*Adv. & Appl. in Stat.* **6**(2) (2006), 241-248

# ON STATISTICAL MODELLING OF FLOODS AND DROUGHT INSURANCE PRODUCTS FOR DEVELOPING COUNTRIES

**R. O. SIMWA**

Department of Mathematics
University of Nairobi
P. O. Box 30197, Nairobi, Kenya
e-mail: rsimwa@yahoo.co.uk

## Abstract

Probabilistic models have been used to measure risk, the chance of occurrence of unfavourable events. The extend of damage may then take the form of a random variable. In this paper we follow such an approach to arrive at methods of obtaining estimates of the monetary economic loss due to the particular risks, disasters due to the floods and drought, with application to product pricing of the relevant possible insurance products.

Environmental natural disasters, which include floods and droughts, are common in the poor developing countries and may lead to destruction of property and even loss of lives. When such disasters occur emergent funds are required to meet immediate needs of the affected population (food, shelter, medicine, among others). The recent Tsunami disaster in East Asia, for instance, leads to urgent need of donor aid support by the affected countries. The financial support is mainly provided for by the rich developed countries. Such support could be availed before the disaster occurs and used to purchase the relevant insurance.



---

2000 Mathematics Subject Classification: 62p05.

Keywords and phrases: expectation and variance of random variables, risk loading factor, insurance, gross premium.

Received January 20, 2006

## 1. Introduction

The recent environmental disaster in Indonesia, the Tsunami disaster caused thousands of deaths and destruction of property through floods. In Kenya there have also been recurrent floods in Western region (Budalangi Floods) and droughts in the North Eastern region. This shows how nature can overwhelm developing economies like Kenya and Indonesia, and leaves substantial funding requirements. The Tsunami disaster leads to massive aid requirements from donors. In essence the main form of financial mitigation in such cases has been hurried transfers of aid (from developed and rich donor countries or organisation like UNICEF) with minimal contribution from the government in the affected country. Alternatively we note that this aid may be used for funding appropriate insurance protection against the accruing losses. For instance one internationally agreed on target for aid by the developed countries is 0.7% of the corresponding country's National Domestic Product such that the amount with respect to United Kingdom is currently £ 6 billion. Some of this money could be allocated to disaster relief. Other supra national bodies like the UNICEF may be willing to contribute towards the funding of the relevant insurance against the disasters. Thus either the donor country or the supra national body may unilaterally arrange for the insurance cover against disaster financial losses in a developing country (or countries) or the developing countries individually arrange for the insurance and then seek relevant funding from the developed country (or supra national body).



The major environmental disasters in Kenya (or Africa) have been drought and floods. In this study we analyse a general insurance model and apply it to drought and floods environmental disasters insurance management. The model is based on the traditional Individual Risk Model (Hossack et al. [3]) which will be referred to as the basic model. The insurance option in the management of these disasters benefits from the advantages of using insurance systems in an economy. The risk is thus budgeted for reducing uncertainty and the eventual financial loss when the disaster occurs.

The basic model is adapted for modelling disaster insurance products by considering the corresponding frequency variable, severity variable and the product variable.

## 2. The Basic Model and the Floods Disaster

The basic model is considered by analysing the corresponding frequency variable, severity variable and the product variable.

### 2.1. Frequency variable

We specify the frequency of occurrence of the floods which could be once in ten years, such that if

$$I = \{1 \text{ if the floods occur; } 0 \text{ otherwise}\}, \tag{2.1}$$

then $I$ is a random variable with a Bernoulli distribution. Let $q$ be the parameter for the distribution. Hence $E[I] = q$ and $Var[I] = q(1 - q)$.

### 2.2. Severity variable

The extent of flooding and the corresponding claim process can be linked through a mathematical relationship. The depth of flooding measured in certain part of the country can be used as an index on which the claims can be based. It is considered objective, easily verifiable and robust against interference. Through studying various cases, it has been noted (Waters [4]) that the cost of refuge, immediate flood relief and medical supplies increase approximately to the square of the depth of flooding, $d$. The severity, to be denoted by $B$ can thus be expressed as



$$\text{Severity, } B = d^2 * f_c, \tag{2.2}$$

where $f_c$ is the cost adjustment factor.

In general we consider $B$ as a random variable, since for the same part of the country where floods have occurred $d$ and $f_c$ may vary. For instance $d$ may vary with the terrain, soil types and vegetation while $f_c$ may depend on the economic level of the flooded area (quality of household properties, economic activity and the infra structure in the area, the level of affluence of the flood victims, among others).

Assuming that $B$ has the Poisson distribution with parameter $b$, we have

$$E[B] = b. \tag{2.3}$$

### 2.3. Product variable

Let $Y$ be the random variable representing the amount, in monetary terms, of the financial loss due to the floods in a given region. Then

$$Y = IB, \tag{2.4}$$

where $I$ and $B$ are as defined above. We introduce a random variable $X$, for the purpose of simplification as

$$X = Y/E[B], \tag{2.5}$$

then

$$X = IE[B] = Ib. \tag{2.6}$$

Thus $E[X] = bq$ and $Var[X] = b^2q(1-q)$.

## 3. Generalisation of the Basic Model



In this section, the generalisation of cases studied in Section 2, is discussed. Initially the case of several locations and the case of more than one year contract term are considered separately. Finally the case of draught is studied along the same lines as the case of floods.

### 3.1. A case of several locations

Let $Y_j$ be the financial loss variable due to the floods in the $j$th location (or country) and $N$ be the total number of locations. Then

$$Y = \sum_{j=1}^{N} Y_j. \tag{3.1}$$

Extending the results of Section 2, we have

$$Y_j = I_j B_j \tag{3.2}$$

and

$$X = Y/\{E[B_1] \cdots E[B_N]\} = \sum I_j b_j, \tag{3.3}$$

where $b_j = E[B_j]$; $j = 1, 2, \ldots, N$. Thus

$$E[X] = \sum b_j q_j \tag{3.4}$$

and

$$Var[X] = \sum b_j^2 q_j (1 - q_j). \tag{3.5}$$

**3.2. A case of more than one year contract term**

Let $i$ be the per annum risk-free interest rate. Then the discount rate for one year denoted by $v$ is given by $v = \frac{1}{1+i}$. Thus

$$Y = \sum_{j=1}^{m} Y_j v^j \text{ and } X = Y/\{E[B_1], \ldots, E[B_m]\}, \tag{3.6}$$

where $B_j$ is the severity contribution to the overall severity $B$ due to the $j$th year $(j = 1, 2, \ldots, m)$. In this manner, we have

$$X = \sum_{j=1}^{m} I_j b_j v^j, \tag{3.7}$$

where $b_j = E[B_j]$; $j = 1, 2, \ldots, m$.

Hence

$$E[X] = \sum_{j=1}^{m} q_j b_j v^j \tag{3.8}$$

and

$$Var[X] = \sum_{j=1}^{m} q_j (1 - q_j) b_j^2 v^{2j}. \tag{3.9}$$

**3.3. Combined case**

By combining Cases 3.1 and 3.2, we obtain a case of several locations and more than one year contract term. It is immediately found that in this case, we have

$$E[X] = \sum_{k=1}^{N} \sum_{j=1}^{m} q_{kj} b_{kj} v^{j} \tag{3.10}$$

and

$$Var[X] = \sum_{k=1}^{N} \sum_{j=1}^{m} q_{kj}(1 - q_{kj}) b_{kj}^{2} v^{2j}, \tag{3.11}$$

where $b_{kj} = E[B_{kj}]$; $B_{kj}$ being the severity level for the $k$th location $j$ years since the inception of the contract with $k = 1, 2, \ldots, N$ and $j = 1, 2, \ldots, m$. Analogous definition for $b_{kj}^2$ and $q_{kj}$ follows.

### 3.4. The generalised model and the drought disaster

The generalised model in Subsection 3.2 is adapted in considering drought disaster insurance modelling. The frequency, the severity and the product variables are as defined in Subsection 3.2 with the stated distributions. However the parameter estimates are derived differently as follows:

#### 3.4.1. The frequency estimate

Assume contract starts just after heavy rains and expected duration for occurrence of a drought is 12 years. Let the season qualify to be a drought if there have been low rains in three consecutive years. Then $q = 0.47$ (Waters [4]).

#### 3.4.2. The severity estimate

It is difficult to choose an appropriate loss index because of the large number of risk factors and their uncertain impact. However the amount insured should equal the expected value of losses resulting from all other factors. For simplicity a non-random severity variable may be assumed growing at say 5% per annum. Thus we have

$$B_j = b_j = \frac{b(1.05)^j}{m}, \tag{3.12}$$

where $b$ is the amount of loss if the drought occurred and $m$ is the term of the contract.

### 3.4.3. The product estimate

Assume a risk-free discount rate of say 5% such that $v = \frac{1}{1.05}$. Then $E[X]$ and $Var[X]$ are as given in equations (3.8) and (3.9), respectively.

## 4. Model Application in Premium Calculations

In this section, two hypothetical scenarios are considered, one on floods and another on drought to illustrate model application. In general the gross single premium for a general insurance cover against some risk, which is also the price of the insurance cover, is given by

$$\text{Gross Premium} = E[X] + k \times \sqrt{Var[X]}, \tag{4.1}$$

where $0 < k < 1$ is the *risk loading factor*. $k$ depends on the risk pricing of the parties involved. This premium may be raised to include brokerage costs and other investment fees.



### 4.1. Scenario A: Floods

The basic model in Section 2 is assumed for a one year contract with the following model parameter estimates:

$q = 0.1$, and $b = £370$ million if we assume that during the flood the claim is £2 per day, for 2 million people affected, and the coverage is three months. Let $k = 0.2$. Thus

$$\text{Gross Premium} = 0.1 \times 370 + 0.2 \times 370 \times \sqrt{0.1 \times 0.9} = £110 \text{ million}. \tag{4.2}$$

### 4.2. Scenario B: Drought

The generalised model in Section 3 is assumed with the following parameter estimates:

$q = 0.47$, $k = 0.2$, $i = 0.05$ per annum and let $b = £2.5$ billion (£2 per person per day, 7 million people, six months coverage). Assume a ten-year cover such that $b_j = \frac{2.5 \times (1.05)^j}{m}$, $j = 1, 2, \ldots, 10$ and $m = 10$.

Note that the amount $b_j$ correspond to year $j$ since the contract was made. Thus

$$E[X] = 0.1 \times 0.47 \times 2.5 \times 10 = £1.2 \text{ billion.}$$

Standard Error $= \sqrt{Var[X]} = 0.1 \times 2.5\sqrt{0.47(1-0.47)} \times 10 = £1.2$ billion and hence

$$\text{Gross Premium} = 1.2 + 0.2 \times 1.2 = £1.44 \text{ billion.} \tag{4.3}$$

## 5. Conclusions

- There is need for the appropriate data of some meteorological products that will provide information on the distribution of the frequency variable and the severity variable.

- Collaboration between meteorologists and actuaries will enhance the quality of the relevant weather risk insurance policies.

- Such models may guide the government (donor country or supra national organisations) in budgeting for the economic losses due to the risks from the environmental disasters.



## References

[1] R. Cowley, Floods, inflation, and under-insurance, The Actuary, October, 2002 ed., pp. 20-21.

[2] GIRO, Actuarial working party, Floods at York, November 2000, www.york.gov.uk/environment/flooding

[3] I. B. Hossack, J. H. Pollard and B. Zehnwirth, Introductory Statistics with Application to General Insurance, 2nd ed., Cambridge University Press, 1999.

[4] J. Waters, Funding for disasters, The Actuary, September, 2000 ed., pp. 30-31.

■