Far East J. Appl. Math. **24**(2) (2006), 163-175

THE PERIODIC WAVE SOLUTIONS FOR THE GENERALIZED HIROTA-SATSUMA SYSTEM AND THE NUTKU-Ö$\tilde{G}$UZ EQUATION

H. A. ABDUSALAM

Mathematics Department, Faculty of Science
Cairo University, Giza, Egypt

Current address
Mathematics Department, College of Science
King Saud University, P. O. Box 2455
Riyadh 11451, Kingdom of Saudi Arabia
e-mail: hosny@operamail.com

Abstract

The periodic wave solution for the generalized Hirota-Satsuma system and the Nutku-Ö$\tilde{g}$uz equation are obtained by using the *F*-expansion method which can be thought of as a generalization of the Jacobi elliptic function method proposed recently. In the limit cases, the solitary wave solutions are obtained.

1. Introduction

It is known that soliton, breather, compaction, etc. come from the robust interaction of the dissipation and dispersion in physics system. The robust interaction can be expressed by the mathematical form of

2000 Mathematics Subject Classification: 35D05, 35J70, 35R35, 49J40.

Keywords and phrases: Jacobi elliptic function, doubly periodic solutions, Hirota-Satsuma system, *F*-expansion method, Nutku-Ö$\tilde{g}$uz equation.

This work was supported by College of Sciences - Research Center project no. (Math/2006/20).

Received May 15, 2006

subtle balance between the nonlinear term and the highest order partial derivative term. So the nonlinear transform can be introduced as follows [6]:

$$u = \sum_{J=0}^{n} a_J G_J, \tag{1}$$

where G_J is an arbitrary function, a_J is a parameter to be determined, n is determined by the subtle balance [8] of the nonlinear term and the highest order partial derivative term [6].

Recently, Jacobi elliptic function expansion method was proposed [1, 2, 3, 5, 9, 11, 12, 13, 14] and improved to F-expansion method [4, 10] as the summary and generalization of the Jacobi elliptic function method. These methods are effective to construct the periodic wave solutions of nonlinear equations, when Jacobi elliptic functions degenerate as hyperbolic functions or trigonometric functions, the solitary wave solutions and singularity solutions can be obtained.

Jacobi elliptic function expansion sets

$$G_J = F^J(\zeta), \tag{2}$$

where $F(\zeta)$ is the Jacobi elliptic function, $\zeta = x + ct$. $F^J(\zeta)$ is the Jth order power of $F(\zeta)$.

F-expansion method demands that $F(\zeta)$ is the solution of the following equation [10]:

$$F'^2(\zeta) = PF^4(\zeta) + QF^2(\zeta) + R. \tag{3}$$

Substituting (1) into the nonlinear equation (3) yields an equation for $F^J(\zeta)$. For the subtle balance in nonlinear transform, the equation for $F^J(\zeta)$ can be solved easily in most cases by setting the coefficient polynomials into zero and getting a series of algebraic equations. Solving these algebraic equations by using *Mathematica* or *Maple* programs, we can get the relations of undetermined parameters a_J. Thus we get the exact solutions of nonlinear partial differential equation (PDE).

The paper is organized as follows: in Section 2, we first search for the periodic wave solutions for the new generalized Hirota-Satsuma coupled system [7] and then in Section 3, we find the periodic solutions for the Nutku-Ög̃uz equation [4, 10]. A conclusion is then given in the final Section 4.

2. The Generalized Hirota-Satsuma System

The new generalized Hirota-Satsuma coupled system is given as [7]:

$$\begin{aligned} u_t &= \frac{1}{2}u_{xxx} - 3uu_x + 3(wv)_x, \\ v_t &= -v_{xxx} + 3uv_x, \\ w_t &= -w_{xxx} + 3uw_x, \end{aligned} \tag{4}$$

which was introduced by Wu et al. recently [14].

First, we seek the travelling wave solution of equation (4) in the form

$$u(x, t) = U(\zeta), \quad v(x, t) = V(\zeta), \quad w(x, t) = W(\zeta), \quad \zeta = x + ct. \tag{5}$$

Substituting (5) into equations (4), we have the following system of ordinary differential equations:

$$\begin{aligned} cU' &= \frac{1}{2}U''' - 3UU' + 3WV' + VW', \\ cV' &= -V''' + 3UV', \\ cW' &= -W''' + 3UW', \end{aligned} \tag{6}$$

where $'$ means differentiation with respect to ζ.

Secondly, based on the subtle balance, $n = 2$, we introduce the following nonlinear transforms:

$$\begin{aligned} U(\zeta) &= a_0 + a_1F(\zeta) + a_2F^2(\zeta), \quad a_2 \neq 0, \\ V(\zeta) &= b_0 + b_1F(\zeta) + b_2F^2(\zeta), \quad b_2 \neq 0, \\ W(\zeta) &= d_0 + d_1F(\zeta) + d_2F^2(\zeta), \quad d_2 \neq 0, \end{aligned} \tag{7}$$

where $a_0, a_1, a_2, b_0, b_1, b_2, d_{0,} d_1, d_2$ are constants to be determined.

Considering (3) and substituting (7) into (6) and collecting all terms with the same degree of $F(\zeta)$ to zero respectively, we obtain a series of algebraic equations corresponding to U, V, W respectively:

F^0 : $-2ca_1 + Qa_1 - 6a_0a_1 + 6b_1d_0 + 6b_0d_1 = 0,$

F^1 : $-6a_1^2 - 4ca_2 + 8Qa_2 - 12a_0a_2 + 12b_2d_0 + 12b_1d_1 + 12b_0d_2 = 0,$

F^2 : $(6a_1(P - 3a_2) + 18b_2d_1 + 18b_1d_2) = 0,$

F^3 : $(24Pa_2 - 12a_2^2 + 24b_2d_2) = 0,$

F^0 : $cb_1 + Qb_1 - 3a_0b_1 = 0,$

F^1 : $-3a_1b_1 + 2cb_2 + 8Qb_2 - 6a_0b_2 = 0,$

F^2 : $6Pd_1 - 3a_2d_1 - 6a_1d_2 = 0,$

F^3 : $6Pb_1 - 3a_2b_1 - 6a_1b_2 = 0,$

F^0 : $cd_1 + Qd_1 - 3a_0d_1 = 0,$

F^1 : $-3a_1d_1 + 2cd_2 + 8Qd_2 - 6a_2d_2 = 0,$

F^2 : $6Pd_1 - 3a_2d_1 - 6a_1d_2 = 0,$

F^3 : $24Pd_2 - 6a_2d_2 = 0.$

Thirdly, solving algebraic equations above by using *Mathematica* or *Maple*, we have the following solution:

$$a_0 = \frac{1}{3}(c + 4Q), \quad a_2 = 4P,$$

$$b_0 = \frac{4P(-3Pd_0 + 2(c + Q)d_2)}{3d_2^2}, \quad b_2 = \frac{4P^2}{d_2},$$

$$a_1 = 0, \quad b_1 = 0, \quad d_1 = 0, \tag{8}$$

with c, d_0, $d_2 \neq 0$ being arbitrary constants.

Substituting (8) into (7), we have a general form of travelling wave solutions of equations (4):

$$U(\zeta) = \frac{1}{3}(c + 4Q) + 4PF^2(\zeta),$$

$$V(\zeta) = \frac{4P[-3Pd_0 + 2(c + Q)d_2]}{3d_2^2} + \left(\frac{4P^2}{d_2}\right)F^2(\zeta),$$

$$W(\zeta) = d_0 + d_2F^2(\zeta). \tag{9}$$

When $F(\zeta) = sn(\zeta, m)$ or $F(\zeta) = cd(\zeta, m) = cn(\zeta, m)/dn(\zeta, m)$, we get $P = m^2$, $Q = -(1 + m^2)$ from equation (3), where m is the modulus of Jacobi elliptic function, $0 < m^2 < 1$. The solution of (4) is obtained as:

$$U_1(\zeta) = \frac{1}{3}[c - 2(1 + m^2)] + 4m^2 sn^2(\zeta, m),$$

$$V_1(\zeta) = \frac{4m^2[-3m^2 d_0 + 2(c - (1 + m^2))d_2]}{3d_2^2} + \left(\frac{4m^4}{d_2}\right)sn^2(\zeta, m),$$

$$W_1(\zeta) = d_0 + d_2 sn^2(\zeta, m). \tag{10}$$

It is known that $sn(\zeta, m) \to \tanh(\xi)$ when $m \to 1$; thus (10) degenerates into the following form:

$$U_1(\zeta) = \frac{1}{3}(c - 8) + 4\tanh^2(\zeta),$$

$$V_1(\zeta) = \frac{-12d_0 + 8(c - 2)d_2}{3d_2^2} + \left(\frac{4}{d_2}\right)\tanh^2(\zeta),$$

$$W_1(\zeta) = d_0 + d_2\tanh^2(\zeta). \tag{11}$$

For $F(\zeta) = cd(\zeta, m) = cn(\zeta, m)/dn(\zeta, m)$, we have

$$U_2(\zeta) = \frac{1}{3}[c - 4(1 + m^2)] + 4m^2 cd^2(\zeta, m),$$

$$V_2(\zeta) = \frac{4m^2[-3m^2 d_0 + 2(c - (1 + m^2))d_2]}{3d_2^2} + \left(\frac{4m^4}{d_2}\right)cd^2(\zeta, m),$$

$$W_2(\zeta) = d_0 + d_2 cd^2(\zeta, m).$$

When $F(\zeta) = cn(\zeta, m)$, we get $P = -m^2$, $Q = 2m^2 - 1$ from equation (3). The nondegenerative elliptic function solution is

$$U_3(\zeta) = \frac{1}{3}[c + 4(2m^2 - 1)] - 4m^2 cn^2(\zeta, m),$$

$$V_3(\zeta) = \frac{-4m^2[3m^2 d_0 + 2(c + (2m^2 - 1))d_2]}{3d_2^2} + \left(\frac{4m^4}{d_2}\right) cn^2(\zeta, m),$$

$$W_3(\zeta) = d_0 + d_2 cn^2(\zeta, m). \tag{12}$$

Allowing $m \to 1$ (12) reduces to the following soliton solution:

$$U_3(\zeta) = \frac{1}{3}(c + 4) - 4\,\mathrm{sech}^2(\zeta),$$

$$V_3(\zeta) = \frac{-4(3d_0 + 2(c + 1)d_2)}{3d_2^2} + \left(\frac{4m^2}{d_2}\right)\mathrm{sech}^2(\zeta),$$

$$W_3(\zeta) = d_0 + d_2\,\mathrm{sech}^2(\zeta). \tag{13}$$

When $F(\zeta) = dn(\zeta, m)$, we get $P = -1$, $Q = (2 - m^2)$ from equation (3), the nondegenerative elliptic function solution is given as:

$$U_4(\zeta) = \frac{1}{3}(c + 4(2 - m^2)) - 4dn^2(\zeta, m),$$

$$V_4(\zeta) = \frac{-4[3d_0 + 2(c + (2 - m^2))d_2]}{3d_2^2} + \left(\frac{4}{d_2}\right) dn^2(\zeta, m),$$

$$W_4(\zeta) = d_0 + d_2 dn^2(\zeta, m). \tag{14}$$

When $m \to 1$, $dn(\zeta, m) \to \mathrm{sech}(\zeta)$ and we get the solution (13).

When $F(\zeta) = ns(\zeta, m) = 1/sn(\zeta, m)$ or $F(\zeta) = dc(\zeta, m) = dn(\zeta, m)/cn(\zeta, m)$, we get $P = 1$, $Q = -(1 + m^2)$ from equation (3). The nondegenerative elliptic function solution is obtained, which is the periodic singularity solution:

$$U_5(\zeta) = \frac{1}{3}(c - 4(1 + m^2)) + 4ns^2(\zeta, m),$$

$$V_5(\zeta) = \frac{4[-3d_0 + 2(c - (1 + m^2))d]}{3d_2^2} + \left(\frac{4}{d_2}\right) ns^2(\zeta, m),$$

$$W_5(\zeta) = d_0 + d_2 ns^2(\zeta, m). \tag{15}$$

Taking $m \to 0$, in equation (15) we get the following singularity solution:

$$U_5(\zeta) = \frac{1}{3}(c - 4) + 4\csc^2(\zeta),$$

$$V_5(\zeta) = \frac{4[-3d_0 + 2(c - 2)d_2]}{3d_2^2} + \left(\frac{4}{d_2}\right)\csc^2(\zeta),$$

$$W_5(\zeta) = d_0 + d_2\csc^2(\zeta). \tag{16}$$

When $m \to 1$, we get the singularity solution:

$$U_5(\zeta) = \frac{1}{3}(c - 8) + 4\coth^2(\zeta),$$

$$V_5(\zeta) = \frac{4[-3d_0 + 2(c - 2)d]}{3d_2^2} + \left(\frac{4}{d_2}\right)\coth^2(\zeta),$$

$$W_5(\zeta) = d_0 + d_2\coth^2(\zeta). \tag{17}$$

For $F(\zeta) = dc(\zeta, m) = dn(\zeta, m)/cn(\zeta, m)$, we have

$$U_6(\zeta) = \frac{1}{3}(c - 4(1 + m^2)) + 4dc^2(\zeta, m),$$

$$V_6(\zeta) = \frac{4[-3d_0 + 2(c - (1 + m^2))d]}{3d_2^2} + \left(\frac{4}{d_2}\right) dc^2(\zeta, m),$$

$$W_6(\zeta) = d_0 + d_2 dc^2(\zeta, m). \tag{18}$$

When $m \to 0$, we get the singularity solution:

$$U_6(\zeta) = \frac{1}{3}(c - 4) + 4\sec^2(\zeta),$$

$$V_6(\zeta) = \frac{4[-3d_0 + 2(c - 2)d]}{3d_2^2} + \left(\frac{4}{d_2}\right)\sec^2(\zeta),$$

$$W_6(\zeta) = d_0 + d_2\sec^2(\zeta). \tag{19}$$

When $F(\zeta) = nc(\zeta, m) = 1/cn(\zeta, m)$, we get $P = (1 - m^2)$, $Q = (2m^2 - 1)$ from equation (3). The nondegenerative solution is

$$U_7(\zeta) = \frac{1}{3}(c + 4(2m^2 - 1)) + 4(1 - m^2)nc^2(\zeta, m),$$

$$V_7(\zeta) = \frac{4(1 - m^2)[-3(1 - m^2)d_0 + 2(c + (2m^2 - 1))d]}{3d_2^2} + \left(\frac{4(1 - m^2)^2}{d_2}\right)nc^2(\zeta, m),$$

$$W_7(\zeta) = d_0 + d_2^2 nc^2(\zeta, m). \tag{20}$$

Taking $m \to 0$ we get solution (19).

When $F(\zeta) = nd(\zeta, m) = 1/dn(\zeta, m)$, we get $P = (m^2 - 1)$, $Q = (2 - m^2)$ from equation (3). The nondegenerative solution is

$$U_8(\zeta) = d_0 + d_2 nd^2(\zeta, m),$$

$$V_8(\zeta) = \frac{4(m^2 - 1)[-3(m^2 - 1)d_0 + 2(c + (2 - m^2))d_2]}{3d_2^2} + \left(\frac{4(m^2 - 1)^2}{d_2}\right)nd^2(\zeta, m),$$

$$W_8(\zeta) = d_0 + d_2 nd^2(\zeta, m). \tag{21}$$

When $F(\zeta) = sc(\zeta, m) = sn(\zeta, m)/cn(\zeta, m)$, we get $P = (1 - m^2)$, $Q = (2 - m^2)$ from equation (3), the nondegenerative solution is

$$U_9(\zeta) = \frac{1}{3}(c + 4(2 - m^2)) + 4(1 - m^2)sc^2(\zeta, m),$$

$$V_9(\zeta) = \frac{4(1 - m^2)[-3(1 - m^2)d_0 + 2(c + (2 - m^2))d]}{3d_2^2} + \left(\frac{4(1 - m^2)^2}{d_2}\right)sc^2(\zeta, m),$$

$$W_9(\zeta) = d_0 + d_2 sc^2(\zeta, m). \tag{22}$$

When $m \to 0$, $sc(\zeta, m) \to \tan(\zeta)$, we get the solution:

$$U_9(\zeta) = \frac{1}{3}(c + 8) + 4\tan^2(\zeta),$$

$$V_9(\zeta) = \frac{-12d_0 + 8(c + 2)d}{3d_2^2} + \left(\frac{4}{d_2}\right)\tan^2(\zeta),$$

$$W_9(\zeta) = d_0 + d_2 \tan^2(\zeta). \tag{23}$$

When $F(\zeta) = sd(\zeta, m) = sn(\zeta, m)/dn(\zeta, m)$, we get $P = m^2(m^2 - 1)$, $Q = (2m^2 - 1)$ from equation (3). The nondegenerative solution is

$$U_{10}(\zeta) = \frac{1}{3}(c + 4(2m^2 - 1)) + 4m^2(m^2 - 1)sd^2(\zeta),$$

$$V_{10}(\zeta) = \frac{4m^2(m^2 - 1)[-3m^2(m^2 - 1)d_0 + 2(c + (2m^2 - 1))d]}{3d_2^2} + \left(\frac{4m^4(m^2 - 1)^2}{d_2}\right)sd^2(\zeta),$$

$$W_{10}(\zeta) = d_0 + d_2 sd^2(\zeta). \tag{24}$$

When $F(\zeta) = cs(\zeta, m) = cn(\zeta, m)/sn(\zeta, m)$, we get $P = 1$, $Q = (2 - m^2)$ from equation (3). The nondegenerative solution is

$$U_{11}(\zeta) = \frac{1}{3}(c + 4(2 - m^2)) + 4cs^2(\zeta, m),$$

$$V_{11}(\zeta) = \frac{4[-3Pd_0 + 2(c + (2 - m^2))d]}{3d_2^2} + \left(\frac{4}{d_2}\right)cs^2(\zeta, m),$$

$$W_{11}(\zeta) = d_0 + d_2 cs^2(\zeta, m). \tag{25}$$

When $m \to 1$, $cs(\zeta, m) \to \operatorname{csch}(\zeta)$, we get the following solution:

$$U_{11}(\zeta) = \frac{1}{3}(c + 4) + 4\operatorname{csch}^2(\zeta),$$

$$V_{11}(\zeta) = \frac{4[-3Pd_0 + 2(c + 1)d_2]}{3d_2^2} + \left(\frac{4}{d_2}\right)\operatorname{csch}^2(\zeta),$$

$$W_{11}(\zeta) = d_0 + d_2^2\operatorname{csch}(\zeta). \tag{26}$$

Taking $m \to 0$, $cs(\zeta, m) \to \cot(\zeta)$, equation (25) reduces to

$$U_{11}(\zeta) = \frac{1}{3}(c+8) + 4\cot^2(\zeta),$$

$$V_{11}(\zeta) = \frac{4[-3Pd_0 + 2(c+2)d_2]}{3d_2^2} + \left(\frac{4}{d_2}\right)\cot^2(\zeta),$$

$$W_{11}(\zeta) = d_0 + d_2\cot^2(\zeta). \tag{27}$$

When $F(\zeta) = ds(\zeta, m) = dn(\zeta, m)/sn(\zeta, m)$, we get $P = 1$, $Q = (2m^2 - 1)$ from equation (3). The nondegenerative solution is

$$U_{12}(\zeta) = \frac{1}{3}(c + 4(2m^2 - 1)) + 4ds^2(\zeta),$$

$$V_{12}(\zeta) = \frac{4[-3d_0 + 2(c + (2m^2 - 1))d]}{3d_2^2} + \left(\frac{4}{d_2}\right)ds^2(\zeta),$$

$$W_{12}(\zeta) = d_0 + d_2 ds^2(\zeta). \tag{28}$$

When $m \to 1$, we get the solution (26) and when $m \to 0$, we get the solution (17).

3. The Nutku-Ög̃uz Equation

The Nutku-Ög̃uz equation [4] is written as

$$u_t = u_{xxx} + 2\lambda uu_x + vv_x + (uv)_x,$$

$$v_t = v_{xxx} + 2\mu vv_x + uu_x + (uv)_x, \tag{29}$$

where λ and μ are real constants satisfying the condition

$$\lambda + \mu = 1. \tag{30}$$

Nutku-Ög̃uz equation [4] pointed out that this system (29) decouples if $\lambda = \mu = 1/2$. It is further shown that subject to the condition (30) the system (29) is a bi-Hamiltonian system with two local Hamiltonian structures. Thus, one has a bi-Hamiltonian system which contains a free parameter. In general, a bi-Hamiltonian system is supposed to be integrable since it has infinite number of conserved quantities. However, it is peculiar that a system is integrable with arbitrary value of parameter. Hu and Liu [4] showed that the parameter can be removed.

The dispersionless version of this system with condition (30) was studied recently by Matsuno [7]. We concentrate here to find solution of the system (29) under the condition (30) for a certain value of parameters.

First, we seek travelling wave solution of equation (29) in the form

$$u(x, t) = U(\zeta), \quad v(x, t) = V(\zeta), \quad \zeta = x + ct. \tag{31}$$

Substituting (31) into equations (29), we have the following system of ordinary differential equations:

$$U''' + (2\lambda U - c)U' + (UV)' + VV' = 0,$$

$$V''' + (2\mu V - c)V' + (UV)' + UU' = 0. \tag{32}$$

Secondly, based on the subtle balance, we introduce the following nonlinear transforms:

$$U(\zeta) = a_0 + a_1 F(\zeta) + a_2 F^2(\zeta), \quad a_2 \neq 0,$$

$$V(\zeta) = b_0 + b_1 F(\zeta) + b_2 F^2(\zeta), \quad b_2 \neq 0, \tag{33}$$

where $a_0, a_1, a_2, b_0, b_1, b_2$ are constants to be determined.

Considering (3) and substituting (33) into (32) and collecting all the terms with the same degree of $F(\zeta)$ to zero respectively, we obtain a series of algebraic equations corresponding to U, V, and W respectively as:

F^0: $\quad -2a_1 + Qa_1 + 2\lambda a_0 a_1 + a_1 b_0 + a_0 b_1 + b_0 b_1 = 0,$

F^1: $\quad 2\lambda a_1^2 - 2ca_2 + 8Qa_2 + 2b_0 a_2 + 2a_1 b_1 + b_1^2 + 2b_0 b_2 + 2a_0(2\lambda a_2 + b_2) = 0,$

F^2: $\quad 6a_1 P + 6\lambda a_1 a_2 + 3a_2 b_1 + 3a_1 a b_2 + 3b_1 b_2 = 0,$

F^3: $\quad 24Pa_2 + 4a_2^2 + 4a_2 b_2 + 2b_2^2 = 0,$

F^0: $\quad a_0 a_1 + a_1 b_0 - cb_1 + Qb_1 + a_0 b_1 + 2\mu b_0 b_1 = 0,$

F^1: $\quad a_1^2 + 2a_2 b_0 + 2a_1 b_1 + 2\mu b_1^2 - 2cb_2 + 8Qb_2 + 4\mu b_0 b_2 + 2a_0(a_2 + b_2) = 0,$

F^2: $\quad 6b_1 P + 3b_1 a_2 + 6\mu b_2 b_1 + 3a_1(a_2 + b_2) = 0,$

F^3: $\quad 24Pb_2 + 2a_2^2 + 4a_2 b_2 + 4\mu b_2^2 = 0. \quad (34)$

Thirdly, solving the algebraic equations above by using *Mathematica*, for example, when $\lambda = 1/3$ and $\mu = 2/3$ we find that

$$a_0 = -3(c-4Q), \quad a_1 = 0, \quad a_2 = 36P,$$

$$b_0 = 3(c-4Q), \quad b_1 = 0, \quad b_2 = -36P. \tag{35}$$

The corresponding solutions are

$$U(\zeta) = -3(c-4Q) + 36PF^2(\zeta),$$

$$V(\zeta) = 3(c-4Q) - 36PF^2(\zeta). \tag{36}$$

Fourthly, choose P, Q and R in ODE (3) such that the corresponding solution $F(\zeta)$ of ODE (3) is one of Jacobi elliptic functions. By the same method used above we can obtain twelve periodic wave solutions to equations (27). For example, when $F(\zeta) = cn(\zeta, m)$, we get $P = -m^2$, $Q = (2m^2 - 1)$ from equation (3). The nondegenerative solution is obtained as:

$$U(\zeta) = -3(c - 4(2m^2 - 1)) - 36m^2 cn^2(\zeta, m),$$

$$V(\zeta) = 3(c - 4(2m^2 - 1)) + 36m^2 cn^2(\zeta, m). \tag{37}$$

When $m \to 1$ we get the solution:

$$U(\zeta) = -3(c-4) - 36\,\mathrm{sech}^2(\zeta),$$

$$V(\zeta) = 3(c-4) + 36\,\mathrm{sech}^2(\zeta). \tag{38}$$

The other wave solutions to the system (29) with the condition (30) can be obtained for different values of λ, μ by the same manner introduced in Section 2, but we omit them here for simplicity.

4. Conclusions

In this paper we have applied the F-expansion method to find a series of 12 exact solutions for the new generalized Hirota-Satsuma system. The same method is also applied for the coupled Nutku-Öğuz equation. The limit cases are obtained for all solutions.

References

[1] H. A. Abdusalam, A new approach to exact solutions for some nonlinear wave equations, Diff. Eq. Dynam. Sys. (to appear).

[2] E. Fan, Extended tanh-function method and its applications to nonlinear equations, Phys. Lett. A 277 (2000), 212.

[3] E. Fan and H. Zhang, A note on the homogeneous balance method, Phys. Lett. A 246 (1998), 403.

[4] H. C. Hu and Q. P. Liu, Decouple a coupled KDV system of Nutku and Ö$\tilde{g}$uz, arXiv:nlin. SISI/0110002 v2 10 Apr 2002.

[5] S. Liu, Z. Fu and Q. Zhao, Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations, Phys. Lett. A 289 (2001), 69.

[6] J. Liu, L. Yang and K. Yang, Nonlinear transform and Jacobi elliptic function solutions of nonlinear equations, Chaos Solitons & Fractals 20(5) (2004), 1157.

[7] Y. Matsuno, Reduction of dispersionless coupled Korteweg-de Vries equations to the Euler-Darbox equation, J. Math. Phys. 42(4) (2001), 1744.

[8] W. Mingliang, Exact solutions for a compound KdV-Burgers equation, Phys. Lett. A 213 (1996), 279.

[9] M. Mingliang and Y. Zhou, The periodic wave solutions for the Klein-Gordon-Schrödinger equations, Phys. Lett. A 318 (2003), 84.

[10] Y. Nutku and O. Ö$\tilde{g}$uz, Bi-Hamiltonian structure of a pair of coupled KdV equations, Nuovo Cimento B (11) 105(12) (1990), 1381.

[11] E. J. Parkes and B. R. Duffy, Travelling solitary wave solutions to a compound KdV-Burgers equation, Phys. Lett. A 229 (1997), 217.

[12] S. Shen and Z. Pan, A note on the Jacobi elliptic function expansion method, Phys. Lett. A 308 (2003), 143.

[13] M. Wang, Report on the workshop of mathematical physics, Lanzhou University, China, August 2002.

[14] Y. T. Wu, X. G. Geng, X. B. Hu and S. M. Zhu, On quasi-periodic solutions of the $2+1$ dimensional Caudrey-Dodd-Gibbon-Kotera-Sawada equation, Phys. Lett. A 255 (1999), 259.

■