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MARTINGALE METHOD FOR RUIN PROBABILITIES IN A CONTROLLED GENERAL RISK PROCESS WITH MARKOV CHAINS

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Abstract

In this paper, we study a controlled general risk process. We assume that claim and rates of interest are homogeneous Markov chains, taking a countable number of non-negative values. Generalized Lundberg inequalities for ruin probability of this process are derived by the martingale approach.

1. Introduction

In classical risk model, the claim number process was assumed to be a Poisson process and the individual claim amounts were described as independent and identically distributed random variables. In recent years, the

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classical risk process has been extended to more practical and real situations. For most of the investigations treated in risk theory, it is very significant to deal with the risks that rise from monetary inflation in the insurance and finance market, and also to consider the operation uncertainties in administration of financial capital. The ruin problem has been studied by many researchers [4, 9, 10]. Sundt and Teugels [16, 17] studied ruin probability under the compound Poisson risk model with the effects of constant rate. Yang [19] gave both exponential and non-exponential upper bounds for ruin probabilities in a risk model with constant interest force and independent premiums and claims. Xu and Wang [18] gave upper bounds for ruin probabilities in a risk model with interest force and independent premiums and claims with Markov chain interest rate. Cai [1, 2] considered the ruin probabilities in two risk models, with independent premiums and claims and used a first-order autoregressive process to model the rates of interest. Cai and Dickson [3] built Lundberg inequalities for ruin probabilities in two discrete-time risk process with a Markov chain interest model and independent premiums and claims. Quang [11] established Lundberg inequalities using the recursive technique for ruin probabilities in two risk models with homogeneous Markov chain premiums when claims and interest rates sequences are independent. Quang [12] used martingale approach to build upper bounds for ruin probabilities in a risk model with interest force and independent interest rates and premiums when the claims form a Markov chain. Quang [13] used martingale approach to build upper bounds for ruin probabilities in a risk model with interest force and independent interest rates and Markov chain claims and Markov chain premiums. Quang [14] used martingale approach to build upper bounds for ruin probabilities in a risk model with interest force and independent claims, Markov chain premiums and Markov chain interests. Quang [15] also used recursive approach to build upper bounds for ruin probabilities in a risk model with interest force and Markov chain premiums, Markov chain claims, with the independent interest rates.

In addition, many papers studied an insurance model where the risk process can be controlled by proportional reinsurance. The performance criterion is to choose reinsurance control strategies to bound the ruin probability of a discrete-time process with a Markov chain interest. Controlling a risk process is a very active area of research, particularly in the last decade; see [4-7], for instance. Nevertheless obtaining explicit optimal solutions is a difficult task in a general setting. Diasparra and Romera [8] obtained generalized Lundberg inequalities for the ruin probabilities in a controlled discrete-time risk process with a Markov chain interest.

In this article, we extend the model considered by Diasparra and Romera [8] to introduce claim and rates of interest as homogeneous Markov chains, taking a countable number of non-negative values. Generalized Lundberg inequalities for ruin probability of this process are derived by the Martingale approach.

2. The Model and Basic Assumptions

Let Y_n be the *n*th claim payment. The random variable Z_n stands for the length of the *n*th period, that is, the time between the occurrences of the claims Y_{n-1} and Y_n. Let $\{I_n\}_{n\geq 0}$ be the interest rate process. We assume that Y_n, Z_n and I_n are defined on the probability space (Ω, A, P). We consider a discrete-time insurance risk process in which the surplus process $\{U_n\}_{n\geq 1}$ with initial surplus *u* can be written as

$$U_n = U_{n-1}(1 + I_n) + C(b_{n-1}) \cdot Z_n - h(b_{n-1}, Y_n), \text{ for } n \geq 1. \tag{2.1}$$

We make several assumptions:

Assumption 2.1. $U_0 = u \geq 0.$

Assumption 2.2. $\{Y_n\}_{n\geq 0}$ is a homogeneous Markov chain, such that for any *n*, the values of Y_n are taken from a set of non-negative numbers $G_Y = \{y_1, y_2, ..., y_n, ...\}$ with $Y_0 = y_i$ and

$$p_{ij} = P[\omega \in \Omega : Y_{n+1}(\omega) = y_j \,|\, Y_n(\omega) = y_i](n \in N,\ y_i \in G_Y,\ y_j \in G_Y), \quad (2.2)$$

where $0 \le p_{ij} \le 1, \sum_{j=1}^{+\infty} p_{ij} = 1.$

Assumption 2.3. $\{I_n\}_{n \ge 0}$ is a homogeneous Markov chain, such that for any n, the values of I_n are taken from a set of non-negative numbers $G_I = \{i_1, i_2, ..., i_m, ...\}$ with $I_0 = i_r$ and

$$q_{rs} = P[\omega \in \Omega : I_{m+1}(\omega) = i_s \,|\, I_m(\omega) = i_r]\ (m \in N,\ i_r \in G_I,\ i_s \in G_I),$$

where $0 \le q_{rs} \le 1, \sum_{s=1}^{+\infty} q_{rs} = 1.$

Assumption 2.4. $\{Z_n\}_{n \ge 0}$ is a sequence of independent and identically distributed non-negative continuous random variables with the same distributive function

$$F(z) = P(\omega \in \Omega;\ Z_0(\omega) \le z)$$

with $F(0) = 0.$

Assumption 2.5. We denote by $C(b)$ the premium left for the insurer if the retention level b is chosen, where

$$0 < C(b) \le c, \quad b \in B.$$

The process can be controlled by reinsurance, that is, by choosing the retention level (or proportionality factor or risk exposure) $b \in B$ of a reinsurance contract for one period, where $B := [b_{\min}, 1],\ b_{\min} \in (0, 1]$ will be introduced below. The premium rate c is fixed.

Assumption 2.6. We denote the function $h(b,\ y)$ with values in $[0,\ y]$ specifies the fraction of the claim y paid by the insurer, and it also depends on the retention level b at the beginning of the period. Hence $y - h(b,\ y)$ is the part paid by the reinsurer. The retention level $b = 1$ stands for control action with no reinsurance. In this article, we consider the case of proportional reinsurance, which means that

$$h(b,\ y) = b \cdot y,\ \text{with}\ b \in B. \tag{2.3}$$

Usually, the constant $b_{\min}$ in Assumption 2.5 is chosen by

$$b_{\min} := \min\{b \in (0,\ 1];\ C(b) > 0\}. \tag{2.4}$$

Assumption 2.7. We suppose that $\{Y_n\}_{n\geq 0}$, $\{Z_n\}_{n\geq 0}$ and $\{I_n\}_{n\geq 0}$ are independent.

Assumption 2.8. We consider Markovian control policies $\pi = \{a_n\}_{n\geq 1}$, which at each time *n* depend only on the current state, that is, $a_n(U_n) := b_n$ for $n \geq 0$. Abusing notation, we identify functions $a : X \to B$, where $X = \mathbb{R} \bigcup \ell$, B is the decision space.

Consider an arbitrary initial state $U_0 = u \geq 0$ and a control policy $\pi = \{a_n\}_{n\geq 1}$. Then, by iteration of (2.1) and assuming (2.2), it follows that for $n \geq 1$, U_n satisfies

$$U_n = \mathcal{U} \prod_{l=1}^{n} (1 + I_l) + \sum_{l=1}^{n} \left(C(b_{n-1}) Z_l - b_{l-1} \cdot Y_l \prod_{m=l+1}^{n} (1 + I_m) \right). \tag{2.5}$$

The ruin probability when using the policy π, given the initial surplus *u*, and the initial claim $Y_0 = y_i$, the initial interest rate $I_0 = i_r$ with Assumptions 2.1 to 2.8 is defined as

$$\psi^{\pi}(u,\ y_i,\ i_r) = P^{\pi}\left(\bigcup_{k=1}^{\infty} (U_k < 0) | U_0 = u,\ Y_0 = y_i,\ I_0 = i_r \right) \tag{2.6}$$

which we can also express as

$$\psi^{\pi}(u,\ y_i,\ i_r) = P^{\pi}(U_k < 0 \text{ for some } k \geq 1 | U_0 = u,\ Y_0 = y_i,\ I_0 = i_r). \tag{2.7}$$

Similarly, the ruin probabilities in the finite horizon case with Assumptions 2.1 to 2.8 are given by

$$\psi_n^\pi(u, y_i, i_r) = P^\pi\left(\bigcup_{k=1}^{n}(U_k < 0)|U_0 = u, Y_0 = y_i, I_0 = i_r\right). \quad (2.8)$$

Firstly, we have

$$\psi_1^\pi(u, y_i, i_r) \leq \psi_2^\pi(u, y_i, i_r) \leq \cdots \leq \psi_n^\pi(u, y_i, i_r) \leq \cdots, \quad (2.9)$$

and with any $n \in N$,

$$\psi_n^\pi(u, y_i, i_r) \leq 1. \quad (2.10)$$

Thus, from (2.7) and (2.8), we obtain

$$\lim_{n\to\infty} \psi_n^\pi(u, y_i, i_r) = \psi^\pi(u, y_i, i_r).$$

We denote by $\prod$ the policy space. A control policy π^* is said to be *optimal* if for any initial $(Y_0, I_0) = (y_i, i_r)$, we have

$$\psi^{\pi^*}(u, y_i, i_r) \leq \psi^\pi(u, y_i, i_r) \text{ for all } \pi \in \Pi.$$

3. Upper Bounds for Ruin Probability by the Martingale Approach

We now construct upper bounds for ruin probabilities by the martingale approach. To this end, let $V_n = U_n\prod_{i=1}^{n}(1 + I_i)^{-1}$ with $n \geq 1$, be the so-called discounted risk process. The ruin probabilities φ_n^π in (2.8) associated to the $\{V_n, n = 1, 2, ...\}$ are

$$\psi_n^\pi(u_0, y_i, i_r) = P^\pi\left(\bigcup_{k=1}^{n}(V_k < 0|U_0 = u_0, Y_0 = y_i, I_0 = i_r)\right).$$

In the classical risk model, process $\{e^{-R_0U_n}\}_{n\geq 1}$ is a martingale. However, for our model (2.5), there is no constant $r > 0$ such that

$\{e^{-rU_n}\}_{n\geq 1}$ is a martingale. Still, there exists a constant $r > 0$ such that $\{e^{-rV_n}\}_{n\geq 1}$ is a supermartingale, which allows us to derive probability inequalities by the optional stopping theorem. Such a constant is defined in the following lemmas:

Lemma 3.1. *Let model* (2.5) *satisfy Assumptions* 2.1 *to* 2.8. *Assume that for each* $y_i \in G_Y = \{y_1, y_2, ..., y_n, ...\}$, $bE^{\pi}(Y_1 | Y_0 = y_i) < C(b)E^{\pi}(Z_1)$ *and* $P^{\pi}(bY_1 - C(b)Z_1 > 0 | Y_0 = y_i) > 0.$ *Then there exists a constant* $R_0 = R_0(b)$ *such that*

$$E^{\pi}[e^{-R_0[C(b)Z_1 - bY_1]} | Y_0 = y_i] = 1. \tag{2.11}$$

Proof. Define

$$f_i(t) = E^{\pi}[e^{-t[C(b)Z_1 - bY_1]} | Y_0 = y_i] - 1, \quad t \in (0; +\infty).$$

We have

$$f_i(0) = -E^{\pi}[C(b)Z_1 - bY_1 | Y_0 = y_i] = -C(b)E^{\pi}(Z_1) + bE^{\pi}(Y_1 | Y_0 = y_i) < 0$$

$$\text{(by independence)} \tag{2.12}$$

and the second derivative is

$$f_i''(t) = E^{\pi}[[C(b)Z_1 - bY_1]^2 e^{-t[C(b)Z_1 - bY_1]} | Y_0 = y_i] > 0.$$

This implies that

$$f_i(t) \text{ is a convex function with } f_i(0) = 0. \tag{2.13}$$

By $P^{\pi}(bY_1 - C(b)Z_1 > 0 | Y_0 = y_i) > 0,$ we can find some constant $\delta > 0$ such that

$$P^{\pi}(bY_1 - C(b)Z_1 > \delta > 0 | Y_0 = y_i) > 0.$$

Then we get

$$f_i(t) = E^{\pi}[e^{-t[C(b)Z_1 - bY_1]} | Y_0 = y_i] - 1$$

$$\geq E^{\pi}(\{e^{-t[C(b)Z_1 - bY_1]} | Y_0 = y_i\} \cdot 1_{\{bY_1 - C(b)Z_1 > \delta | Y_0 = y_i\}}) - 1$$

$$\geq e^{t\delta} P^{\pi}(bY_1 - C(b)Z_1 > \delta | Y_0 = y_i) - 1.$$

This implies that

$$\lim_{t \to +\infty} f_i(t) = +\infty. \tag{2.14}$$

From (2.12), (2.13) and (2.14), there exists a unique positive constant R_i satisfying $f_i(R_i) = 0$.

Let $R_0 = \inf\{R_i > 0 : E^{\pi}[e^{-R_0[C(b)Z_1 - bY_1]} | Y_0 = y_i] = 1\}$. Then R_0 satisfies (2.11).

Lemma 3.2. *Let model* (2.5) *satisfy Assumptions* 2.1 *to* 2.8.

Assume that for each $y_i \in G_Y = \{y_1, y_2, ..., y_n, ...\}$, $i_r \in G_I = \{i_1, i_2, ..., i_m, ...\}$,

$$P^{\pi}([bY_1 - C(b)Z_1](1 + I_1)^{-1} > 0 | Y_0 = y_i, I_0 = i_r) > 0$$

and

$$E^{\pi}(-[C(b)Z_1 - bY_1](1 + I_1)^{-1} | Y_0 = y_i, I_0 = i_r) < 0, \tag{2.15}$$

there exists $\rho_{ir} > 0$ *satisfying that*

$$E^{\pi}(e^{-\rho_{ik}[C(b)Z_1 - bY_1](1 + I_1)^{-1}} | Y_0 = y_i, I_0 = i_r) = 1. \tag{2.16}$$

Then

$$R_1 = \min \rho_{ir} \geq R_0. \tag{2.17}$$

Furthermore, for all $y_i \in G_Y = \{y_1, y_2, ..., y_n, ...\}$, $i_r \in G_I = \{i_1, i_2, ..., i_m, ...\}$,

$$E^{\pi}(e^{-R_1[C(b)Z_1 - bY_1](1+I_1)^{-1}} \,|\, Y_0 = y_i,\ I_0 = i_r) \le 1. \tag{2.18}$$

Proof. For each $y_i \in G_Y = \{y_1, y_2, ..., y_n, ...\},\ i_r \in G_I = \{i_1, i_2, ..., i_m, ...\}$, let

$$l_{ir}(t) = E^{\pi}(e^{-r[C(b)Z_1 - bY_1](1+I_1)^{-1}} \,|\, Y_0 = y_i,\ I_0 = i_r), \text{ for } t > 0.$$

Then the first derivative of $l_{ir}(t)$ at $t = 0$ is

$$l'_{ir}(0) = E^{\pi}(-[C(b)Z_1 - bY_1](1 + I_1)^{-1} \,|\, Y_0 = y_i,\ I_0 = i_r) < 0$$

and the second derivative is

$$l''_{ir}(t)$$

$$= E^{\pi}(([C(b)Z_1 - bY_1](1 + I_1)^{-1})^2 e^{-r[C(b)Z_1 - bY_1][(1+I_1)^{-1}]^2} \,|\, Y_0 = y_i,\ I_0 = i_r)$$

$$> 0.$$

This shows that $l_{ir}(t)$ is a convex function. From (2.15), it implies that $\lim_{t \to +\infty} f_{ir}(t) = +\infty$.

Let ρ_{ir} be the unique positive root of the equation $l_{ir}(t) = 0$ on $(0, +\infty)$ and $0 < \rho < \rho_{ir}$. Then

$$E^{\pi}(e^{-R_0[C(b)Z_1 - bY_1](1+I_1)^{-1}} \,|\, Y_0 = y_i,\ I_0 = i_r)$$

$$= \sum_{i,\,j} \sum_{r,\,s} p_{ij} q_{rs} e[e^{-R_0[C(b)Z_1 - by_j](1+i_s)^{-1}}] \text{ (by Jensen's inequality)}$$

$$\le \sum_{s} q_{rs} E[e^{-R_0[C(b)Z_1 - bY_1]} \,|\, Y_0 = y_i]^{(1+i_s)^{-1}}.$$

Consequently, by Lemma 3.1, we have $E[e^{-R_0[C(b)Z_1 - bY_1]} \,|\, Y_0 = y_i] = 1$. Hence, since $\sum_s p_{rs} = 1$,

$$E^{\pi}(e^{-R_0[C(b)Z_1 - bY_1](1+I_1)^{-1}} \,|\, Y_0 = y_i,\, I_0 = i_r) \le 1.$$

This implies that $l_{ir}(R_0) \le 0.$ Moreover, $R_0 \le \rho_{ir}$ for i, r and so

$$R_1 := \min_{i,\, r} \rho_{ir} \ge R_0.$$

Thus, (2.13) holds. In addition, $R_1 \le \rho_{ir}$ for all i, r, which implies that $l_{ir}(R_1) \le 0.$ This yields (2.14).

Theorem 3.1. *Under the hypotheses of Lemma* 3.1 *and Lemma* 3.2, *for all* $y_i \in G_Y = \{y_1, y_2, ..., y_n, ...\}$, $i_r \in G_I = \{i_1, i_2, ..., i_m, ...\}$ *and* $u \ge 0$,

$$\psi^{\pi}(u,\, y_i,\, i_r) \le e^{-R_1 u}. \tag{2.19}$$

Proof. Let $V_k = U_k \prod_{l=1}^{k} (1 + I_l)^{-1}.$ Then

$$V_k = u + \sum_{l=1}^{k} \left((C(b)Z_1 - bY_l) \prod_{t=1}^{l} (1 + I_t)^{-1} \right). \tag{2.20}$$

Let $S_n = e^{-R_1 V_n}.$ Then

$$S_{n+1} = S_n e^{-R_1(C(b)Z_{n+1} - bY_{n+1}) \prod_{t=1}^{n+1} (1+I_t)^{-1}}.$$

Thus, for any $n \ge 1$,

$$E^{\pi}[S_{n+1} \,|\, Y_1, ..., Y_n,\, Z_1, ..., Z_n,\, I_1, ..., I_n]$$

$$= S_n E^{\pi}[e^{-R_1(C(b)Z_{n+1} - bY_{n+1}) \prod_{t=1}^{n+1} (1+I_t)^{-1}} \,|\, Y_1, ..., Y_n,\, Z_1, ..., Z_n,\, I_1, ..., I_n]$$

$$= S_n E^{\pi}[e^{-R_1(C(b)Z_{n+1} - bY_{n+1}) \prod_{t=1}^{n+1} (1+I_t)^{-1}} \,|\, Y_n,\, I_1, ..., I_n].$$

From $0 \le \prod_{t=1}^{n}(1+I_t)^{-1} \le 1$ and Jensen's inequality, we have

$$S_n E^{\pi}[e^{-R_1(C(b)Z_{n+1}-bY_{n+1})\prod_{t=1}^{n+1}(1+I_t)^{-1}} \mid Y_n, I_1, \ldots, I_n]$$

$$\le S_n E^{\pi}[e^{-R_1(C(b)Z_{n+1}-bY_{n+1})(1+I_{n+1})^{-1}} \mid Y_n, I_1, \ldots, I_n]^{\prod_{t=1}^{n}(1+I_t)^{-1}}.$$

In addition,

$$E^{\pi}[e^{-R_1(C(b)Z_{n+1}-bY_{n+1})(1+I_{n+1})^{-1}} \mid Y_n, I_1, \ldots, I_n]$$

$$= E^{\pi}[e^{-R_1(C(b)Z_{n+1}-bY_{n+1})(1+I_{n+1})^{-1}} \mid Y_n, I_n]$$

$$= E^{\pi}[e^{-R_1(C(b)Z_n-bY_1)(1+I_1)^{-1}} \mid Y_0, I_0] \le 1.$$

Thus, we have

$$E^{\pi}[S_{n+1} \mid Y_1, \ldots, Y_n, Z_1, \ldots, Z_n, I_1, \ldots, I_n] \le S_n.$$

This implies that $\{S_n\}_{n\ge 1}$ is a supermartingale.

Let $T_i = \min\{n : V_n < 0 \mid I_0 = i\}$, where V_n is given by (2.20). Then T_i is a stopping time and $n \wedge T_i = \min\{n, T_i\}$ is a finite stopping time. Thus, by the optional stopping theorem for martingale, we get

$$E^{\pi}(S_{n\wedge T_i}) \le E^{\pi}(S_0) = e^{-R_1 u}.$$

Hence,

$$e^{-R_1 u} \ge E^{\pi}(S_{n\wedge T_i}) \ge E^{\pi}((S_{n\wedge T_i}) \cdot 1_{(T_i \le n)}) \ge E^{\pi}((S_{T_i}) \cdot 1_{(T_i \le n)})$$

$$= E^{\pi}(e^{-R_1 V_{T_i}} . 1_{(T_i \le n)}) \ge E^{\pi}(1_{(T_i \le n)}) \ge \psi_n^{\pi}(u, y_i, i_r), \qquad (2.21)$$

where (2.21) follows because $V_{T_i} < 0$. Thus, by letting $n \to +\infty$ in (2.19), we obtain the result.

4. Conclusion

We studied a controlled general risk process when claim and rates of interest are homogeneous Markov chains taking a countable number of non-negative values. Using Lemma 3.1 and Lemma 3.2, Theorem 3.1 provides a probability inequality for $\psi^{\pi}(u, y_i, i_0)$ by the martingale approach.

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