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INTEGER LINEAR PROGRAMMING APPROACH FOR DETECTION LEARNING OUTCOMES ACHIEVEMENT

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Abstract

In this paper, we develop a model of integer linear programming that can be used in detection of the maximum of students' mathematical learning outcomes. The model is tested by using test result data from 997 participants, which show that the model could be used to solve the problem of learning outcomes in education.

1. Introduction

In accordance with the development of science, a problem can be modeled mathematically called the *mathematical model* of the problem. Mathematical modeling is a step taken to obtain solution of the problem by utilizing the tools of mathematics through real world context. A mathematical model is a description of the conditions presented by a real

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world problem. It consists of the purpose function and some constraints in the form of a linear or nonlinear function. Mathematical models that have objective functions and constraints linear are called *linear programming* (*LP*) [1, 2]. Integer linear programming (ILP) is a linear programming model with one additional restriction that all variable values must be integers. Some ILP applications have been used which were successful in solving optimization problems in various sectors including education, such as the allocation of student majors [3], scheduling problems [4-6], marking the location of mathematics test material [7], examination timetabling [8], and mapping students' cognitive capability [9].

The cognitive aspect of learning outcomes is the ultimate goal in educational activities to be used as a measuring tool in several objectives for instance, to know the quality of education [10], the quality and achievement of knowledge, skills and attitudes [11], the quality of learning outcomes and teachers [12, 13], and an overview of student learning outcomes [14], the results and success rates of learning [15, 16].

In general, the cognitive aspects of learning outcomes are measured based on six aspects, namely knowledge, comprehension, application, analysis, synthesis, and evaluation for each topic of teaching material on educational unit. The number of material topics, cognitive aspects, the number of students and the extent of the area being tested will make a problem of determining the material that has been achieved and thoroughly completed. Therefore, it needs a tool to overcome the problem. This paper aims to develop a model of ILP that can be used in detection of the maximum of students' mathematical learning outcomes.

2. Integer Linear Programming

ILP model is an extension of the LP model with one additional restriction that is all variables are integer valued. The ILP model can be expressed as follows:

$$\text{Maximum } F(x) = c^T x \tag{1}$$

$$\text{Constraint } Ax \leq b \tag{2}$$

$$1 \leq x \leq u \tag{3}$$

$$x_i \text{ integer, } j \in J' \subset J,$$

where A is a matrix of order $m \times n$, c is a vector of order $n \times 1$, c^T is the transpose of c, and $J = \{1, 2, ..., n\}$.

The basic approaches to solve the problems of ILP are the branch-and-bound methods [17-19], and cutting plane [20]. Another approach is the integrating process [21, 22], and the use of software [23]. Intgerizing process at a component of the optimal basic feasible vector $(x_B)_k$, to linear programming can be written as

$$(x_B)_k = \beta_k - \alpha_{k1}(x_N)_1 - \cdots - \alpha_{kj}(x_N)_j - \cdots - \alpha_{kn} - m(x_N)_N n - m. \tag{4}$$

If $(x_B)_k$ is an integer variable and we assume that β_k is not an integer, then the partitioning of β_k into the integer and fractional components is given

$$\beta_k = [\beta_k] + f_k, \quad 0 \leq f_k \leq 1. \tag{5}$$

Increase $(x_B)_k$ to its nearest integer, $([\beta_k] + 1)$. Based on the idea of suboptimal solutions, we may elevate a particular nonbasic variable, say $(x_N)_{j^*}$, above its bound of zero, provided α_{kj^*}, as one of the elements of the vector α_{j^*}, is negative. Let Δ_{j^*} be an amount of movement of the non-variable $(x_N)_{j^*}$, such that the numerical value of scalar $(x_B)_k$ is an integer. Referring to equation (4), Δ_{j^*} can then be expressed as

$$\Delta_{f^*} = \frac{1 - f_k}{-\alpha_{kj^*}} \tag{6}$$

while the remaining nonbasics stay at zero. It can be seen that after substituting (5) into (6) for $(x_N)_{j^*}$ and taking into account the partitioning of β_k, we obtain

$$(x_B)_k = [\beta_k] + 1. \tag{7}$$

Thus, $(x_B)_k$ is now an integer.

3. Learning Outcomes

Learning outcomes are the ultimate goal in each lesson. Learning outcomes are expressions of the purpose of education which is a statement of what is expected to be known and understood by learners after completing a period of study [24]. Watson [25] defines that learning outcomes are the changes in people as a result of learning experience [26]. Learning outcomes can encapsulate a wide range of knowledge, skills and behaviours [27-29].

In general, the size and level of ability of student learning outcomes are based on the cognitive domain of Bloom's taxonomy developed by educational psychologist Bloom [30], namely knowledge, comprehensive, application, analysis, synthesis and evaluation [31]. The result of cognitive learning is acquired through the activity of knowing, understanding, applying, analyzing and evaluating [32]. In general, the measure of the learning outcomes of education in Indonesia is stated by completeness of study, that is the percentages achievement of competence with the maximum of 100 as the ideal thoroughness. Based on the national agency of educational standards [33] that the criteria for learning completeness ranged from 0-100% and thoroughness for each indicator of at least 75%. Target of thoroughness is expected to reach at least 75% nationally [34].

4. The Mathematical Model

The measurement of cognitive aspects of learning outcomes refers to Bloom's taxonomy, i.e., knowledge (C1), comprehensive (C2), application

(C3), analysis (C4), synthesis (C5) and evaluation (C6). The number of items tested depends on the evaluation objectives, materials and aspects to be achieved in an evaluation activity conducted at the school. Creating a mathematical model of learning outcomes problem is done in two stages. First, create variables and parameters as symbols for the components to be used. Second, create objective function, that is to determine students' mathematics learning outcomes of the maximum number of items that have been mastered by students and make some constraints from problems that can maximize objective function. Form of data based on the test results for the measurement of learning outcomes can be expressed as in the form of Table 1 below.

Table 1. The preliminary data form test results

Topics	Aspect						Σ
	C1	C2	C3	C4	C5	C6	
1	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	p_1
2	c_{21}	c_{22}	c_{23}	c_{24}	c_{25}	c_{26}	p_2
3	c_{31}	c_{32}	c_{33}	c_{34}	c_{35}	c_{36}	p_3
.	.	.	.	.	.	.	.
m	c_{m1}	c_{m2}	c_{m3}	c_{m4}	c_{m5}	c_{m6}	p_m
Σ	r_1	r_2	r_3	r_4	r_5	r_6	

where C_j's are cognitive aspects to $-j$, $j = 1, ..., 6$, c_{ij} is the number of students who correctly answer on each topic to $-i$, $i = 1, ..., m$ on aspect C_j, p_i is the number of students who correctly answer topic to $-i$ for all C_j, and r_j is the number of students who correctly answer topic to $-i$ on each C_j.

Based on the data in Table 1, is then formed the variables and parameters needed in the modeling process for problem solving.

Set:

Variable:

X_i : variable items to $-i$,

X_{ij} : variable items to $-i$ (X_i) on cognitive to $-j$ (C_j).

Parameter:

p_i : a number of students who answer correctly X_i for all C_j,

q_i : the maximum number if all students answer correctly X_i for all C_j,

r_j : a number of students who answer correctly X_i on each C_j,

s_j : the maximum number if all students answer correctly X_i on each C_j,

S_{ij} : a number of students who answer correctly on each X_i for all C_j,

T_{ij} : the maximum number of students who answer correctly on each X_i for all C_j.

Having established the required variables and parameters so the initial data table test results can be expressed in Table 2 below:

Table 2. Preparation of modeling

Topics	Variable	Aspect						Σ
		C1	C2	C3	C4	C5	C6	
1	X_1	$c_{11}X_{11}$	$c_{12}X_{12}$	$c_{13}X_{13}$	$c_{14}X_{14}$	$c_{15}X_{15}$	$c_{16}X_{16}$	p_1
2	X_2	$c_{21}X_{21}$	$c_{22}X_{22}$	$c_{23}X_{23}$	$c_{24}X_{24}$	$c_{25}X_{25}$	$c_{26}X_{26}$	p_2
3	X_3	$c_{31}X_{31}$	$c_{32}X_{32}$	$c_{33}X_{33}$	$c_{34}X_{34}$	$c_{35}X_{35}$	$c_{36}X_{36}$	p_3
.	.	.	.	.	.	.	.	.
m	X_m	$c_{m1}X_{m1}$	$c_{m2}X_{m2}$	$c_{m3}X_{m3}$	$c_{m4}X_{m4}$	$c_{m5}X_{m5}$	$c_{m6}X_{m6}$	p_m
Σ		r_1	r_2	r_3	r_4	r_5	r_6	

Furthermore, to solve the problem of learning outcomes, the mathematical model can be expressed as follows.

The objective function is to know learning outcomes of mathematics students about the maximum number of items mastered by them. The objective function can be given by

$$Z = Max \sum_{i=1}^{m} \sum_{j=1}^{6} X_{ij}. \tag{8}$$

Some constraints of problems can maximize the objective function.

The constraints on the number of all X_i, and the number of X_i for each C_j are expressed as follows:

$$\sum_{i=1}^{m} \sum_{j=1}^{6} X_{ij} \leq M, \tag{9}$$

$$\sum_{j=1}^{6} X_{ij} \leq 6, \quad i = 1, \ldots, m, \tag{10}$$

$$\sum_{i=1}^{m} X_{ij} \leq m, \quad j = 1, \ldots, 6. \tag{11}$$

The constraints on the number of students who answer correctly all X_i and each C_j are stated below:

$$\sum_{j=1}^{6} S_{ij} X_{ij} \leq 6_i, \quad i = 1, \ldots, m, \tag{12}$$

$$\sum_{i=1}^{m} S_{ij} X_{ij} \leq r_j, \quad j = 1, \ldots, 6. \tag{13}$$

The constraints on the number of students who answer correctly every X_i and the maximum of all students answering correctly each C_j are described as follows:

$$\sum_{j=1}^{6} S_{ij} X_{ij} \leq q_i, \quad j = 1, \ldots, m, \tag{14}$$

$$\sum_{i=1}^{m} T_{ij} X_{ij} \leq s_i, \quad j = 1, \ldots, 6. \tag{15}$$

For the constraints to the variable difference of students who answer correctly but not each X_i and all domains C_j's, the capability aspect is stated as follows:

$$\sum_{j=1}^{6} (S_{ij} X_{ij} - X_{ij}) \leq p_i, \quad i = 1, \ldots, m, \tag{16}$$

$$\sum_{i=1}^{m} (S_{ij} X_{ij} - X_{ij}) \leq r_j, \quad j = 1, \ldots, 6. \tag{17}$$

The constraints for the difference of maximum amount if all the students answer correctly and the number of students who answer correctly all X_i C_j can be expressed as follows:

$$\sum_{j=1}^{6} (T_{ij} X_{ij} - S_{ij} X_{ij}) \leq p_i, \quad i = 1, \ldots, m, \tag{18}$$

$$\sum_{i=1}^{m} (T_{ij} X_{ij} - S_{ij} X_{ij}) \leq r_j, \quad j = 1, \ldots, 6. \tag{19}$$

The constraints or the difference of maximum if all students answer correctly X_i with the number of students based on test results for each domain C_j can be stated as follows:

$$\sum_{j=1}^{6}(T_{ij}X_{ij} - S_{ij}X_{ij}) \le T_{ij}, \quad i = 1, \ldots, m, \tag{20}$$

$$\sum_{i=1}^{m}(T_{ij}X_{ij} - S_{ij}X_{ij}) \le s_j, \quad j = 1, \ldots, 6. \tag{21}$$

The constraints for the difference of maximum if all students answer correctly X_i with the number of students who answer correctly the results of tests for each domain C_j can be stated as follows:

$$\sum_{j=1}^{3}(T_{ij}X_{ij} - S_{ij}X_{ij}) \le q_i - p_i, \quad i = 1, \ldots, m, \tag{22}$$

$$\sum_{i=1}^{m}(T_{ij}X_{ij} - S_{ij}X_{ij}) \le s_j - r_j, \quad j = 1, \ldots, 6, \tag{23}$$

where X_{ij} is an integer.

5. Test Data

Data learning outcomes with participants of 997 students are from 8 schools grade 12 public and private high school. Data collection technique is conducted by using 48 items to the students as a sample. The used instrument is an objective test taken from document of mathematics national examination. The selection of test items is performed by recapitulation of teaching materials, so it is taken 16 pieces of topic with 3 aspect, i.e., knowledge (C1), comprehension (C2) and application (C3). Test data are presented in Table 3.

Table 3. Test data

Topics	Variable	Aspect			∑
		C1	C2	C3	
1	X1	614	767	326	1707
2	X2	357	598	336	1291
3	X3	616	625	793	2034
4	X4	475	406	398	1278
5	X5	385	376	510	1271
6	X6	555	754	550	1859
7	X7	700	471	385	1556
8	X8	301	613	444	1358
9	X9	633	506	316	1454
10	X10	634	506	550	1689
11	X11	730	415	368	1513
12	X12	579	242	323	1144
13	X13	325	458	442	1225
14	X14	343	435	410	1279
15	X15	280	155	472	907
16	X16	333	532	319	1184
∑		7951	7859	6942	22752

6. Result and Discussion

Based on the data in Table 3, the replacement of all parameter values in equations (9) to (23) provides 1 objective function and 139 linear constraints. Due to all the variables must be integers, the prepared model is an integer linear programming (ILP). To solve ILP model by application of LINDO 6.1 packages, we have to follow:

LP OPTIMUM FOUND AT STEP 45

OBJECTIVE VALUE = 32.4264412

SET X111 TO <= 0 AT1, BND= 23.00 TWIN=-0.1000E+3182

NEW INTEGER SOLUTION OF 23.0000038 AT BRANCH 1 PIVOT 82

BOUND ON OPTIMUM: 23.00000

DELETE X111 AT LEVEL 1

ENUMERATION COMPLETE. BRANCHES = 1 PIVOTS = 82

LAST INTEGER SOLUTION IS THE BEST FOUND

RE-INSTALLING BEST SOLUTION...

OBJECTIVE FUNCTION VALUE

(1) $Z = 23.00000$

VARIABLE	VALUE	REDUCED COST	VARIABLE	VALUE	REDUCED COST
X11	1.000000	–1.000000	X91	1.000000	–1.000000
X12	1.000000	–1.000000	X92	1.000000	–1.000000
X13	0.000000	–1.000000	X93	0.000000	–1.000000
X21	0.000000	–1.000000	X101	1.000000	–1.000000
X22	1.000000	–1.000000	X102	0.000000	–1.000000
X23	0.000000	–1.000000	X103	0.000000	–1.000000
X31	1.000000	–1.000000	X111	1.000000	–1.000000
X32	1.000000	–1.000000	X112	0.000000	–1.000000
X33	1.000000	–1.000000	X113	0.000000	–1.000000
X41	1.000000	–1.000000	X121	1.000000	–1.000000
X42	0.000000	–1.000000	X122	0.000000	–1.000000
X43	0.000000	–1.000000	X123	0.000000	–1.000000
X51	0.000000	–1.000000	X131	0.000000	–1.000000
X52	0.000000	–1.000000	X132	1.000000	–1.000000
X53	1.000000	–1.000000	X133	0.000000	–1.000000
X61	1.000000	–1.000000	X141	0.000000	–1.000000
X62	1.000000	–1.000000	X142	0.000000	–1.000000
X63	0.000000	–1.000000	X143	1.000000	–1.000000
X71	1.000000	–1.000000	X151	1.000000	–1.000000
X72	1.000000	–1.000000	X152	0.000000	–1.000000
X73	0.000000	–1.000000	X153	0.000000	–1.000000
X81	0.000000	–1.000000	X161	0.000000	–1.000000
X82	1.000000	–1.000000	X162	1.000000	–1.000000
X83	1.000000	–1.000000	X163	0.000000	–1.000000

Based on the output of calculation, the value of the objective function is $(Z = 23)$. It means that the maximum number of items mastered by the students is 23 out of 48 items or 47.92%. It shows that it has not yet reached the national completeness criteria, which is at least 75% [33, 34]. Therefore, learning can be improved on the location of the variable 0 to obtain maximum learning outcomes.

7. Conclusion

Learning outcomes is an essential unsure to determine the quality of learning by student. The completeness of learning is one of essential unsure to measure learning outcomes. We developed an integer linear programming model in order to detect the maximum of students' mathematical learning outcomes. The model could be used to solve the problem of learning outcomes in education.

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