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OPTIMAL ORDERING POLICIES FOR EXPONENTIALLY DETERIORATING ITEMS UNDER SCENARIO OF PROGRESSIVE CREDIT PERIOD

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Abstract

The concept of progressive credit period given by supplier for settling the account is as follows: If the retailer settles outstanding amount by M, then the supplier does not charge any interest. If the retailer pays after M but before $N\,(N > M)$, then the supplier charges the retailer an unpaid balance at the rate I_{c_1}. If retailer settles the account after N, then he will have to pay an interest rate of $I_{c_2}\,(I_{c_2} > I_{c_1})$. Here an attempt is made to develop mathematical model, when units in inventory are subject to constant rate of deterioration and supplier provides two progressive credit periods. An easy-to-use algorithm is given to find the optimal solution to the presented mathematical formulation.

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1. Introduction

In practice, the credit period offered by the supplier is advantageous in two folds: viz (i) it encourages retailers to buy more and attracts new retailers; and (ii) it is the best substitute to price discounts. Brigham [3] denoted credit period as “net 30”, i.e., the supplier offers a retailer a delay period of 30 days for settling the account.

Goyal [6] gave mathematical model when the supplier offers the retailer a permissible delay period in settling the account. Shah [12, 13, 14] and Aggarwal and Jaggi [1], then extended Goyal’s model for exponentially deteriorating items. Jamal et al. [8] gave generalized model to allow for shortages. Hwang and Shinn [7] developed optimal pricing and lot-sizing for the retailer under the scheme of permissible delay in payments. Liao et al. [10] considered an inventory model for stock dependent demand rate when a delay in payments is permissible. Chang and Dye [4] considered backlogging rate to be inversely proportional to waiting time in Jamal et al. 1997’s model. Other related articles are by Arcelus et al. [2], Chang et al. [5], Jamal et al. [9], Sarker et al. [11], Shah [13, 14] and Teng [15].

This article deals with an *EOQ* model when units in inventory are subject to constant rate of deterioration and supplier offers two progressive credit periods to the retailer to settle the account. An algorithm is given to explore computational flow.

2. Assumptions and Notations

The following assumptions are used to develop aforesaid model:

- The inventory system deals with single item.
- The demand of R-units for an item is constant during the cycle time.
- Shortages are not allowed and lead-time is zero.
- Replenishment is instantaneous. Replenishment rate is infinite.
- The units inventory deteriorate at a constant rate (say), θ, $0 \leq \theta < 1$, during the cycle time.

- The deteriorated units can neither be repaired nor replaced during the period under review.
- If the retailer pays by M, then supplier does not charge to the retailer. If the retailer pays after M and before $N\,(M < N)$, then he can keep difference in unit sale price and unit cost in an interest bearing account at the rate of I_e per unit per year. During $[M, N]$, the supplier charges the retailer an interest rate of I_{c_1} per unit per year. If the retailer pays after N, then supplier charges the retailer an interest rate of I_{c_2} per unit per annum with $I_{c_2} > I_{c_1}$.

The mathematical development of the model is under following notations:

- R : the demand rate per annum.
- h : the inventory holding cost per unit per year excluding interest charges.
- p : the selling price per unit.
- C : the unit purchase cost, with $C < p$.
- M : the first permissible credit period in settling the account without any extra charges.
- N : the second permissible delay period in settling the account with an interest charge of I_{c_1} and $N > M$.
- I_{c_1} : the interest charged per \$ in stock per year by the supplier when the retailer pays after M but before N.
- I_{c_2} : the interest charged per \$ in stock per year by the supplier when the retailer pays after N.
- I_e : the interest earned per \$ per year.
- A : the ordering cost per order.
- Q : the procurement quantity (a decision variable).
- T : the replenishment cycle time (a decision variable).

- $Q(t)$: the on-hand inventory level at time $t\,(0 \le t \le T)$.
- $D(T)$: the number of units deteriorated during the cycle time T.
- $K(T)$: the total inventory cost per time unit which is sum of: (a) ordering cost; OC, (b) inventory holding cost (excluding interest charges); IHC, (c) purchase cost; PC, (d) interest charges; IC, for unsold items after the allowable delay period M or N, minus (e) interest earned from the sales revenue during the permissible delay period $[0, M]$.

3. Mathematical Formulation

The on-hand inventory depletes due to demand and deterioration of units at a constant rate θ. The instantaneous state of inventory at any time t is governed by the differential equation

$$\frac{dQ(t)}{dt} + Q(t) = -R, \;\; 0 \le t \le T \tag{1}$$

with the boundary condition $Q(0) = Q$ and $Q(T) = 0$. The solution of equation (1) is given by

$$Q(t) = \frac{R}{\theta}(e^{\theta(T-t)} - 1), \;\; 0 \le t \le T \tag{2}$$

and the procurement quantity is

$$Q = \frac{R}{\theta}(e^{\theta T} - 1). \tag{3}$$

The components of total inventory cost of the system per time unit are as follows:

(a) ordering cost;

$$OC = \frac{A}{T}, \tag{4}$$

(b) inventory holding cost;

$$IHC = \frac{h}{T}\int_0^T Q(t)dt = \frac{hR}{\theta^2 T}(e^{\theta T} - \theta T - 1), \tag{5}$$

(c) purchase cost;

$$PC = \frac{CR}{\theta T}(e^{\theta T} - 1). \tag{6}$$

The computations of interest charged and interest earned depend on the length of cycle time T. There are three possibilities:

Case 1. $T \leq M$.

Case 2. $M < T < N$.

Case 3. $T \geq N$.

Now, we discuss each case in detail.

Case 1. $T \leq M$.

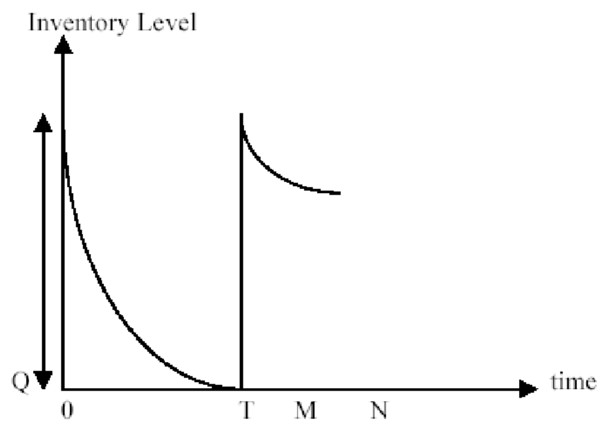

Figure 3.1

Here, the retailer sells Q-units in cycle time T and is paying CQ to the supplier in full at time $M \geq T$. So interest charges are zero, i.e.,

$$I_{c_1} = 0. \tag{7}$$

During $[0, T]$, the retailer sells products at price p per unit and deposits the revenue into an interest earning account at the rate of I_e per \$ per year. In the period $[T, M]$, the retailer only deposits the total revenue into an account that earns I_e per \$ per year. Hence, interest earned per unit time is

$$IE_1 = \frac{pI_e}{T}\left(\int_0^T Rtdt + RT(M - T)\right) = pI_eR\left(M - \frac{T}{2}\right). \tag{8}$$

Using (4)-(8), the total cost of an inventory system per time unit is

$$K_1(T) = OC + IHC + PC + IC_1 - IE_1. \tag{9}$$

The optimum value of $T = T_1$ is the solution of non-linear equation

$$\frac{dK_1(T)}{dT} = \frac{pI_eR}{2} + \frac{CRe^{\theta T} - \dfrac{hR(\theta - \theta e^{\theta T})}{\theta^2} + \dfrac{hR}{\theta}}{T} + \frac{-A - \dfrac{CR(e^{\theta T} - 1)}{\theta} + \dfrac{hR(1 - e^{\theta T})}{\theta^2}}{T^2} = 0. \tag{10}$$

The obtained $T = T_1$ minimizes the total cost because

$$\frac{d^2K_1(T)}{dT^2} = \frac{CR\theta R^{\theta T} + hRe^{\theta T}}{T} + \frac{-2CRe^{\theta T} + \dfrac{hR(\theta - \theta e^{\theta T})}{\theta^2} - \dfrac{hR}{\theta} - \dfrac{hRe^{\theta T}}{\theta}}{T^2} + \frac{2A + 2\dfrac{CR(e^{\theta T} - 1)}{\theta} - 2\dfrac{hR(1 - e^{\theta T})}{\theta^2}}{T^3} > 0 \,\forall\, T. \tag{11}$$

Case 2. $M < T < N$.

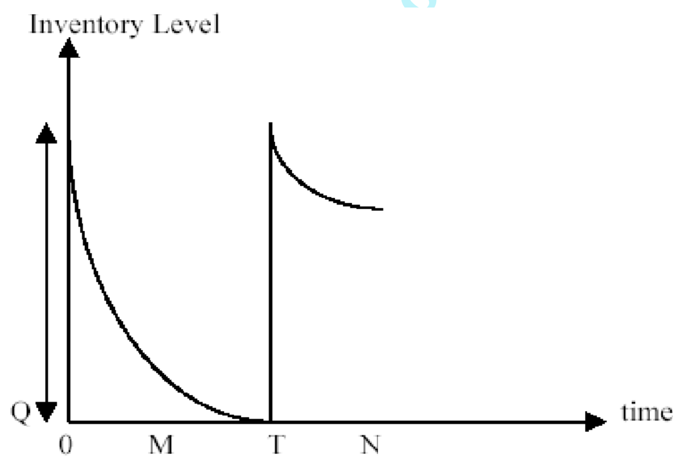

Figure 3.2

Here, interest earned, IE_2, during $[0, M]$ is $IE_2 = pI_e\int_0^M Rtdt = \frac{pI_eRM^2}{2}$.

Buyer has to pay for Q-unit at time $t = 0$ at the rate of C \$ per unit to the supplier up to time M, the retailer sells RM-units and has pRM plus interest earned IE_2 to pay the supplier. Depending on the difference between the total purchase cost; CQ, and the revenue; $pRM + IE_2$, two sub-cases may arise:

Sub-case 2.1. Let $pRM + IE_2 \geq CQ$.

Here, the retailer has sufficient amount in his account to pay off total purchase cost at M. Then interest charges,

$$IC_{2.1} = 0 \tag{12}$$

and interest earned,

$$IE_{2.1} = \frac{pI_e}{T}\int_0^M Rtdt = \frac{pI_eRM^2}{2T}. \tag{13}$$

Therefore, the total cost of an inventory system per time unit is

$$K_{2.1}(T) = OC + IHC + PC + IC_{2.1} - IE_{2.1}. \tag{14}$$

The optimum value of $T = T_{2.1}$ is a solution of non-linear equation

$$\frac{dK_{2.1}(T)}{dT} = \frac{CRe^{\theta T} - \dfrac{hR(\theta - \theta e^{\theta T})}{\theta^2} + \dfrac{hR}{\theta}}{T} + \frac{-A - \dfrac{CR(e^{\theta T} - 1)}{\theta} + \dfrac{hR(1 - e^{\theta T})}{\theta^2} + \dfrac{pI_eRM^2}{2}}{T^2} = 0 \tag{15}$$

and $T = T_{2.1}$ minimizes the total cost $K_{2.1}$ of an inventory system because

$$\frac{d^2K_{2.1}(T)}{dT^2} = \frac{CR\theta R^{\theta T} + hRe^{\theta T}}{T} - \frac{CRe^{\theta T} - \dfrac{hR(\theta - \theta e^{\theta T})}{\theta^2} + \dfrac{hR}{\theta}}{T^2} + \frac{-CRe^{\theta T} - \dfrac{hRe^{\theta T}}{\theta}}{T^2} - 2\,\frac{-A - \dfrac{CR(e^{\theta T} - 1)}{\theta} + \dfrac{hR(1 - e^{\theta T})}{\theta^2} + \dfrac{pI_eRM^2}{2}}{T^3} > 0 \;\forall\; T. \tag{16}$$

Sub-case 2.2. Let $pRM + IE_2 < CQ$.

In this case, the retailer does not have sufficient money in his account to do payment at given permissible credit period, M, then supplier charges retailer on the unpaid balance, $U_1 = CQ - [pRM + IE_2]$ at the interest rate I_{c_1} at time M, therefore interest charges; $IC_{2.2}$ per time unit is

$$IC_{2.2} = \frac{U_1^2 I_{c_1}}{2pRT} \int_M^T Q(t)dt$$

$$= \left(\frac{CR(e^{\theta T} - 1)}{\theta} - pRM - \frac{1}{2} pI_e RM^2\right)^2 I_{c_1}\left(\frac{-(1+\theta T)R}{\theta^2} + \frac{(e^{-\theta(-T+M)} + M\theta)R}{\theta^2}\right) \Big/ pRT \tag{17}$$

and interest earned,

$$IE_{2.2} = \frac{pI_e}{T}\int_0^M Rtdt = \frac{pI_e RM^2}{2T}. \tag{18}$$

Therefore, the total cost of an inventory system per time unit is

$$K_{2.2}(T) = OC + IHC + PC + IC_{2.2} - IE_{2.2}. \tag{19}$$

The optimum value of $T = T_{2.2}$ can be obtained by solving non-linear equation

$$\frac{dK_{2.2}(T)}{dT} = \frac{pIeRM^2 - 2A}{2T^2} + \frac{CRe^{\theta T}(T-1)}{\theta T^2} + \frac{C + hR(e^{\theta T} + e^{-\theta T})}{p\theta^2 T} - \frac{2U_1 I_{c_1}(-e^{\theta(T-M)} - M\theta + 1 + T\theta)CRe^{\theta T}}{p\theta^2 T} + \frac{U_1^2 I_{c_1}(e^{\theta(T-M)}(\theta T - 1) + 1 - M\theta)}{p\theta^2 T} = 0 \tag{20}$$

with suitable iterative method. The sufficiency condition is

$$\frac{d^2K_{2.2}(T)}{dT^2} = \frac{2A - pIeRM^2}{T^3} - \frac{2CRe^{\theta T}}{T^2} + \frac{Re^{\theta T}(C\theta + h)}{T} + \frac{2CR(e^{\theta T} - 1)}{\theta T^3} + \frac{2hR(e^{\theta T}(1 - \theta T) - 1)}{\theta^2 T^3}$$
$$- \frac{2CRe^{\theta T} I_{c_1}(-e^{\theta(T-M)} - M\theta + 1 + \theta T)(CRe^{\theta T} + U_1\theta)}{p\theta^2 T}$$
$$+ \frac{2U_1 I_{c_1}(4CRe^{\theta T} - U_1)(e^{\theta(T-M)}(\theta T - 1) + 1 - M\theta)}{p\theta^2 T^3}$$
$$+ \frac{U_1^2 I_{c_1} e^{\theta(T-M)}}{pT} > 0 \ \forall \ T. \qquad (21)$$

Case 3. $T \geq N$.

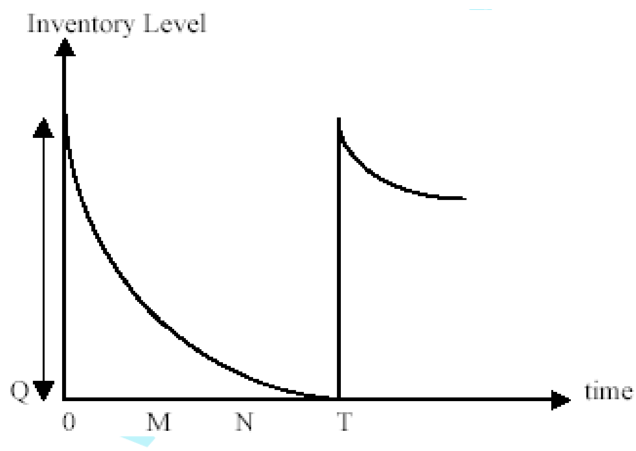

Figure 3.3

We proceed as Case 2. Total purchase cost of Q-units is CQ, the amount of money in retailer's account at M is $pRM + \frac{pI_eRN^2}{2}$. The following three sub-cases arise:

Sub-case 3.1. Let $pRM + IE_2 \geq CQ$.

This sub-case is same as Sub-case 2.1.

Here, interest charges,

$$IC_{3.1} = 0 \tag{22}$$

and interest earned,

$$IE_{3.1} = \frac{pI_e}{T}\int_0^M Rtdt = \frac{pI_eRM^2}{2T}. \tag{23}$$

Therefore, the total cost of an inventory system per time unit is

$$K_{3.1}(T) = OC + PC + IHC + IC_{3.1} - IE_{3.1}. \tag{24}$$

Sub-case 3.2. Let $pRM + IE_2 < CQ$ and

$$pR(N-M) + \frac{pI_eR(N^2 - M^2)}{2} \geq CQ - (pRM + IE_2).$$

Here, retailer does not have enough money in his account to settle the payment at time M but he can do it before or at N. At M, retailer pays $pRM_1 + IE_2$ and supplier charges for the unpaid balance $U_1 = CQ - (pRM + IE_2)$ with interest rate I_{c_1}. This situation is same as Sub-case 2.2. The total cost $K_{3.2}(T)$, of an inventory system per time unit is

$$K_{3.2}(T) = OC + PC + IHC + IC_{3.1} - IE_{3.2}. \tag{25}$$

Sub-case 3.3. Let $pRM + IE_2 < CQ$ and

$$pR(N-M) + \frac{pI_eR(M_2 - M_1)^2}{2} < CQ - (pRM + IE).$$

Here, retailer does not have money in his account to pay off total purchase cost at time N, he pays $pRM + IE_2$ at M and $pR(N-M) + \frac{pI_eR}{2}(N^2 - M^2)$ at N. Here, retailer will have to pay interest charges on the unpaid balance $U_1 = CQ - (pRM + IE_2)$ with interest rate I_{c_1} during $[M, N]$ and unpaid balance,

$$U_2 = U_1 - \left(pR(N-M) + \frac{pI_eR}{2}(N^2 - M^2)\right)$$

with interest rate I_{c_2} during $[N, T]$. Hence total interest payable per time unit is

$$IC_{3.3} = \frac{U_1 I_{c_1}(N - M)}{T} + \frac{U_2^2}{pRT} I_{c_2} \int_N^T Q(t)dt \tag{26}$$

and interest earned,

$$IE_{3.3} = \frac{pI_e}{T} \int_0^M Rtdt = \frac{pI_e RM^2}{2T}. \tag{27}$$

Therefore the total cost of an inventory system per time unit is

$$K_{3.3} = OC + PC + IHC + IC_{3.3} - IE_{3.3}. \tag{28}$$

The first order condition for $K_{3.3}(T)$ to be minimum is

$$\frac{dK_{3.3}(T)}{dT} = \frac{pI_e RM^2 - 2A}{2T^2} + \frac{CRe^{\theta T}(\theta T - 1) + CR}{\theta T^2}$$

$$+ \frac{hR(e^{\theta T}(\theta T - 1) + 1)}{\theta^2 T^2} - \frac{U_1 I_{c_1}(N - M)}{T^2} + \frac{CRe^{\theta T} I_{c_1}(N - M)}{T}$$

$$+ \frac{2\%1 I_{c_2}(e^{-\theta(N-T)} + N\theta - 1 - \theta T)CRe^{\theta T}}{p\theta^2 T}$$

$$+ \frac{\%1^2 I_{c_2}(e^{-\theta(N-T)}(\theta T - 1) - N\theta + 1)}{p\theta^2 T^2} = 0$$

$$\%1 = \left(U_1 - pR(N - M) - \frac{pI_e R(N^2 - M^2)}{2}\right) \tag{29}$$

and the sufficiency condition is

$$\frac{d^2 K_{3.3}(T)}{dT^2} = \frac{2A - pI_e RM^2}{T^3} + \frac{CRe^{\theta T}(\theta T - 2)}{T^2}$$

$$+ \frac{2CR(e^{\theta T} - 1)}{\theta T^3} + \frac{hRe^{\theta T}}{T} + \frac{2hR(e^{\theta T}(1 - \theta T) - 1)}{\theta^2 T^3}$$

$$+\frac{2U_1 I_{c_1}(N-M)}{T^3}+\frac{CRe^{\theta T} I_{c_1}(N-M)(T-2)}{T^2}$$

$$+\frac{2I_{c_2}(e^{-\theta(N-T)}+N\theta-1-\theta T)((CRe^{\theta T})^2+\%1^2)}{p\theta^2 T^3}$$

$$+\frac{2\%1 I_{c_2}(e^{-\theta(N-T)}-1)(2CRTe^{\theta T}-\%1)}{p\theta T^2}+\frac{\%1^2 I_{c_2} e^{-\theta(N-T)}}{pT}$$

$$+\frac{2\%1 I_{c_2}(e^{-\theta(N-T)}(\theta T-1)-N\theta+1)CRe^{\theta T}(\theta T-2)}{p\theta^2 T^2}>0\ \forall\, T,$$

where

$$\%1=\left(U_1-pR(N-M)-\frac{pI_e R(N^2-M^2)}{2}\right). \tag{30}$$

In the next section, we present computational algorithm to search for optimal solution.

4. Computational Algorithm

- **Step 1.** Given parametric values of R, h, I_{c_1}, I_{c_2}, I_e, C, p, M, N in proper units.
- **Step 2.** Compute $T = T_1$ from (10).

If $T_1 \le M$, then find $K_1(T_1)$ using (10) and go to Step 3.

Otherwise,

If $M < T < N$, then

If $pRM + \frac{pI_e RM^2}{2} \ge CQ$

Then compute $T = T_{2.1}$ from (15)

and $K_{2.1}(T_{2.1})$ from (14). Go to Step 3.

Otherwise

Compute $T = T_{2.2}$ from (20)

and $K_{2.2}(T_{2.2})$ from (19). Go to Step 3.

Otherwise

If $pRM + \frac{pI_e RM^2}{2} \geq CQ$

Then compute $T = T_{3.1}$ from (15)

and $K_{3.1}(T_{3.1})$ from (14). Go to Step 3.

Otherwise

If $pRM + \frac{pI_e RM^2}{2} < CQ$ and

$$pR(N - M) + \frac{pI_e R(N - M)^2}{2} < CQ - \left(pRM + \frac{pI_e RM^2}{2}\right)$$

Then compute $T = T_{3.2}$ from (20)

and $K_{3.2}(T_{3.2})$ from (25). Go to Step 3.

Otherwise

Compute $T = T_{3.3}$ from (29) and $K_{3.3}(T_{3.3})$

from (28). Go to Step 3.

• **Step 3.**

$$K(T) = \text{mini}\{K_1(T_1), K_{2.1}(T_{2.1}), K_{2.2}(T_{2.2}), K_{3.1}(T_{3.1}), K_{3.2}(T_{3.2}), K_{3.3}(T_{3.3})\}$$

and corresponding optimum T and Q.

5. Theoretical Results

Proposition 5.1. *For* $T \leq M$, $K_1(T)$ *is minimum.*

Proof. $\dfrac{d^2 K_1(T)}{dT^2}$ given by equation (11) is non-negative for all $T \leq M$.

Proposition 5.2. *For* $T \leq M$, $K_1(T)$ *is decreasing function of allowable credit period M.*

Proof. $\dfrac{d^2 K_1(T)}{dM} = -pI_e R < 0.$

Proposition 5.3. *For* $M < T < N$, *if* $pRM + IE_2 > CQ$, *then* $K_{2.1}(T_{2.1})$ *is minimum otherwise* $K_{2.2}(T_{2.2})$ *is minimum.*

Proof. Clearly, from equation (16), $\dfrac{d^2 K_{2.1}(T)}{dT^2} > 0$. Otherwise from equation (21), $\dfrac{d^2 K_{2.2}(T)}{dT^2} > 0$.

Proposition 5.4. *For* $M < T < N$, $K_{2.1}(T)$ (*or* $K_{2.2}(T)$) *is decreasing function of credit period M, for all T.*

Proof. $\dfrac{dK_{2.1}(T)}{dM} = -\dfrac{pI_e RM}{T} < 0, \forall T$ and

$$\frac{dK_{2.2}(T)}{dM} = -\frac{2U_1 I_{c_1}(e^{\theta(T-M)} - M\theta + 1 + \theta T)(-R - RI_e M)}{\theta^2 T} - \frac{pI_e RM}{T}$$

$$-\frac{U_1^2 I_{c_1}(\theta e^{\theta(T-M)} - \theta)}{p\theta^2 T} < 0, \forall T.$$

Proposition 5.5. *For* $T > N$, $K_{3.2}(T)$ *is minimum if* $pRM + IE_2 < CQ$ *and*

$$pR(N - M) + \frac{PI_e R}{2}(N^2 - M^2) \geq CQ - (pRM + IE_2)$$

and $K_{3.3}(T)$ *is minimum if*

$$pR(N-M)+\frac{pI_eR}{2}(N^2-M^2)<CQ-(pRM+IE_2).$$

Proof. Obvious from equations (20) and (27).

Proposition 5.6. *For* $T>N$, $K_{3.2}(T)$ *and* $K_{3.3}(T)$ *are decreasing functions of M. Also* $K_{3.3}(T)$ *is increasing function of N.*

Proof.

$$\frac{dK_{3.2}(T)}{dM}=-\frac{2U_1I_{c_1}(e^{\theta(T-M)}-M\theta+1+\theta T)(-R-RI_eM)}{\theta^2T}$$

$$-\frac{pI_eRM}{T}-\frac{U_1^2I_{c_1}(\theta e^{\theta(T-M)}-\theta)}{p\theta^2T}<0,\ \forall T,$$

$$\frac{dK_{3.3}(T)}{dM}=-\frac{(pR+pI_eRM)I_{c_1}(N-M)}{T}$$

$$-\frac{U_1I_{c_1}}{T}-\frac{pI_eRM}{T}<0,$$

$$\%1=\left(U_1-pR(N-M)-\frac{pI_eR(N^2-M^2)}{2}\right),$$

$$\frac{dK_{3.3}(T)}{dN}=\frac{U_1I_{c_1}}{T}+\frac{2\%1I_{c_2}(e^{-\theta(N-T)}+N\theta-1-\theta T)(R-I_eRN)}{\theta^2T}$$

$$+\frac{\%1^2I_{c_2}(-\theta e^{-\theta(N-T)}+\theta)}{p\theta^2T}>0$$

$$\%1=\left(U_1-pR(N-M)-\frac{pI_eR(N^2-M^2)}{2}\right).$$

6. Conclusion

In this paper, an attempt is made to come up with mathematical formulation and analytic theoretical results. When supplier offers progressive credit periods, if retailer could not pay his unpaid balance, when units in inventory system are subject to constant rate of deterioration. The computational easy-to-use algorithm is given to search for optimal policy.

The derived model can be extended to a two parameter Weibull distribution. It can be extended by taking different forms of demand functions and other things.

References

[1] S. P. Aggarwal and C. K. Jaggi, Ordering policies of deteriorating items under permissible delay in payments, J. Oper. Res. Soc. 46 (1995), 658-662.

[2] F. Arcelus, Nita H. Shah and G. Srinivasan, Retailer's response to special sales: price discount vs trade credit, OMEGA 29(5) (2001), 417-428.

[3] E. F. Brigham, Fundamentals of Financial Management, The Dryden Press, Florida, 1995.

[4] H. J. Chang and C. Y. Dye, An inventory model for deteriorating items with partial back-logging and permissible delay in payments, Internat. J. Systems Sci. 32 (2001), 345-352.

[5] C. T. Chang, L. Y. Ouyang and J. T. Teng, An EOQ model for deteriorating items under supplier credits linked to order quantity, Appl. Math. Model. 27 (2003), 983-996.

[6] S. K. Goyal, Economic order quantity under conditions of permissible delay in payments, J. Oper. Res. Soc. 36 (1985), 335-338.

[7] H. Hwang and S. W. Shinn, Retailer's pricing and lot-sizing policy for exponentially deteriorating products under the conditions of permissible delay in payments, Comput. Oper. Res. 24 (1997), 539-547.

[8] A. M. M. Jamal, B. R. Sarker and S. Wang, An ordering policy for deteriorating items with allowable shortages and permissible delay in payment, J. Oper. Res. Soc. 48 (1997), 826-833.

[9] A. M. M. Jamal, B. R. Sarker and S. Wang, Optimal payment time for a retailer under permitted delay of payments by the wholesaler, Internat. J. Prod. Econo. 66 (2000), 59-66.

[10] H. C. Liao, C. H. Tsai and C. T. Su, An inventory model for deteriorating items under inflection when a delay in payments is permissible, Internat. J. Prod. Econ. 63 (2000), 207-214.

[11] B. R. Sarker, A. M. M. Jamal and S. Wang, Optional payment time under permissible delay for production with deterioration, Production Planning and Control 11 (2000), 380-390.

[12] Nita H. Shah, A lot – size model for exponentially decaying inventory when delay in payments is permissible, Cahiers du Centre D' Etudes de Recherche Operationell Operations Research, Statistics and Applied Mathematik 35 (1993a), 1-9.

[13] Nita H. Shah, A probabilistic order level system when delay in payments is permissible, J. Korean Oper. Res. Manage. Sci. 18(2) (1993b), 175-183.

[14] Nita H. Shah, Probabilistic time scheduling model for exponentially decaying inventory when delay in payments is permissible, Internat. J. Prod. Econ. 32 (1993c), 77-82.

[15] J. T. Teng, On the economic order quantity under conditions of permissible delay in payments, J. Oper. Res. Soc. 53 (2002), 915-918.

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