ with } \omega = 0.75.$$

(v) Fuller [5] modified the classical limited information maximum likelihood (LIML) estimator, when

$$k_1 = k_2 = \lambda_* - \frac{\alpha}{T - K} \text{ for } \alpha = 1, 4,$$

where

$$\lambda_* = \min_{\gamma} \frac{(y - Y_2\gamma)' Q_{X_1} (y - Y_2\gamma)}{(y - Y_2\gamma)' Q_X (y - Y_2\gamma)}$$

and it is computed using the LIML estimate.

We can express the random coefficients of double k-class estimator for the structural coefficient γ with characterizing scalars k_1 and k_2 as follows:

$$\hat{\gamma}_{RDKC} = \hat{\gamma}_{RKC} + (k_1 - k_2)(Y_2' A Y_2)^{-1} Y_2' Q_X y, \tag{17}$$

where $\hat{\gamma}_{RKC}$ is the random k-class estimator of γ with characterizing scalar k_1 and

$$A = (1 - k_1)[I - X_1(X_1'X_1)^{-1} X_1'] + k_1 X_2(X_2'X_2)^{-1} X_2'.$$

We can also derived the exact expression for bias and MSE of the random coefficients of double k-class estimator of γ. When $T - \ell_1 \geq 1$, the bias is

$$E(\hat{\gamma}_{RDKC}) - \gamma = \left(\gamma - \frac{\Omega_{21}' + \gamma'\Omega_{22}}{\Omega_{22}}\right)[\zeta F_0(1;\ 0;\ 1) - 1]$$

$$+ (k_1 - k_2)\left[\frac{\Omega_{21}' + \gamma'\Omega_{22}}{\Omega_{22}} F_0(1;\ 1;\ 1)\right], \tag{18}$$

where

$$n = (T - \ell_1)/2,$$

$$m = (T - K)/2,$$

$$\zeta = (\Pi_2' X_2' X_2 \Pi_2)/2\Omega_{22}$$

and for non-negative integers a, b, c, d,

$$F_d(a;\, b;\, c) = e^{-\zeta} \sum_{r=0}^{\infty} \sum_{j=1}^{\infty} (r \cdot d + 1) k_1^r \, \frac{\Gamma(n + j - a - 1) \cdot \Gamma(m + r + b)}{\Gamma(n + j + r + c) \cdot \Gamma(m)} \cdot \frac{\zeta^j}{j!}, \quad (19)$$

when $2n \succ 3$, the MSE of the random coefficients of double k-class estimator of γ is given by

$$E(\hat{\gamma}_{RDKC} - \gamma)^2$$

$$= \left(\gamma - \frac{\Omega_{21}' + \gamma'\Omega_{22}}{\Omega_{22}}\right)^2 + (k_1 - k_2)^2 \left(\frac{\Omega_{21}' + \gamma'\Omega_{22}}{\Omega_{22}}\right)^2 F_1(1;\, 2;\, 2)$$

$$+ \frac{\Omega_{11 \cdot 2}}{2\Omega_{22}} [(1 - k_2)^2 F_1(0;\, 1;\, 1) + (m - n) F_1(0;\, 0;\, 1) + \delta F_1(1;\, 0;\, 2)]$$

$$+ \delta \left(\gamma - \frac{\Omega_{21}' + \gamma'\Omega_{22}}{\Omega_{22}}\right)^2 \left[\frac{1}{2} F_1(0;\, 0;\, 1) + \delta F_1(1;\, 0;\, 2) - 2F_0(1;\, 0;\, 1)\right]$$

$$+ 2\, \frac{\Omega_{21}' + \gamma'\Omega_{22}}{\Omega_{22}} \left(\gamma - \frac{\Omega_{21}' + \gamma'\Omega_{22}}{\Omega_{22}}\right) (k_1 - k_2) [\delta F_1(1;\, 1;\, 2) - F_0(1;\, 1;\, 1)], \quad (20)$$

where

$$\Omega_{11 \cdot 2} = (\Omega_{11} + 2\gamma'\Omega_{21} + \gamma'\Omega_{22}\gamma) - \frac{(\Omega_{21}' + \gamma'\Omega_{22})^2}{\Omega_{22}}.$$

From equations (18) and (20), we can see that the bias and MSE of $\hat{\gamma}_{RDKC}$ depend not only on Ω_{22} but also on Ω_{11} and Ω_{12} which have a covariance matrix of the random coefficients. This bias and MSE are similarly defined as given by Dwivedi and Srivastava [4] when there is not a random coefficient in the model.

Dwivedi and Srivastava [4] derived two interesting results for the value of k_1 and k_2. First, for a given k_1, the double k-class estimator is unbiased if the value of k_2 is set as

$$K_U = k_1 + \frac{\Omega_{22}}{\Omega_{21}' + \gamma\Omega_{22}} \left(\gamma - \frac{\Omega_{21}' + \gamma\Omega_{22}}{\Omega_{22}}\right) \left[\frac{\zeta \cdot F_0(1,\, 0,\, 1) - 1}{F_0(1,\, 1,\, 1)}\right].$$

Second, the MSE of the double k-class estimator is less than that of k-class estimator with the value of k_2 is between k_1 and k^*, where k^* defined as

$$K^* = k_1$$

$$+\left[\frac{(1-k_1)\frac{\Omega_{11\cdot 2}}{\Omega_{22}}F_1(0,1,1)+2\frac{\Omega'_{21}+\gamma\Omega_{22}}{\Omega_{22}}\left(\gamma-\frac{\Omega'_{21}+\gamma\Omega_{22}}{\Omega_{22}}\right)[\zeta\cdot F_1(1,1,2)-F_0(1,1\,1)]}{\left[\left(\frac{\Omega'_{21}+\gamma\Omega_{22}}{\Omega_{22}}\right)^2 F_1(1,2,2)+\frac{\Omega_{11\cdot 2}}{2\Omega_{22}}F_1(0,1,1)\right]}\right].$$

For a given k_1, we suggest four different values of k_2 that can be used in random coefficients of double k-class estimators.

1. $K_{U^*} = \dfrac{k_1 + k_u}{2}$,

2. $K_M = \dfrac{k_1 + k^*}{2}$,

3. $K_G = \sqrt{|k_1 \times k^*|}$,

4. $K_H = \begin{cases} k^* & \text{if } k_1 = 0 \\ \dfrac{2}{\left(\dfrac{1}{k_1} + \dfrac{1}{k^*}\right)} & \text{if } k_1 \neq 0. \end{cases}$

We look from the above expression for the optimal value of k_2 that minimizing the MSE of random coefficients of double k-class estimator of γ. In order to shed some light about the best value of k_2 from the alternative values of K_U, K_{U^*}, K_M, K_G and K_H, we set $T = 10$, $\gamma = 1$, $K = 5$, $\ell_1 = 2$, $\zeta = 10$, $\Omega_{11} = 1.0$, $\Omega_{21} = 0.4$ and $\Omega_{22} = 1.0$. All values of K_2 and the resulting of bias and MSE of the random coefficients of double k-class estimator of γ under our specification are reported in the following table.

Table 1. An alternative values of K_2 with the Bias and MSE for the estimator γ

Values of K_2	K_1				
	–1.0	–0.5	0.0	0.5	1.0
K_U	207.74 0 (214.523)	185.487 0 (211.75)	162.487 0 (209.468)	137.354 0 (208.842)	115.965 0 (269.29)
K^*	16.481 0.364 (1.532)	12.158 0.37 (1.07)	8.039 0.378 (0.658)	4.26 0.386 (0.339)	1.431 0.398 (0.185)
K_{U^*}	103.371 0.199 (53.265)	92.587 0.199 (52.728)	81.244 0.199 (52.343)	68.927 0.198 (52.414)	58.482 0.2 (67.87)
K_M	7.74 0.381 (0.478)	5.829 0.384 (0.378)	4.019 0.387 (0.288)	2.38 0.391 (0.217)	1.216 0.399 (0.179)
K_G	4.06 0.388 (0.201)	2.466 0.391 (0.211)	0 0.397 (0.175)	1.46 0.394 (0.185)	1.196 0.399 (0.179)
K_H	–2.129 0.4 (0.201)	–1.043 0.399 (0.185)	8.039 0.378 (0.658)	0.895 0.396 (0.176)	1.177 0.399 (0.179)

From the above table, each cell has the estimated value of K_2, the estimated bias, and the estimated value of MSE is in brackets. Comparing the estimated MSE with characterizing scalars K_1 and alternative values of K_2, we found that our suggested values for K_M, K_G and K_H are better, in the sense of MSE, than K_U and K^* which proposed by Dwivedi and Srivastava [4]. We also found that the value of K_{U^*} has MSE between K_U and K^*. The optimal value of K_1 and K_2 when we have a random coefficient needs some further research.

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