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A RESULT ON ONE PLAN SUSPENSION SYSTEM

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Abstract

This paper is concerned with acceptance sampling systems when small sample size are necessary or desirable, for example, when production quantities are small or, when inspection is either costly or destructive. Under these conditions, a sampling plan with a small sample size is not very effective, since discrimination between good and bad quality is not sufficient. Nor does the lot-by-lot inspection provide an incentive for the producer to turn out consistently good quality. Hence it is intended to adopt one plan suspension system with Special Type of Double Sampling Plan (STDS) as the reference plan. Matching Special Type of Double Sampling Plan with Single Sampling Plan is also explained. Tables are provided for easy selection of the plan. Illustrations are also provided for practical usage of tables.

Cone and Dodge [1] have first shown that the effectiveness of a small sample lot-by-lot sampling system can be greatly improved by using cumulative results as a basis for suspending inspection.

Troxell [3] has applied this suspension principle to acceptance sampling system incorporating a suspension rule to suspend inspection on the basis of unfavourable lot history, when small sampling plans are

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necessary or desirable. A suspension system is a combination of a suspension rule and a single lot-by-lot sampling plan, or pair of plans. A suspension rule determines at each sample whether inspection shall be continued or evidence which dictates that sampling shall be suspended.

When single plan is used with a suspension rule it is called as *One Plan* (*OP*) *suspension system*. In OP suspension system, a lot-by-lot sampling plan is used in the usual way to decide whether individual lots shall be accepted or rejected. Similarly when two plans, tightened and normal are used it is called *Two Plan* (*TP*) *suspension system*. A suspension rule, which is designated as (j, k), $2 < j < k$ is a rule used for suspending inspection based on finding j lot rejections in k or less lots. Here suspension rule is a stopping time random variable and a suspension system is a rule used with a single sampling plan or a pair of normal and tightened plans.

Conditions for application

1. Production is steady, so that the results on current and preceding lots are broadly indicative of a continuos process.

2. Samples are taken from lots substantially in the order of production so that observed variations in quality of product reflect process performance.

3. Inspection is performed close to the production source so the inspection information can be made available promptly.

4. Inspection by attributes with quality measured in terms of fraction defective p.

5. A single sample of size, n or double or multiple samples of equal size n, taken from each sampled lot.

Operating procedure for suspension system

∗ For the product under consideration establish a Reference Quality Level (RQL). This RQL represents the desired quality at delivery considering the need of service and cost of production.

∗ Consider the established RQL, select a suspension system.

∗ Apply the suspension rule to the first, second, …, k-th lot, then to each successive group of k lots.

∗ If any lot is rejected, declare the lot nonconforming and dispose it in accordance with standard procedures.

∗ If for any lot, the suspension rule occurs, declare the current lot nonconforming and also declare the process nonconforming.

∗ When the process is judged to be nonconforming:

- Notify the submitting agency that no additional lots may be submitted for inspection until that agency has furnished evidence satisfactory to the inspection agency that action has been taken to assure the submission of satisfactory material.

- Dispose the current nonconforming lot in accordance with standard procedures.

- When satisfactory evidence of corrective action is furnished, start inspection again with the next succeeding lot and with this lot being cumulation.

- If it becomes necessary to refuse lot submission a second time, then advise an appropriate higher authority and notify the submitting agency that the further submission will be refused until evidence satisfactory to the higher authority has been approved.

Performance measures of one plan suspension system

Average Run Length (ARL)

According to Troxell [3] the expected time to suspension or average run length of a rule is important in the evaluation of a suspension system. The average run length of the suspension rule (j, k) designated $\text{ARL}(j, k)$ can be calculated in the following way.

First, the expected number of lot rejections until suspension is calculated. Since lot rejections are interspaced with lot acceptances, the second step is to find the total expected number of lots inspected, including the rejection lot, between successive lot rejections, the ARL equals the sum of the total number of inspected lots until suspension. In fact the total number of inspected lots between consecutive rejections are independently and identically distributed for all rejections so that:

ARL(j, k) = (Total expected number of inspected lots between two rejections) $*$ (Expected number of rejections until suspension).

(i) ARL for the rule (f, j), $j > 2$ is

$$\text{ARL}(f, j) = 1 - (1 - P_a).$$

(ii) ARL for the rule (j, ∞) is

$$\text{ARL}(j, \infty) = j/(1 - P_a).$$

(iii) ARL for the rule $(2, k)$ is

$$\text{ARL}(2, k) = \frac{2 - P_a^{(k-1)}}{(1 - P_a)(1 - P_a)} \quad \text{for} \quad k \geq 2. \tag{1}$$

(iv) ARL for the rule $(3, k)$ is

$$\text{ARL}(3, k) = 1/((1 - P_a) * (1 + 1/b_1 + 1/b_2 + 1/b_3 + \cdots + 1/b_{k-1} + 1)). \tag{2}$$

The coefficients are defined below:

$$b_1 = -P_a^{(k-1)}$$

$$b_{2n+1} = -(1 - P_a)P_a^{(k-n-2)}$$

$$b_{2n} = -(1 - P_a)P_a^{(n-1)},$$

where if k is even, $n = 1, 2, \ldots, (k-2)/2$; if k is odd $n = 1, 2, \ldots, (k-1)/2$ but b_k is not defined. Here P_a represents the probability of acceptance of an individual lot.

Special type double sampling plan

When sampling plans are set for product characteristics that involve costly or destructive testing by attributes, it is usual practice to use a Single Sampling Plan with acceptance number $c = 0$ and $c = 1$. But the OC curves of SSP with $c = 0$ and $c = 1$, lead to conflicting interest between the producer and the consumer. The plan $c = 0$ favours consumer while the plan $c = 1$ favours producer. Govindaraju [2] has proposed the STDS plan to avoid such shortcomings.

Operating procedure for STDS plan

1. From a lot, select a random sample of size n, and observe the number of defectives d_1. If $d_1 > 1$, then reject the lot. If $d_1 = 0$, then select a random sample of size n_2 and observe the number of defective d_2. If $d_2 < 1$, accept the lot, otherwise (if $d_2 > 2$), reject the lot.

The average fraction of lots for which the process is acceptable or the probability of accepting the process is given in equation (5). This value is designated as P_A. Figure 1 is an example of OC curves for the suspension systems using the rules (2, 2), (2, 3), (2, 4), (2, 5) and STDS plan. The unusual feature of suspension system OC curves is noted in Figure 1 the fact that the values of P_A is not smaller than 1/2. This arises because the minimal attainable ARL is 2.

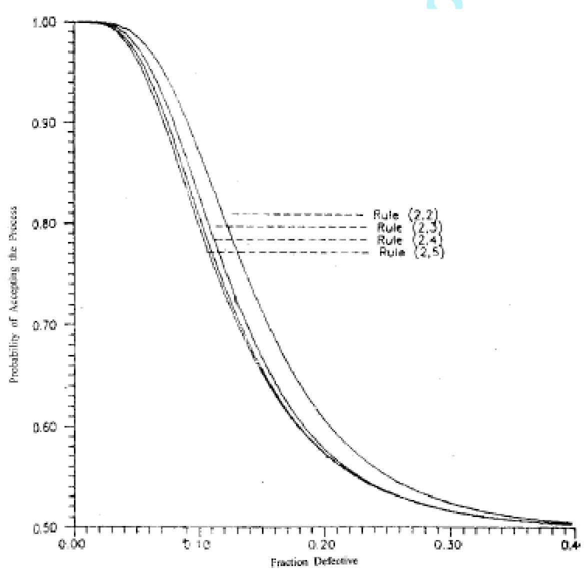

Figure 1. OC curves of the suspension system with the STDS plan (10; 0, 1)

Table 1. OR values of $(2, k)$ suspension system for two different STDS values

	$n_1 = 10, n_2 = 20$			$n_1 = 25, n_2 = 40$		
	$P_{0.98}$	$P_{0.80}$	OR	$P_{0.98}$	$P_{0.80}$	OR
2	0.01026	0.04417	4.2885	0.00472	0.02032	4.30508
3	0.00762	0.03584	4.7158	0.03352	0.01649	4.68466
4	0.00645	0.03277	5.0806	0.00298	0.01512	5.07483
5	0.00576	0.03123	5.4288	0.00266	0.01441	5.41729

Table 1 shows that $\text{OR}(2, 2) < \text{OR}(2, 3) < \text{OR}(2, 4) < \text{OR}(2, 5)$

According to this rule $(2, 2)$ discriminates the best in the $(2, k)$ class of rules, that is the ratio of the two fraction defectives is smaller than the ratio for any other value of R.

Designing of suspension system

Procedure A. RQL_1 and n are specified.

1. Select the desired values of RQL_1, and n.

2. Choose ARL, from one of the reference values in Table 2.

3. Use tables to find the rules $(2, 2)$, $(2, \text{last})$ which have fraction defective $p > \text{RQL}_1$.

4. (a) Use $(2, 2)$ for best discrimination.

(b) Use (2, last) for the rule having actual fraction defective closest to RQL_1.

5. If no $(2, k)$ exists use the class $(3, k)$ and follow step 1 through 4.

Example 1. For given STDS plan $(n = 11, c_1 = 0, c_2 = 1)$, $\text{RQL}_1 = 0.045$ and $\text{ARL}_1 = 10$ from Table 3 the rules (2, 2), (2, 3), (2, 4), and (2, 5) qualify for $p > 0.045$ at $\text{ARL}_1 = 10$. Hence the rule (2, 2) is used for the best discrimination and the rule (2, 5) for closeness at RQL_1.

Procedure B. RQL_1, RQL_2 and n are specified.

1. Select the desired values of RQL_1, RQL_2, and n.

2. Choose ARL_1 and ARL_2.

3. Use tables to find the rules (2, first) … (2, last) which have $p > \mathrm{RQL}_1$ at ARL_1 and $p < \mathrm{RQL}_2$ at ARL_2.

4. (a) Use (2, first) for best discrimination.

(b) Use (2, last) for closeness at RQL_1.

5. If no $(2, k)$ rule exists, use the class $(3, k)$ and follow step 1 through 4.

Example 2. For given STDS plan $(n = 8, c_1 = 0, c_2 = 1)$, $\mathrm{RQL}_1 = 0.06$, $\mathrm{RQL}_2 = 0.1$, $\mathrm{ARL}_1 = 10$ and $\mathrm{ARL}_2 = 5$, from Table 3 the rules (2, 2), (2, 3), (2, 4) and (2, 5) qualify for $p \geq 0.06$ at $\mathrm{ARL}_1 = 10$ and from Table 2 the rules (2, 4) and (2, 5) qualify for $p \leq 0.1$ for $\mathrm{ARL}_2 = 5$. Therefore, the rules (2, 4) and (2, 5) satisfy step 3 of Procedure B and hence the rule (2, 4) is used for best discrimination and (2, 5) for closeness at RQL_1.

Procedure C. RQL_1, and ARL are specified.

1. Select the desired value of RQL_1.

2. Choose ARL_1 from the table.

3. Use tables for each rule to find the smallest value of n such that $p \geq \mathrm{RQL}_1$.

4. (a) Use (2, 2) for best discrimination.

(b) Use (2, last) for the rule having smallest sample size.

Example 3. Given $\mathrm{RQL}_1 = 0.29$ and $\mathrm{ARL}_1 = 5$ from Table 2 the rules (2, 2), (2, 3), (2, 4) and (2, 5) qualify for $p \geq 0.29$, i.e., satisfy step 3 of Procedure C. Hence the rule (2, 2) is used for the best discrimination and rule (2, 5) is having the smallest sample size.

Procedure D. RQL_1 and RQL_2 are specified.

Select the desired values of RQL_1 and RQL_2.

1. Choose ARL_1 and ARL_2.

2. Use tables for each rule to find integers n_1 and n_2 such that $p \geq RQL_1$ for n, and $p \leq RQL_2$ for n_2.

3. (a) Use n_1 to match the suspension system as closely as possible to RQL_1.

(b) Use n_2 to match the suspension system as closely as possible to RQL_2.

Example 4. For given $RQL_1 = 0.04$ and $RQL_2 = 0.1$ select $ARL_1 = 10$ and $ARL_2 = 5$ from the reference values. From Table 2 for each rule 2 integers n_1 and n_2 are found such that $p \geq 0.07$ for n_1 and $p \leq 0.1$ for n_2.

For the rule

(2, 2)	$n_1 = 19$	$n_2 = 8$
(2, 3)	$n_1 = 15$	$n_2 = 7$
(2, 4)	$n_1 = 12$	$n_2 = 6.$

Construction of tables

The probability of acceptance $P_a(p)$ under binomial and Poisson models are given in equations (3) and (4), respectively,

$$P_a(p) = (1 - p)^n + n_2 p(1 - p)^{n-1} \tag{3}$$

$$P_a(p) = e^{-np} + n_2 p e^{-np}. \tag{4}$$

Using equation (3), ARL equations and the equation (1) the fraction defective value or RQL value is obtained for $n = 2(1)\ 20$ by the method of successive approximation. In Table 2, these p values are tabulated for given n and ARL for different rules. Using the equation (4), ARL equation and the np values are attained for given ARL by the method of successive approximation and are tabulated in Table 7. This 'np' value may be used to find suspension rule when n exceeds 20.

Comparison between one plan suspension system SSP $(n, c = 0)$ as reference plan and STDS $(n = n_1 + n_2, c_1 = 0, c_2 = 1)$ as reference plan

1. Comparison with respect to OC curve

Table 8 gives the value of fraction defective p for given n and ARL using SSP $(n, 0)$ and STDS $(\phi = 0.5, c_1 = 0, c_2 = 1)$ as reference plan, respectively. Let $p_1(j, k)$ be the fraction defective corresponding to the rule (j, k) using SSP as reference plan and $p_2(j, k)$ be the fraction defective corresponding to the rule (j, k) using STDS as the reference plan. For fixed n and ARL, it is seen that give $p_2(j, k) > p_1(j, k)$, where $j \le k \le \infty$, $j = 2 \approx 3$. So for fixed n, the OC curve of suspension system using STDS as reference plan is always below the OC curve of suspension system using SSP as reference plan. Hence for fixed P_a, STDS always gives less defective than the SSP in suspension system.

2. Comparison with reference to OR

From Table 9 one can find OR (j, k) in SSP is always greater than OR (j, k) of STDS in suspension rule (j, k) when $2 \le j \le k \le \infty$. Hence the discriminating power of STDS is better than SSP $(n, 0)$ as reference plan in suspension system.

References

[1] A. F. Cone and H. F. Dodge, A Cumulative Results Plan for Small Sample Inspection, Sandia Corporation Reprint, SCR-678, Albuquerque, NM, 1962.

[2] K. Govindaraju, Contribution to the study of certain special purpose plan, Ph.D. Thesis, Bharathiar University, Coimbatore, Tamil Nadu, India, 1984.

[3] J. R. Troxell, An investigation of suspension system for small sample inspection Ph.D. Thesis, Rutgers University, NJ, 1972.

[4] J. R. Troxell, Suspension system for small sample inspection, Technometrics 22(14) (1980), 517-533.

Table 2. Values of fraction defective p given $n = 2(1)20$, ARL$(2, k) = 5$, where $k = 2(1)5$

Sample size n	(2, 2)	(2, 3)	(2, 4)	(2, 5)
2	0.55826	0.47578	0.44204	0.42461
3	0.39229	0.32906	0.30406	0.29134
4	0.30265	0.25185	0.23207	0.22204
5	0.24639	0.20405	0.18770	0.17944
6	0.20777	0.17151	0.15750	0.15057
7	0.17962	0.14793	0.13581	0.12971
8	0.15818	0.13005	0.11932	0.11393
9	0.14132	0.11603	0.10640	0.10157
10	0.12770	0.10474	0.09602	0.09163
11	0.11648	0.09545	0.08747	0.08347
12	0.10707	0.87676	0.08032	0.07664
13	0.09907	0.81073	0.07426	0.07085
14	0.09218	0.07539	0.69045	0.06587
15	0.08619	0.70450	0.64575	0.06154
16	0.08092	0.06613	0.60543	0.05774
17	0.07627	0.06230	0.57031	0.05439
18	0.07212	0.05889	0.53905	0.05141
19	0.06840	0.05583	0.05110	0.04873
20	0.06504	0.05309	0.04858	0.04632

Table 3. Values of fraction defective p given $n = 2(1)20$, ARL$(2, k) = 10$, where $k = 2(1)5$

Sample size n	(2, 2)	(2, 3)	(2, 4)	(2, 5)
2	0.37016	0.29745	0.26559	0.24728
3	0.25211	0.20101	0.17897	0.16638
4	0.19142	0.05197	0.13509	0.12549
5	0.15433	0.12220	0.10852	0.10075
6	0.12929	0.10219	0.09069	0.08418
7	0.11126	0.08783	0.07790	0.67228
8	0.09764	0.07699	0.06827	0.06334
9	0.08699	0.06855	0.06077	0.05636
10	0.07849	0.06178	0.05474	0.05077
11	0.07141	0.05621	0.04981	0.04619
12	0.06555	0.05157	0.04569	0.04237
13	0.06057	0.04764	0.04220	0.03913
14	0.05629	0.04427	0.03920	0.03635
15	0.05258	0.04338	0.03661	0.03394
16	0.04934	0.03877	0.03434	0.03183
17	0.04646	0.03651	0.03233	0.02997
18	0.04391	0.03449	0.03054	0.02831
19	0.04162	0.03268	0.02894	0.02683
20	0.03956	0.03106	0.02750	0.02519

Table 4. Values of fraction defective p given $n = 2(1)20$, ARL$(2, k) = 50$, where $k = 2(1)5$

Sample size n	(2, 2)	(2, 3)	(2, 4)	(2, 5)
2	0.15177	0.11344	0.09633	0.08612
3	0.10154	0.07575	0.06430	0.05747
4	0.07631	0.05690	0.04827	0.04313
5	0.06113	0.04556	0.03864	0.03452
6	0.05100	0.03798	0.03221	0.02878
7	0.04375	0.03257	0.02762	0.02468
8	0.03830	0.02852	0.02417	0.02159
9	0.03406	0.02335	0.02149	0.01919
10	0.03066	0.02282	0.01934	0.01728
11	0.02789	0.02075	0.01758	0.01571
12	0.02556	0.01901	0.01613	0.01440
13	0.02361	0.01756	0.01489	0.01329
14	0.02192	0.01630	0.01382	0.01235
15	0.02046	0.01522	0.01290	0.01152
16	0.01919	0.01427	0.01209	0.01080
17	0.18063	0.01343	0.01139	0.01017
18	0.01706	0.01268	0.01075	0.00960
19	0.01616	0.01202	0.10099	0.00910
20	0.015358	0.01142	0.00968	0.00864

Table 5. Values of fraction defective p given $n = 2(1)20$, ARL$(3, k) = 5$, where $k = 3(1)5$

Sample size n	(3, 3)	(3, 4)	(3, 5)
2	0.78237	0.68604	0.64266
3	0.59006	0.49870	0.46112
4	0.47112	0.39116	0.35940
5	0.39149	0.32161	0.29440
6	0.33467	0.27360	0.24927
7	0.29217	0.23712	0.21613
8	0.25920	0.20957	0.19075
9	0.14142	0.18775	0.17072
10	0.12775	0.17004	0.15448
11	0.11650	0.15538	0.14060
12	0.10708	0.14304	0.12978
13	0.09907	0.13252	0.12018
14	0.09218	0.12344	0.11190
15	0.08619	0.11552	0.10468
16	0.08093	0.10856	0.09834
17	0.07627	0.10239	0.09273
18	0.07212	0.09688	0.08772
19	0.06840	0.09193	0.08322
20	0.06504	0.08747	0.07916

Table 6. Values of fraction defective p given $n = 2(1)20$, ARL$(3, k) = 10$, where $k = 3(1)5$

Sample size n	(3, 3)	(3, 4)	(3, 5)
2	0.575582	0.467354	0.413845
3	0.406043	0.322779	0.283497
4	0.313856	0.246864	0.215904
5	0.255803	0.199919	0.174396
6	0.215875	0.167992	0.146292
7	0.186727	0.144864	0.125995
8	0.164513	0.127336	0.110647
9	0.147022	0.113593	0.098632
10	0.132892	0.102529	0.088973
11	0.12124	0.093429	0.081037
12	0.111466	0.085812	0.074401
13	0.103150	0.079345	0.068770
14	0.095989	0.073783	0.063932
15	0.089758	0.068951	0.059729
16	0.084286	0.064712	0.056045
17	0.079443	0.060965	0.052789
18	0.075128	0.057628	0.049891
19	0.071255	0.054637	0.047294
20	0.067762	0.051941	0.044955

Table 7. Values of fraction defective np given
$(2, k) = 2(1)9, (3, k) = 3(1)5$

Rule	10	20	30	40	50	80	100
(2, 2)	0.29985	0.18862	0.14687	0.12377	0.10872	0.08328	0.07358
(2, 3)	0.23056	0.14139	0.10889	0.10118	0.07973	0.0606	0.05338
(2, 4)	0.20213	0.12129	0.09254	0.07706	0.0547	0.05073	0.04454
(2, 5)	0.18628	0.10967	0.08300	0.06878	0.05108	0.04485	0.03931
(2, 6)	0.17617	0.10197	0.07661	0.06321	0.04603	0.03798	0.03576
(2, 7)	0.1692	0.09644	0.07196	0.05915	0.05357	0.03575	0.03316
(2, 8)	0.16417	0.09226	0.06843	0.05605	0.04828	0.03555	0.03114
(2, 9)	0.19042	0.08899	0.06564	0.05355	0.04603	0.03394	0.02953
(3, 3)	0.54132	0.3651	0.29846	0.26083	0.2359	0.19253	0.17540
(3, 4)	0.40413	0.26621	0.21553	0.18737	0.16882	0.13694	0.12445
(3, 5)	0.3447	0.22183	0.17798	0.15391	0.13819	0.11139	0.10100

Table 8. Values of fraction defective p for given N and ARL = 5

Sample size N	SSP (2, 3)	STDS (2, 3)
2	0.27597	0.47578
3	0.19369	0.32906
4	0.14910	0.25185
5	0.12118	0.20405
6	0.10215	0.17151
7	0.08814	0.14793
8	0.07756	0.13005
9	0.06925	0.11603

10	0.06254	0.10474
11	0.05702	0.09545
12	0.05240	0.87676
13	0.04847	0.81073
14	0.04508	0.07539
15	0.04214	0.70450
16	0.03956	0.06613
17	0.03728	0.06230
18	0.03524	0.05889
19	0.03342	0.05583
20	0.03178	0.05309

Table 9. OR values of $(2, k)$ suspension system for SSP and STDS plans

	SSP for $n = 5$			STDS for $n = 5$		
	$P_{0.98}$	$P_{0.80}$	OR	$P_{0.98}$	$P_{0.80}$	OR
2	0.032	0.151	4.65	0.06153	0.26500	4.30
3	0.024	0.121	5.09	0.04573	0.21530	4.71
4	0.020	0.110	5.49	0.03872	0.19660	5.08
5	0.018	0.105	5.86	0.0346	0.18732	5.41

■