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# ON *c*-SPACES AND HYPERGRAPHS

**A. K. Sruthi and P. T. Ramachandran**

Department of Mathematics
University of Calicut
Calicut University P.O., 673635
Kerala, India

## Abstract

In this paper, we examine the *c*-structure generated by the edge set of a hypergraph and prove that the elements of this *c*-structure induced by the hypergraph are the vertex sets of the connected hypersubgraphs. Further, we try to find some interrelations between a hypergraph and the *c*-space induced by that hypergraph.

## 1. Introduction

The concept of connectedness has applications in the field of Digital Topology and Image Processing. A set with a *c*-structure on it is called *c-space*. In 1983, Börger [3] proposed an axiomatic approach to connectivity, known as the theory of connectivity class or *c*-structures. A systematic study of *c*-spaces was further carried out by Serra [12] and further extended by Heijmans [7], Ronse [10], Muscat and Buhagiar [8], Dugowson [6], Santhosh [11], etc. In this paper, we are trying to study the theory of *c*-spaces comparable with the theory of hypergraphs, which is relevant because hypergraphs too have applications in the field of image processing. Hope

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that this will help us to develop the theory of $c$-spaces that has more applicability in the field of image processing.

## 2. Preliminaries

A $c$-structure on a set $X$ is a collection $\mathcal{C}$ of subsets of $X$ such that the following properties hold:

(i) $\varnothing \in \mathcal{C}$ and $\{x\} \in \mathcal{C}$ for every $x \in X$.

(ii) If $\{C_i : i \in I\}$ is a nonempty collection of members of $\mathcal{C}$ with $\bigcap_{i \in I} C_i \neq \varnothing$, then $\bigcup_{i \in I} C_i \in \mathcal{C}$.

The set $X$ together with a $c$-structure $\mathcal{C}$, that is, $(X, \mathcal{C})$ is called a *c-space* [8] and elements of $\mathcal{C}$ are called *connected sets* of $(X, \mathcal{C})$. The empty set and singleton sets of a $c$-structure are called *trivial connected sets* and the elements which are neither empty nor singleton of a $c$-space are called *non-trivial connected sets*.

For any set $X$, let $\mathcal{D} = \{\varnothing\} \cup \{\{x\} : x \in X\}$. Clearly, $\mathcal{D}$ is a $c$-structure on $X$.

Let $X$ and $Y$ be two $c$-spaces and $f : X \to Y$ be a function. $f$ is called *c-continuous* [8] or *catenuous* [8], if it maps connected sets of $X$ to connected sets of $Y$. Also, a bijection $f$ is said to be a *c-isomorphism* or *catenomorphism* if both $f$ and $f^{-1}$ are $c$-continuous.

Let $X$ be a set and $\mathcal{B} \subseteq \mathcal{P}(X)$. Then the intersection of all $c$-structures on $X$ containing $\mathcal{B}$ is a $c$-structure on $X$, called the *c-structure* generated by $\mathcal{B}$ and is denoted by $\langle \mathcal{B} \rangle$. It is the smallest $c$-structure on $X$ containing $\mathcal{B}$.

The non-trivial connected sets of a $c$-structure generated by $\mathcal{B}$ are characterized by the condition that any two points of such a connected set $C$ can be joined by a finite chain of elements of $\mathcal{B}$. That is, for all $x, y \in C$, we can find elements $B_i$, $i = 0$ to $n$ in $\mathcal{B}$ such that $B_i \subseteq C$, $B_i \cap B_{i+1} \neq \emptyset$ for $i = 0$ to $n - 1$ and $x \in B_0$, $y \in B_n$ for some positive integer $n$.

A hypergraph [13] $H$ is an ordered pair $(X, \mathcal{E})$, where $X$ is a set and $\mathcal{E} = \{E_i : i \in I\}$ is a family of nonempty subsets of $X$. The elements of $X$ are called *vertices* and the elements of $\mathcal{E}$ are called the *edges* or *hyper edges*.

Consider the hypergraph $H = (X, \mathcal{E})$. Then a hypergraph $H' = (X', \mathcal{E}')$ is said to be a *hypersubgraph* [1] or *strong subhypergraph* [4] of $H$ whenever $X' \subseteq X$ and $\mathcal{E}' \subseteq \mathcal{E}$.

In a hypergraph $H = (X, \mathcal{E})$, a chain [2] from the vertex $x_1$ to the vertex $x_{q+1}$ is an alternated vertex-edge sequence $(x_1, E_1, x_2, E_2, ..., E_q, x_{q+1})$ of distinct vertices and edges of $H$ such that for $i = 1, 2, ..., q$, $\{x_i, x_{i+1}\} \subseteq E_i$, where $q$ is called the *length* of the chain.

Let $H = (X, \mathcal{E})$ be a hypergraph. Then the vertices $a, b \in X$ are said to be *connected* in $H$ if there exists a chain from $a$ to $b$. The hypergraph $H$ is said to be *connected* if every pair of distinct vertices is connected in $H$.

Two hypergraphs $H = (X, \mathcal{E})$ and $H' = (X', \mathcal{E}')$ are said to be *isomorphic* [2] if there exists a bijection $\phi : X \to X'$ such that, for every $E \subseteq X$, $E \in \mathcal{E}$ if and only if $\phi(E) \in \mathcal{E}'$.

## 3. Hypergraph Induced *c*-spaces

The edge set $\mathcal{E}$ of a hypergraph $H = (X, \mathcal{E})$ is a collection of nonempty subsets of $X$. Therefore, $\mathcal{E}$ cannot be a $c$-structure on the set $X$. But there always exists a smallest $c$-structure on $X$ containing $\mathcal{E}$.

**Example 3.1.** Consider the hypergraph $H = (X, \mathcal{E})$, where $X = \{1, 2, 3, 4, 5\}$ and $\mathcal{E} = \{\{1, 2\}, \{2, 3, 4\}, \{4, 5\}\}$. Here the edge set $\mathcal{E}$ contains neither empty set nor singleton sets, therefore $\mathcal{E}$ is not a $c$-structure on $X$.

The $c$-structure on $X$ generated by the edge set $\mathcal{E}$ is given by $\mathcal{C} = \langle \mathcal{B} \rangle = \mathcal{D} \bigcup \{\{1, 2\}, \{2, 3, 4\}, \{4, 5\}, \{1, 2, 3, 4\}, \{2, 3, 4, 5\}, \{1, 2, 3, 4, 5\}\}$.

**Definition 3.1.** Consider the hypergraph $H = (X, \mathcal{E})$ and let $\mathcal{C} = \langle \mathcal{E} \rangle$ be the $c$-structure generated by the edge set of $H$. Then $\mathcal{C}$ is called the *c-structure* induced by the hypergraph $H$ and the corresponding $c$-space $(X, \mathcal{C})$ is called the *c-space* induced by the hypergraph $H$.

**Remark 3.1.** Any $c$-space can be considered as an induced $c$-space of some hypergraph. For any $c$-space $(X, \mathcal{C})$, let $\mathcal{E} = \mathcal{C} - \{\varnothing\}$. Then the $c$-space induced by the hypergraph $H = (X, \mathcal{E})$ is same as the $c$-space $(X, \mathcal{C})$. But a $c$-space may be considered as an induced $c$-space of more than one hypergraph. Consider the following example:

**Example 3.2.** Consider the $c$-space $(X, \mathcal{C})$, where $X = \{a, b, c, d\}$ and $\mathcal{C} = \mathcal{D} \bigcup \{\{a, b\}, \{b, c\}, \{a, b, c\}\}$. Let $\mathcal{E}_1 = \{\{a, b\}, \{b, c\}\}$, $\mathcal{E}_2 = \{\{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{E}_3 = \mathcal{C} - \{\varnothing\}$. Then the hypergraphs $H_1 = (X, \mathcal{E}_1)$, $H_2 = (X, \mathcal{E}_2)$ and $H_3 = (X, \mathcal{E}_3)$ have the property that $\langle \mathcal{E}_1 \rangle = \langle \mathcal{E}_2 \rangle = \langle \mathcal{E}_3 \rangle = \mathcal{C}$.

**Theorem 3.1.** *Let $H = (X, \mathcal{E})$ be a hypergraph and let $(X, \mathcal{C})$ be the c-space induced by the hypergraph $H$. Then the members of $\mathcal{C}$ are the vertex sets of the connected hypersubgraphs of $H$.*

**Proof.** Let $\mathcal{V}_H$ be the collection of all vertex sets of the connected hypersubgraphs of $H$. Suppose $C$ is a trivial connected set of $(X, \mathcal{C})$. Then $H' = (C, \varnothing)$ is a connected hypersubgraph of $H$ and hence $C \in \mathcal{V}_H$. Now suppose $C$ is a non-trivial connected set of $(X, \mathcal{C})$. Consider the hypersubgraph $H' = (C, \{E_i \in \mathcal{E} : E_i \subseteq C\})$ of $H$. Let $a, b \in C$. Then there exist $E_{k1}, E_{k2}, ..., E_{km}$ such that $E_{ki} \subseteq C$ for $i = 1, 2, ..., m$ and $E_{ki} \bigcap E_{k(i+1)} \neq \varnothing$ for $i = 1, 2, ..., m-1$, $a \in E_{k1}$ and $b \in E_{km}$. Now let $x_i \in E_{ki} \bigcap E_{k(i+1)}$. Then $(a, E_{k1}, x_1, E_{k2}, ..., E_{k(m-1)}, x_{m-1}, E_{km}, b)$ is a chain from $a$ to $b$. Therefore, $a$ and $b$ are connected in $H'$. This is true for every $a, b \in C$. Therefore, $H'$ is a connected hypersubgraph of $H$ and hence $C \in \mathcal{V}_H$. This implies $\mathcal{C} \subseteq \mathcal{V}_H$.

To prove $\mathcal{C} = \mathcal{V}_H$, if possible suppose that there exists $A \in \mathcal{V}_H$ such that $A \notin \mathcal{C}$. Since $A \in \mathcal{V}_H$, there exists a connected hypersubgraph $H' = (A, \mathcal{E}')$ for some $\mathcal{E}' \subseteq \mathcal{E}$. Then for every $x, y \in A$, there exists a chain from $x$ to $y$, say, $(x_1, E_1, x_2, E_2, ..., E_q, x_{q+1})$, where $x_1 = x, x_{q+1} = y$ and for $k = 1, 2, ..., q$ and $x_k, x_{k+1} \in E_k$. Clearly, $E_i \subseteq A$ for $i = 1, 2, ..., q$. Since $x_{i+1} \in E_i \cap E_{i+1}$, $E_i \cap E_{i+1} \neq \emptyset$ for $i = 1, 2, ..., q-1$. That is, for every $x, y \in A$, there exist basis elements $E_1, E_2, ..., E_q$ such that $x \in E_1$, $y \in E_q$, $E_i \subseteq A$ for $i = 1, 2, ..., q$ and $E_i \cap E_{i+1} \neq \emptyset$ for $i = 1, 2, ..., q-1$. This contradicts the assumption that $A \notin \mathcal{C}$, therefore $\mathcal{C} = \mathcal{V}_H$. □

**Remark 3.2.** In simple graphs, the members of the $c$-structure induced by the edge set are the vertex sets of connected subgraphs of the given graph.

**Theorem 3.2.** *If $H = (X, \mathcal{E})$ and $G = (Y, \mathcal{F})$ are two isomorphic hypergraphs, then the $c$-spaces induced by the hypergraphs $H$ and $G$ are $c$-isomorphic.*

**Proof.** Let $(X, \mathcal{C}_{\mathcal{E}})$ and $(Y, \mathcal{C}_{\mathcal{F}})$ be the $c$-spaces induced by the hypergraphs $H$ and $G$, respectively, and let $\phi$ be the hypergraph isomorphism.

Clearly, $\phi : X \to Y$ is a bijection. To prove $\phi : (X, \mathcal{C}_{\mathcal{E}}) \to (Y, \mathcal{C}_{\mathcal{F}})$ is $c$-continuous, let $C \in \mathcal{C}_{\mathcal{E}}$. If $C$ is a trivial connected set of $(X, \mathcal{C}_{\mathcal{E}})$, then clearly $\phi(C) \in \mathcal{C}_{\mathcal{F}}$. Now suppose $C$ is a nontrivial connected set of $(X, \mathcal{C}_{\mathcal{E}})$. For $y, y' \in \phi(C)$, there exist $x, x' \in C$ such that $\phi(x) = y$ and $\phi(x') = y'$. But $C \in \mathcal{C}_{\mathcal{E}}$ and $\mathcal{C}_{\mathcal{E}} = \langle \mathcal{E} \rangle$ implies the existence of the elements $E_i$, $i = 0$ to $n$ in $\mathcal{E}$ such that $E_i \subseteq C$, $E_i \cap E_{i+1} \neq \emptyset$ for $i = 0$ to $n-1$ and $x \in E_0$, $x' \in E_n$ for some positive integer $n$. Take $F_i = \phi(E_i)$ for $i = 0$ to $n$, then

$F_i \in \mathcal{F}$ for $i = 0$ to $n$, $F_i \subseteq \phi(C)$, $F_i \cap F_{i+1} \neq \varnothing$ for $i = 0$ to $n-1$ and $y \in F_0$, $y' \in F_n$. This implies $\phi(C) \in \mathcal{C}_{\mathcal{F}}$ and hence $\phi$ is $c$-continuous. Similarly we can prove that $\phi^{-1}$ is $c$-continuous. Therefore, $\phi : (X, \mathcal{C}_{\mathcal{E}}) \to (Y, \mathcal{C}_{\mathcal{F}})$ is a $c$-isomorphism. □

**Note 3.1.** Let $\mathcal{C}$ be a $c$-structure on $X$ and $\mathcal{B} \subseteq \mathcal{C}$ be such that $\langle \mathcal{B} \rangle = \mathcal{C}$. Then $(X, \mathcal{B})$ is a hypergraph if and only if $B \neq \varnothing$ for each $B \in \mathcal{B}$.

**Theorem 3.3.** *Let $(X, \mathcal{C})$ and $(Y, \mathcal{C}')$ be two $c$-spaces and $f : (X, \mathcal{C}) \to (Y, \mathcal{C}')$ be a $c$-isomorphism and let $\mathcal{B} = \{B_i : i \in I\} \subseteq \mathcal{C}$ be such that $\mathcal{C} = \langle \mathcal{B} \rangle$.*

(i) *Then $\mathcal{C}' = \langle f(\mathcal{B}) \rangle$.*

(ii) *If $(X, \mathcal{B})$ is a hypergraph, then $(Y, f(\mathcal{B}))$ is a hypergraph. Also, the hypergraphs $(X, \mathcal{B})$ and $(Y, f(\mathcal{B}))$ are isomorphic.*

**Proof.** (i) Consider $f(\mathcal{B}) = \{f(B_i) : i \in I\}$. Since $f$ is $c$-continuous and $B_i \in \mathcal{B} \subseteq \mathcal{C}$, we get $f(B_i) \in \mathcal{C}'$, $\forall i \in I$. This implies $f(\mathcal{B}) \subseteq \mathcal{C}'$. Consider a nontrivial connected set $C' \in \mathcal{C}'$ and let $c_1', c_2' \in C'$. Then $C = f^{-1}(C')$ is a nontrivial connected set of $\mathcal{C}$ and $f^{-1}(c_1'), f^{-1}(c_2') \in C$. Then there exist $B_i$, $i = 0$ to $n$ in $\mathcal{B}$ such that $B_i \subseteq C$, $B_i \cap B_{i+1} \neq \varnothing$ for $i = 0$ to $n-1$ and $f^{-1}(c_1') \in B_0$, $f^{-1}(c_2') \in B_n$ for some positive integer $n$. This implies $c_1' \in f(B_0)$, $c_2' \in f(B_n)$ and $f(B_i) \cap f(B_{i+1}) \neq \varnothing$ for $i = 0$ to $n-1$. Therefore, $C' \in \langle f(\mathcal{B}) \rangle$ and hence $\mathcal{C}' = \langle f(\mathcal{B}) \rangle$.

(ii) Consider the $c$-isomorphism $f : (X, \mathcal{C}) \to (Y, \mathcal{C}')$. Suppose that $(X, \mathcal{B})$ is a hypergraph. Then $B_i \neq \varnothing$ for each $i \in I$. But this implies $f(B_i) \neq \varnothing$ for every $i \in I$. Therefore, $(Y, f(\mathcal{B}))$ is a hypergraph. Now consider the map $\phi : X \to Y$ defined by $\phi(x) = f(x)$. Clearly, $\phi$ is an isomorphism between the hypergraphs $(X, \mathcal{B})$ and $(Y, f(\mathcal{B}))$. □

**Remark 3.3.** By the above theorem, if we have two *c*-isomorphic *c*-spaces $(X, \mathcal{C})$ and $(Y, \mathcal{C}')$, consider the collection of nonempty connected sets $\mathcal{B} \subseteq \mathcal{C}$ such that $\mathcal{C} = \langle \mathcal{B} \rangle$. Such a collection always exists, for example $\mathcal{B} = \mathcal{C} - \{\emptyset\}$. Then the *c*-spaces induced by the isomorphic hypergraphs $H = (X, \mathcal{B})$ and $G = (Y, f(\mathcal{B}))$ are $(X, \mathcal{C})$ and $(Y, \mathcal{E})$, respectively, where *f* is the given *c*-isomorphism.

## 4. Isolated Edge and *t*-closed Set

Here we examine the interrelation between the isolated edges of a hypergraph and the *t*-closed sets of the *c*-space induced by that hypergraph.

**Definition 4.1** [8]**.** Let $(X, \mathcal{C})$ be a *c*-space and $A \subseteq X$. A point $x \in X$ is said to *touch* the set *A* if there is a nonempty $C \subseteq A$ such that $\{x\} \bigcup C$ is connected. The set of all points touching the set *A* is denoted by $t(A)$. If $A \subseteq X$ contains all of its touching points, then it is said to be *t-closed*.

**Definition 4.2** [5]**.** Consider the hypergraph $H = (X, \mathcal{E})$ and let $E \in \mathcal{E}$. Then *E* is said to be an *isolated edge* if for all $E' \in \mathcal{E}$ with $E' \neq E$, $E \bigcap E' \neq \emptyset$ implies that $E' \subseteq E$.

**Theorem 4.1.** *Let* $H = (X, \mathcal{E})$ *be a hypergraph and let* $E \in \mathcal{E}$ *be an isolated edge. Then E is t-closed in the c-space induced by the hypergraph H.*

**Proof.** Suppose $(X, \mathcal{C})$ is the *c*-space induced by the hypergraph *H*. To prove *E* is *t*-closed in the *c*-space $(X, \mathcal{C})$, it is enough to show that $t(E) = E$. It is clear that $E \subseteq t(E)$. Let *x* be a touching point of *E*. Then there exists a nonempty subset $C \subseteq E$ such that $A = \{x\} \bigcup C \in \mathcal{C}$. If $x \in C$ then $x \in E$. If $x \notin C$, then take $y \in C$ which exists since *C* is nonempty. Then there exists $E_i$, $i = 0$ to $n$ in $\mathcal{E}$ such that $E_i \subseteq C$, $E_i \bigcap E_{i+1} \neq \emptyset$ for $i = 0$ to $n - 1$ and $x \in E_0$, $y \in E_n$ for some positive integer $n$. But $x \in E_0$ implies $x \in E$. Thus we get $x \in E$ whenever $x \in t(E)$ and hence $E$ is $t$-closed. □

**Remark 4.1.** Let $A \subseteq X$ be a $t$-closed set of a $c$-space $(X, \mathcal{C})$ and let $\mathcal{B} \subseteq \mathcal{C}$ be such that $\langle \mathcal{B} \rangle = \mathcal{C}$. Then $A$ need not be an isolated edge of the hypergraph $H = (X, \mathcal{B})$, whenever $H$ is a hypergraph. Consider the following example:

**Example 4.1.** Consider the $c$-space $(X, \mathcal{C})$, where $X = \{a, b, c, d\}$ and $\mathcal{C} = \mathcal{D} \cup \{\{a, b\}, \{c, d\}, \{b, c, d\}, \{a, b, c, d\}\}$. Since $t(\{a, b\}) = \{a, b\}$, we have $\{a, b\}$ is $t$-closed in the $c$-space $(X, \mathcal{C})$. Let $\mathcal{B} = \{\{a, b\}, \{c, d\}, \{b, c, d\}\}$. Clearly, $(X, \mathcal{B})$ is a hypergraph. But $\{a, b\}$ is not an isolated edge of the hypergraph $H = (X, \mathcal{B})$, since $\{a, b\} \cap \{b, c, d\} \neq \emptyset$ and $\{b, c, d\} \nsubseteq \{a, b\}$.

## 5. α-generated *c*-space and α-uniform Hypergraphs

In this section, we analyze the relation of α-generated $c$-spaces and α-uniform hypergraphs.

**Definition 5.1** [9]**.** Let $X$ be any set and $\alpha$ be any cardinal with $\alpha \leq |X|$. Then a $c$-structure $\mathcal{C}$ on $X$ is said to be α-*generated* if there is a subcollection $\mathcal{B} \subseteq \{A \in \mathcal{C} : |A| \leq \alpha\}$ such that $\mathcal{C} = \langle \mathcal{B} \rangle$.

**Definition 5.2** [2]**.** Consider the hypergraph $H = (X, \mathcal{E})$ and let $|E| = r$ for all $E \in \mathcal{E}$. Then the hypergraph $H = (X, \mathcal{E})$ is called *r-uniform*.

**Theorem 5.1.** *Let $H = (X, \mathcal{E})$ be an α-uniform hypergraph. Then the c-space induced by the hypergraph H is α-generated.*

**Proof.** Let $(X, \mathcal{C})$ be the $c$-space induced by the hypergraph $H$. To prove $(X, \mathcal{C})$ is α-generated, it is enough to show that there exists $\mathcal{B} \subseteq \{A \in \mathcal{C} : |A| \leq \alpha\}$ such that $\mathcal{C} = \langle \mathcal{B} \rangle$. Take $\mathcal{B} = \{A \in \mathcal{C} : |A| = \alpha\}$. Then $\mathcal{B} = \mathcal{E}$ and hence $\langle \mathcal{B} \rangle = \langle \mathcal{E} \rangle = \mathcal{C}$. □

**Remark 5.1.** Converse of the above result is not true. That is, if the *c*-space induced by the hypergraph $H = (X, \mathcal{B})$ is α-generated, then $H$ need not be α-uniform. This is shown by the following example:

**Example 5.1.** Consider the *c*-space $(X, \mathcal{C})$, where $X = \{1, 2, 3, ..., 10\}$ and

$$\mathcal{C} = \mathcal{D} \cup \{\{1, 2\}, \{3, 6, 7\}, \{4, 9\}, \{5, 6\}, \{8, 9, 10\}, \{3, 5, 6, 7\}, \{4, 8, 9, 10\}\}.$$

Then $\mathcal{B} = \{\{1, 2\}, \{3, 6, 7\}, \{4, 9\}, \{5, 6\}, \{8, 9, 10\}\}$ generates $\mathcal{C}$. Here *c*-space $(X, \mathcal{C})$ is 3-generated, but the hypergraph $H = (X, \mathcal{B})$ is not 3-uniform.

**Note 5.1.** Let $(X, \mathcal{C})$ be a *c*-space such that $\mathcal{C} \neq \mathcal{D}$. Then the following are equivalent:

(i) there exists $\mathcal{B} \subseteq \{A \in \mathcal{C} : |A| \leq 2\}$ such that $\langle \mathcal{B} \rangle = \mathcal{C}$,

(ii) there exists $\mathcal{B}' \subseteq \{A \in \mathcal{C} : |A| = 2\}$ such that $\langle \mathcal{B}' \rangle = \mathcal{C}$.

**Note 5.2.** Consider a 2-generated *c*-space $(X, \mathcal{C})$ with $\mathcal{C} \neq \mathcal{D}$. Then $\{A \in \mathcal{C} : |A| = 2\} \neq \varnothing$.

**Theorem 5.2.** *Let $(X, \mathcal{C})$ be a 2-generated c-space. Then there exists $\mathcal{B} \subseteq \mathcal{C}$ with $\langle \mathcal{B} \rangle = \mathcal{C}$ such that the hypergraph $(X, \mathcal{B})$ is 2-uniform.*

**Proof.** Consider the 2-generated *c*-space $(X, \mathcal{C})$. If $\mathcal{C} = \mathcal{D}$, take $\mathcal{B} = \varnothing$. Then $\langle \mathcal{B} \rangle = \mathcal{D} = \mathcal{C}$ and clearly the hypergraph $H = (X, \mathcal{B})$ is 2-uniform. Now suppose that $\mathcal{C} \neq \mathcal{D}$. Since $(X, \mathcal{C})$ is 2-generated, there exists $\mathcal{B} \subseteq \{A \in \mathcal{C} : |A| \leq 2\}$ such that $\langle \mathcal{B} \rangle = \mathcal{C}$. Then there exists $\mathcal{B}' \subseteq \{A \in \mathcal{C} : |A| = 2\}$ such that $\langle \mathcal{B}' \rangle = \mathcal{C}$ and clearly $\mathcal{B}' \neq \varnothing$. But the hypergraph $(X, \mathcal{B}')$ is 2-uniform. □

**Remark 5.2.** Since 2-uniform hypergraphs are graphs, we can say that corresponding to every 2-generated *c*-space, there exists a graph such that the *c*-structure induced by that graph coincides with the given *c*-structure.

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A. K. Sruthi: sruthipremanak@gmail.com