Advances and Applications in Statistics
© 2018 Pushpa Publishing House, Allahabad, India
http://www.pphmj.com
http://dx.doi.org/10.17654/AS052020073
Volume 52, Number 2, 2018, Pages 73-95 **ISSN: 0972-3617**

OPTIMAL UTILIZATION OF RESOURCES IN ORGANIZATIONS USING DATA ENVELOPMENT ANALYSIS

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Abstract

Resource reallocation methods often assign fixed resources to all the operating units of an organization, even those with inefficient input-oriented production. However, inefficient units do not use all of their available resources. In a centralized organizational setting, optimal use of the excess resources of inefficient units is an important issue. This study analyzes how to transfer all the resources from inefficient units to efficient units at low cost. Data envelopment analysis (DEA) applied to proposed ideas was not addressed in previous resource reallocation approaches: first, decreased unit efficiency after resource reallocation is prevented; second, changing the inputs of efficient units

Received: June 25, 2017; Accepted: August 24, 2017

2010 Mathematics Subject Classification: 90C05, 90C90.

Keywords and phrases: DEA, resource reallocation, efficiency, centralized management system.

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simultaneously changes their outputs; third, the amount added to outputs and inputs is proportional to units' capacity to increase production and consumption levels. Reasonable feasible changes in inputs and outputs lead to fair reallocation in which production plans can be used in real-world applications. The applicability of the proposed method is illustrated with a real-world example of bank branches in Iran.

1. Introduction

Organizations' strategic planning of their goals involves resource reallocation. In the economic sphere, resources should be optimally reallocated to yield products and services that meet market demand at the lowest possible production costs. Extensive theoretical research on and many applications of resource reallocation have been conducted [15, 8].

Nesterenko and Zelenyuk [15] develop a resource reallocation model when the reallocation of inputs to all individual units is possible. The model improves the efficiency of each group even if its individual units are efficient. Inklaar et al. [8] analyze the German banking market structure to determine the impact of bank market power on the aggregate industry output growth of small and medium-sized enterprises. The researchers also assess the effect of bank market power on resource reallocation.

Some researchers investigate the resource reallocation problem from the efficiency perspective of data envelopment analysis (DEA) developed by Charnes et al. [3]. DEA is widely used as an empirical method to evaluate the relative performance of peer decision-making units (DMU). This approach estimates an efficient frontier using specific assumptions about the shape of the production possibility set, or the set of feasible activities. This method deals with mathematical models and the modifications and extensions needed for use in actual applications [4, 9]. Various applications (performance analysis of highways, public schools, restaurant chains, etc.) are in the published literature [17, 14, 6].

Many contributions to resource reallocation from efficiency perspective examine to what extent reallocating resources drives production expansion

[7, 10, 16, 18-20]. Golany et al. [7] presented an input-oriented DEA-based model for reallocating resources; the model is subject to total input availability limitations and bounds on changes in the solution for each DMU. The existence of the bounds is important because without these constraints, the model would allocate all inputs to only one DMU. Kumar and Shina [10] provide two models for production plans in which every period is considered to be a DMU and can be associated with other periods through the existing constraints. Pachkova [16] proposes a restricted reallocation approach based on three cases: full reallocation, restricted allocation, and when reallocation is impossible. The model reallocates resources by controlling given transfer costs.

Lozano and Villa [12] introduce the concept of centralized resource allocation and propose both radial and non-radial centralized resource allocation with the aim of reducing the radial and the separate consumption of total input. Asmild et al. [2] reconsiders one of Lozano and Villa's [12] models, modifies it to adjust inefficient units, and extends it with non-discretionary and non-transferable variables. Fang [5] extends the models of Lozano and Villa [12] and Asmild et al. [2] and reduces the total input by adjusting the input and output values. Amirteimoori and Kordrostami [1] assume that changes in supply and demand are predicted by the central decision-maker (DM). Their model prevents decreased efficiency of production plans and defines units' ability as the size of the operational units in the production plan. Wu et al. [17] develop a DEA centralized model for reallocating the inputs of DMUs. The study presents a concept of satisfaction degree of DMUs to adjust DMUs in the new production period. López-Torres and Prior's [11] work is concerned with a human resource reallocation problem in schools with a centralized management mechanism. The approach develops an iterative procedure to make an effort to increase the efficiency of schools.

All research mentioned above assigns inputs to all individual units, even inefficient units. Performing resource reallocation so that the excess resources of inefficient units are transferred to efficient units, Gimenez-Garcia et al. [6] present a three-step method. First, the approach recognizes the resource excesses of inefficient units, second, these amounts are allocated to efficient units, and finally, the output targets are modified for inefficient units.

The current research is concerned with optimal reallocation in a centralized management system. The aim of this approach is to produce a fair, practical production plan. In this study, similar to the research of Gimenez-Garcia et al. [6], the excess resources of inefficient units for reallocation are determined. However, the innovation of this paper addresses problems neglected in the research of Gimenez-Garcia et al. [6]. First, the efficiency of the units does not decline after the reallocation. Second, due to the increased inputs of the efficient units, their outputs also increase after the reallocation. Third, the amounts added to the inputs and outputs of efficient units are proportional to the units' ability to increase production and consumption. In general, although allocation methods are often faced with dramatic changes in inputs and outputs, the changes made in this study are reasonable and practical.

This paper is organized as follows. Section 2 provides an overview of the definitions of the units' abilities and the resource reallocation method. Section 3 proposes a model that deals with the reallocation of resources. Section 4 examines the application of the proposed method to commercial bank branches in Iran and interprets the obtained results, while the final section presents the conclusions of the study.

2. Preliminaries

To describe the DEA models in this paper, assume a production process of n DMU_k, $k \in K = \{1, 2, ..., n\}$, each consuming m inputs x_{ik}, $i \in I = \{1, 2, ..., m\}$ to yield s outputs y_{rk}, $r \in R = \{1, 2, ..., s\}$. The following symbols are used in the formulations:

Indexes

i index for inputs $(i \in I)$

i_v index for allocable inputs $(i_v \in I_v)$

i_f	index for non-allocable inputs $(i_f \in I_f)$
k, o	index for DMUs $(k, o \in K)$
k_1, o_1	index for efficient DMUs $(k_1, o_1 \in K_1)$
k_2, o_2	index for inefficient DMUs $(k_2, o_2 \in K_2)$
r	index for outputs $(r \in R)$

Decision variables

$a_{ik_1}^{+}, a_{ik_1}^{-}, b_{rk_1}^{+}, b_{rk_1}^{-}$	deviation variables that exhibit deviations above and below a goal
d_{rk_1}	amount to be added to the present *r*th output of DMU_{k_1}
$q_{ik_2k_1}$	allocated amount of the *i*th input from unit k_2 to k_1
$s_i^{-(o)}$	slack variable for the *i*th input of the *o*th unit
$s_r^{+(o)}$	slack variable for the *r*th output for the *o*th unit
u_r	weight of the *r*th output
v_i	weight of the *i*th input
$\theta^{(o)}$	input-oriented radial efficiency of the *o*th unit
$\lambda_k^{(o)}$	intensity variable for the *o*th unit (the intensity levels at which DMUs operate) $k = 1, \ldots, n$
$\phi^{(k_1)}$	output-oriented efficiency of the k_1th unit achieved after reallocating inputs

Parameters

$c_{ik_2k_1}$	cost of reallocating one unit of the *i*th input from the k_2th unit to the k_1th unit

D_r	demand change for rth output
$rem_i^{(o_2)}$	ith excess resource for the o_1th unit
x_{ik}	ith input measure for the kth unit
y_{rk}	rth output measure for the kth unit
w_1, w_2, w_3	the preference of the objectives
α_o	proportion of demand change in DMU_o
β_o	proportion of input excess in DMU_o
ε	a non-Archimedean infinitesimal number

It seems reasonable that the amount added to outputs and inputs should be proportional to the ability of operational units as defined by the size of the operational units. Amirteimoori and Kordrostami [1] define take units' ability as follows.

Definition 1. The magnitude size of DMU_k, on the input side, denoted by MSI_k, is defined as the optimal objective value of the following linear programming (LP) problem:

$$\text{Max } MSI_o = \sum_{i \in I} v_i x_{io}$$

$$\text{s.t. } \sum_{i \in I} v_i x_{ik} \leq 1, \quad k \in K,$$

$$v_i \geq \varepsilon, \quad i \in I. \tag{1}$$

DMU_o is said to be greater than DMU_k on the input side if and only if $MSI_o > MSI_k$.

Definition 2. The magnitude size of DMU_o on the output side, denoted by MSO_o, is defined as the optimal objective value of the following LP problem:

$$\begin{aligned} &\text{Max } MSO_o = \sum_{r \in R} u_r y_{ro} \\ &\text{s.t. } \sum_{r \in R} u_r y_{rk} \leq 1, \quad k \in K, \\ &u_r \geq \varepsilon, \quad r \in R. \end{aligned} \tag{2}$$

Similarly, DMU_o is said to be greater than DMU_k in the output side if and only if $MSO_o > MSO_k$.

For each DMU_o, $\alpha_o = MSO_o \Big/ \sum_{o \in K} MSO_o$ and $\beta_o = MSI_o \Big/ \sum_{o \in K} MSI_o$ are considered as an output production, the proportion of input consumption, respectively.

Performing resource reallocation so that the excess resources of inefficient units are transferred to efficient units, Gimenez-Garcia et al. [6] present a three-step method. First, efficient and inefficient units are identified by the following LP, and the excess resources of the inefficient units are determined: $rem_i^{(o_2)} = (1 - \theta^{(o_2)*} x_{io_2}) + s_i^{-(o_2)*}$; $\forall i \in I$, $\forall o_2 \in K_2$, where $\theta^{(o_2)*}$ and $s_i^{-(o_2)*}$ are the optimal solutions to LP (3):

$$\begin{aligned} &\text{Min } \sum_{o \in K} \theta^{(o)} - \varepsilon \left(\sum_{o \in K} \sum_{i \in I} s_i^{-(o)} + \sum_{o \in K} \sum_{r \in R} s_r^{-(o)} \right) \\ &\text{s.t. } \sum_{k \in K} \lambda_k^{(o)} x_{ik} + s_i^{-(o)} = x_{io} \theta^{(o)}; \quad o \in K, \quad i \in I, \\ &\sum_{k \in K} \lambda_k^{(o)} y_{rk} - s_i^{+(o)} = y_{ro}; \quad o \in K, \quad r \in R, \\ &\theta^{(o)} \leq 1, \quad \lambda_k^{(o)} \geq 0; \quad o, k \in K; \quad s_i^{-(o)}, \quad s_r^{+(o)} \geq 0, \quad i \in I, \quad r \in R. \end{aligned} \tag{3}$$

LP (3) serves to unify the phase I ($\theta^{(o)}$ is minimized) and phase II (the slacks are maximized by employing $\theta^{(o)*})$ procedures in a single LP. The efficiency scores are calculated through a single LP problem instead of solving *n*LP for evaluating all units.

Second, the excess resources are allocated among the efficient units by LP (4). In the current study, a number of constraints in the resource transformation in the fast food chain is omitted, as illustrated by [6],

$$\begin{aligned}
\text{Max} \quad & \sum_{o_1 \in K_1} \phi^{(o_1)} - \varepsilon\left(\sum_{o_1 \in K_1}\sum_{o_2 \in K_2}\sum_{i \in I_v} q_{io_2o_1} c_{io_2o_1}\right) \\
\text{s.t.} \quad & \sum_{k \in K} \lambda_k^{(o_1)} y_{rk} - s_r^{+(o_1)} = \phi^{(o_1)} y_{ro_1}, \quad r \in R, \quad o_1 \in K_1, \\
& \sum_{k \in K} \lambda_k^{(o_1)} x_{ik} + s_i^{-(o_1)} = x_{io_1} + \sum_{k_2 \in K_2} q_{ik_2o_1}, \quad i \in I_v, \quad o_1 \in K_1, \\
& \sum_{k \in K} \lambda_k^{(o_1)} x_{ik} + s_i^{-(o_1)} = x_{io_1}, \quad i \in I_f, \quad o_1 \in K_1, \\
& \sum_{o_1 \in K_1} q_{ik_2o_1} \le rem_i^{(k_2)}, \quad i \in I_v, \quad k_2 \in K_2, \\
& \phi^{(o_1)} \ge 1, \quad \lambda_k^{(o_1)}, s_r^{+(o_1)}, s_i^{-(o_1)}, q_{ik_2o_1} \ge 0, \quad i \in I, \quad r \in R, \\
& o_1 \in K_1, \quad k_2 \in K_2.
\end{aligned} \tag{4}$$

Third, the output targets are modified for inefficient units by LP (5),

$$\begin{aligned}
\text{Max} \quad & \sum_{o_2 \in K_2}\left(\phi^{(o_2)} + \varepsilon\left(\sum_{i \in I} s_i^{-(o_2)} + \sum_{r \in R} s_r^{+(o_2)}\right)\right) \\
\text{s.t.} \quad & \sum_{k \in K} \lambda_k^{(o_2)} y_{rk} - s_r^{+(o_2)} = \phi^{(o_2)} y_{ro_2}, \quad r \in R, \quad o_2 \in K_2, \\
& \sum_{k \in K} \lambda_k^{(o_2)} x_{ik} + s_i^{-(o_2)} = x_{io_2} - \sum_{k_1 \in K_1} q_{io_2k_1}, \quad i \in I_v, \quad o_2 \in K_2,
\end{aligned}$$

$$\sum_{k \in K} \lambda_k^{(o_2)} x_{ik} + s_i^{-(o_2)} = x_{io_2}, \quad i \in I_f, \quad o_2 \in K_2,$$

$$\lambda_k^{(o_2)}, s_r^{+(o_2)}, s_i^{-(o_2)} \geq 0, \quad i \in I, \quad r \in R, \quad o_2 \in K_2, \quad k \in K. \tag{5}$$

This method has some limitations. First, the resource reallocation is performed without considering the ability of operational units. As seen in the following empirical examples, it is possible that resources will not be assigned to an efficient unit when this unit has the same transfer cost as other units. Second, it is expected that efficient units increase production by increasing resources, but this is not considered in this method. This limitation is also illustrated in a practical example. Also, in this method, the outputs remain unchanged by increasing the inputs. Third, this method has no measure for maintaining the efficiency of DMUs, so efficient units might become inefficient after reallocation. This limitation is also illustrated in the example.

3. Proposed Model

The proposed method is based on DEA, which deals with resource reallocation and target setting in two steps. In a centralized management system, the method identifies efficient and inefficient units and transfers excess input from inefficient units to efficient units. The proposed method addresses three observations neglected by Gimenez-Garcia et al. [6]. First, resource reallocation is performed taking into account the potential of operational units. Second, the proposed model prevents decreased efficiency among new production units. Third, efficient units increase production with increased resources. The central DM is responsible for monitoring the units and determines the total amount of demand changes. The model modifies the outputs according to the central managers' expectations and the units' ability.

To formulate a mathematical model, $y_{rk_1} + d_{rk_1}$ is the rth revised output, $x_{ik_1} + \sum_{k_2 \in K_2} q_{ik_2k_1}$ is the ith revised input of efficient unit k_1, $\sum_{k_2 \in K_2} q_{ik_2k_1}$ is the ith allocated input to this unit, $x_{ik_2} - \sum_{k_1 \in K_1} q_{ik_2k_1}$ is the ith new input of inefficient unit k_2, and $\sum_{k_1 \in K_1} q_{ik_2k_1}$ is the sum of the *i*th transferred input from this unit to all efficient units. Note that $x_{ik_2} - \sum_{k_1 \in K_1} q_{ik_2k_1} > 0$ because $\sum_{k_1 \in K_1} q_{ik_2k_1} < x_{ik_2}, \forall i \in I, k_2 \in K_2$. The main axiom in this analysis is that increased input usage leads to increased output production. The proposed approach to resource reallocation is aimed at transforming resources. The proposed model is as follows:

$$\text{Min } w_1 \sum_{o_1 \in K_1} \sum_{o_2 \in K_2} \sum_{i \in I} q_{io_2o_1} c_{io_2o_1} + w_2 \sum_{o_1 \in K_1} \sum_{i \in I} (a^+_{io_1} + a^-_{io_1}) + w_3 \sum_{o_1 \in K_1} \sum_{r \in R} (b^+_{ro_1} + b^-_{ro_1})$$

$$\text{s.t.} \quad \sum_{k_1 \in K_1} \lambda_{k_1}^{(o_2)} \left(x_{ik_1} + \sum_{k_2 \in K_2} q_{ik_2k_1} \right) + \sum_{k_2 \in K_2} \lambda_{k_2}^{(o_2)} \left(x_{ik_2} - \sum_{k_1 \in K_1} q_{ik_2k_1} \right) \le x_{io_2} - \sum_{k_1 \in K_1} q_{io_2k_1}, \quad o_2 \in K_2, \quad i \in I, \tag{6a}$$

$$\sum_{k_1 \in K_1} \lambda_{k_1}^{(o_2)} (y_{rk_1} + d_{rk_1}) + \sum_{k_2 \in K_2} \lambda_{k_2}^{(o_2)} y_{rk_2} \ge y_{ro_2}, \quad o_2 \in K_2, \quad r \in R, \tag{6b}$$

$$\sum_{k_1 \in K_1} \lambda_{k_1}^{(o_1)} (y_{rk_1} + d_{rk_1}) + \sum_{k_2 \in K_2} \lambda_{k_2}^{(o_1)} y_{rk_2} \ge y_{ro_1} + d_{ro_1}, \quad o_1 \in K_1, \quad r \in R, \tag{6c}$$

$$\sum_{k_1 \in K_1} \lambda_{k_1}^{(o_1)} \left(x_{ik_1} + \sum_{k_2 \in K_2} q_{ik_2k_1} \right) + \sum_{k_2 \in K_2} \lambda_{k_2}^{(o_1)} \left(x_{ik_2} - \sum_{k_1 \in K_1} q_{ik_2k_1} \right) = x_{io_1} + \sum_{k_2 \in K_2} q_{ik_2o_1}, \quad o_1 \in K_1, \quad i \in I, \tag{6d}$$

$$\frac{\sum_{k_1 \in K_1} \lambda_{k_1}^{(o_2)} \left(x_{ik_1} + \sum_{k_2 \in K_2} q_{ik_2k_1} \right) + \sum_{k_2 \in K_2} \lambda_{k_2}^{(o_2)} \left(x_{ik_2} - \sum_{k_1 \in K_1} q_{ik_2k_1} \right)}{x_{io_2} - \sum_{k_1 \in K_1} q_{io_2k_1}}$$

$$\geq \theta^{(o_2)}, \quad o_2 \in K_2, \quad i \in I, \tag{6e}$$

$$\sum_{k_1 \in K_1} q_{ik_2k_1} = rem_i^{(k_2)}, \quad i \in I, \quad k_2 \in K_2, \tag{6f}$$

$$\sum_{k_1 \in K_1} d_{rk_1} = D_r, \quad r \in R, \tag{6g}$$

$$d_{ro_1} \leq M \sum_{k_2 \in K_2} \sum_{i \in I} q_{ik_2o_1}, \quad o_1 \in K_1, \quad r \in R, \tag{6h}$$

$$\sum_{k_2 \in K_2} q_{ik_2o_1} - \beta_{o_1} \left(\sum_{k_2 \in K_2} rem_i^{(k_2)} \right) = a_{io_1}^{+} - a_{io_1}^{-}, \quad o_1 \in K_1, \quad i \in I, \tag{6i}$$

$$d_{ro_1} - \alpha_{o_1} D_r = b_{ro_1}^{+} - b_{ro_1}^{-}, \quad o_1 \in K_1, \quad r \in R, \tag{6j}$$

$$\lambda_{k_1}^{(o_1)}, \lambda_{k_2}^{(o_1)}, \lambda_{k_1}^{(o_2)}, \lambda_{k_2}^{(o_2)}, d_{rk_1}, q_{ik_2k_1}, b_{ro_1}^{+}, b_{ro_1}^{-}, a_{io_1}^{+}, a_{io_1}^{-} \geq 0,$$

$$k_1, o_1 \in K_1, \quad k_2, o_2 \in K_2, \quad r \in R, \quad i \in I,$$

where $w_1 + w_2 + w_3 = 1$. In this model, constraints (6a), (6b), (6c) and (6d) ensure that the new inputs and outputs are technologically feasible (under the assumption of constant returns to scale). Constraints (6d) and (6e) ensure that the efficiency of units does not decrease. Constraints (6f) and (6g) are established based on the assumption that all excess resources are allocated, and all output demand is met. Constraint (6h), where M is a large number, states that if resources are not allocated to a DMU, its outputs remain unchanged. Constraints (6i) and (6j) describe the following: the target setting for the rth output and the input usage for the ith input of current efficient unit k_1 are considered to be $\alpha_{o_1} D_r$ and $\beta_{o_1} \left(\sum_{k_2 \in K_2} rem_i^{(k_2)} \right)$, respectively, and

α_{o_1} and β_{o_1} are adapted through the definitions mentioned in preceding section. Goal achievement variables are introduced to take into consideration the target setting for the rth output and the ith input. Deviations from the targeted levels are minimized to achieve desired solutions. Let $\sum_{k_2 \in K_2} q_{ik_2 o_1} - \beta_{o_1}\left(\sum_{k_2 \in K_2} rem_i^{(k_2)}\right) = a_{io_1}^{+} - a_{io_1}^{-}$, and $d_{ro_1} - \alpha_{o_1} D_r = b_{ro_1}^{+} - b_{ro_1}^{-}$. The nonnegative variables $a_{io_1}^{+}$, $b_{ro_1}^{+}$, $a_{io_1}^{-}$ and $b_{ro_1}^{-}$ indicate deviations higher and lower than the goals.

Since $\lambda_{k_1}^{(o_1)}, \lambda_{k_2}^{(o_1)}, \lambda_{k_1}^{(o_2)}, \lambda_{k_2}^{(o_2)}, d_{rk_1}$ and $q_{ik_2k_1}$ are decision variables, the equations system of model (6) is obviously nonlinear. Hence, we make the change of variables to create a linear system. These changes are imposed as follows: $q_{ik_2k_1}(\lambda_{k_1}^{(o_1)} - \lambda_{k_2}^{(o_1)})$ and $q_{ik_2k_1}(\lambda_{k_1}^{(o_2)} - \lambda_{k_2}^{(o_2)})$ are the multiple of $q_{ik_2k_1}$, therefore we can write them as $q_{ik_2k_1}(\lambda_{k_1}^{(o_1)} - \lambda_{k_2}^{(o_1)}) + \eta_{ik_2k_1}^{(o_1)} = q_{ik_2k_1}$ and $q_{ik_2k_1}(\lambda_{k_1}^{(o_2)} - \lambda_{k_2}^{(o_2)}) + \eta_{ik_2k_1}^{(o_2)} = q_{ik_2k_1}$ so we replace $q_{ik_2k_1} - \eta_{ik_2k_1}^{(o_1)}$ with $q_{ik_2k_1}(\lambda_{k_1}^{(o_1)} - \lambda_{k_2}^{(o_1)})$ and $q_{ik_2k_1} - \eta_{ik_2k_1}^{(o_2)}$ with $q_{ik_2k_1}(\lambda_{k_1}^{(o_2)} - \lambda_{k_2}^{(o_2)})$ also $\lambda_{k_1}^{(o_1)} d_{rk_1}$ and $\lambda_{k_1}^{(o_2)} d_{rk_1}$ are the multiple of d_{rk_1} then $\lambda_{k_1}^{(o_1)} d_{rk_1} + \gamma_{rk_1}^{(o_1)} = d_{rk_1}$ and $\lambda_{k_1}^{(o_2)} d_{rk_1} + \gamma_{rk_1}^{(o_2)} = d_{rk_1}$ hence $\lambda_{k_1}^{(o_1)} d_{rk_1} = d_{rk_1} - \gamma_{rk_1}^{(o_1)}$ and $\lambda_{k_1}^{(o_2)} d_{rk_1} = d_{rk_1} - \gamma_{rk_1}^{(o_2)}$, that $\eta_{ik_2k_1}^{(o_1)}, \eta_{ik_2k_1}^{(o_2)}, \gamma_{rk_1}^{(o_1)}$ and $\gamma_{rk_1}^{(o_2)}$ are unrestricted in sign. Therefore, nonlinear programming model (6) is converted into LP model (7):

$$\text{Min } w_1 \sum_{o_1 \in K_1} \sum_{o_2 \in K_2} \sum_{i \in I} q_{io_2o_1} c_{io_2o_1} + w_2 \sum_{o_1 \in K_1} \sum_{i \in I} (a_{io_1}^{+} + a_{io_1}^{-})$$
$$+ w_3 \sum_{o_1 \in K_1} \sum_{r \in R} (b_{ro_1}^{+} + b_{ro_1}^{-})$$

s.t. $$\sum_{k_1 \in K_1} \lambda_{k_1}^{(o_2)} x_{ik_1} + \sum_{k_2 \in K_2} \lambda_{k_2}^{(o_2)} x_{ik_2} + \sum_{k_1 \in K_1} \sum_{k_2 \in K_2} (q_{ik_2k_1} - \eta_{ik_2k_1}^{(o_2)})$$

$$\leq x_{io_2} - \sum_{k_1 \in K_1} q_{io_2k_1}, \quad o_2 \in K_2, \quad i \in I,$$

$$\sum_{k_1 \in K_1} \lambda_{k_1}^{(o_2)} y_{rk_1} + \sum_{k_2 \in K_2} \lambda_{k_2}^{(o_2)} y_{rk_2} + \sum_{k_1 \in K_1} (d_{rk_1} - \gamma_{rk_1}^{(o_2)}) \geq y_{ro_2},$$

$$o_2 \in K_2, \quad r \in R,$$

$$\sum_{k_1 \in K_1} \lambda_{k_1}^{(o_1)} y_{rk_1} + \sum_{k_2 \in K_2} \lambda_{k_2}^{(o_1)} y_{rk_2} + \sum_{k_1 \in K_1} (d_{rk_1} - \gamma_{rk_1}^{(o_1)}) \geq y_{ro_1} + d_{ro_1},$$

$$o_1 \in K_1, \quad r \in R,$$

$$\sum_{k_1 \in K_1} \lambda_{k_1}^{(o_1)} x_{ik_1} + \sum_{k_2 \in K_2} \lambda_{k_2}^{(o_1)} x_{ik_2} + \sum_{k_1 \in K_1} \sum_{k_2 \in K_2} (q_{ik_2k_1} - \eta_{ik_2k_1}^{(o_1)})$$

$$= x_{io_1} + \sum_{k_2 \in K_2} q_{ik_2k_1}, \quad \forall o_1 \in K_1, \quad \forall i \in I,$$

$$\sum_{k_1 \in K_1} \lambda_{k_1}^{(o_2)} x_{ik_1} + \sum_{k_2 \in K_2} \lambda_{k_2}^{(o_2)} x_{ik_2} + \sum_{k_1 \in K_1} \sum_{k_2 \in K_2} (q_{ik_2k_1} - \eta_{ik_2k_1}^{(o_1)})$$

$$\geq \theta^{(o_2)} \left(x_{io_2} - \sum_{k_1 \in K_1} q_{io_2k_1} \right), \quad o_2 \in K_2, \quad i \in I,$$

$$\sum_{k_1 \in K_1} q_{ik_2k_1} = rem_i^{(k_2)}, \quad i \in I, \quad k_2 \in K_2,$$

$$\sum_{k_1 \in K_1} d_{rk_1} = D_r, \quad r \in R,$$

$$d_{ro_1} \leq M \sum_{k_2 \in K_2} \sum_{i \in I} q_{ik_2o_1}, \quad o_1 \in K_1, \quad r \in R,$$

$$\sum_{k_2 \in K_2} q_{ik_2o_1} - \beta_{o_1}\left(\sum_{k_2 \in K_2} rem_i^{(k_2)}\right) = a_{io_1}^{+} - a_{io_1}^{-}, \quad o_1 \in K_1, \quad i \in I,$$

$$d_{ro_1} - \alpha_{o_1} D_r = b_{ro_1}^{+} - b_{ro_1}^{-}, \quad o_1 \in K_1, \quad r \in R,$$

$$\lambda_{k_1}^{(o_1)}, \lambda_{k_2}^{(o_1)}, \lambda_{k_1}^{(o_2)}, \lambda_{k_2}^{(o_2)}, d_{rk_1}, q_{ik_2k_1}, b_{ro_1}^{+}, b_{ro_1}^{-}, a_{io_1}^{+}, a_{io_1}^{-} \geq 0,$$

$$k_1, o_1 \in K_1, \quad k_2, o_2 \in K_2, \quad r \in R, \quad i \in I. \tag{7}$$

Model (7) is developed by the changes imposed on the variables.

Theorem 1. *There always exists a feasible solution to model* (7).

Proof. Clearly, we have the following feasible solutions to (7):

$$a_{io_1}^{+} = a_{io_1}^{-} = 0; \quad i \in I, o_1 \in K_1; \quad b_{ro_1}^{+} = b_{ro_1}^{-} = 0; \quad r \in R, k_1 \in K_1,$$

$$d_{ro_1} = \alpha_{o_1} D_r; \quad r \in R, k_1 \in K_1; \quad q_{ik_2k_1} = \beta_{k_1} rem_{k_2}^{(i)}; \quad k_1 \in K_1, k_2 \in K_2,$$

$$\eta_{ik_2k_1}^{(o_1)} = q_{ik_2k_1}; \quad i \in I, k_1 \in K_1, k_1 \neq o_1, k_2 \in K_2,$$

$$\eta_{ik_2o_1}^{(o_1)} = 0; \quad i \in I, o_1 \in K_1, k_2 \in K_2,$$

$$\eta_{ik_2k_1}^{(o_2)} = q_{ik_2k_1}; \quad i \in I, k_1 \in K_1, k_2, o_2 \in K_2, k_2 \neq o_2,$$

$$\eta_{io_2k_1}^{(o_2)} = q_{io_2k_1}; \quad i \in I, k_1 \in K_1, o_2 \in K_2,$$

$$\gamma_{rk_1}^{(o_1)} = d_{rk_1}; \quad r \in R, k_1, o_1 \in K_1, k_1 \neq o_1,$$

$$\gamma_{ro_1}^{(o_1)} = 0; \quad r \in R, o_1, k_1 \in K_1,$$

$$\gamma_{rk_1}^{(o_2)} = d_{rk_1}; \quad r \in R, k_1 \in K_1, o_2 \in K_2,$$

$$\lambda_{o_1}^{(o_1)} = 1,\ \lambda_{k_1}^{(o_1)} = 0,\ \lambda_{k_2}^{(o_1)} = 0;\quad k_1,\ o_1 \in K_1,\ k_2 \in K_2,\ k_1 \neq o_1,$$

$$\lambda_{k_1}^{(o_2)} = 0,\ \lambda_{k_2}^{(o_2)} = 0,\ \lambda_{o_2}^{(o_2)} = 1;\quad k_1,\ o_1 \in K_1,\ k_2,\ o_2 \in K_2,\ k_2 \neq o_2.$$

Hence, the proof is completed. □

Theorem 2. *If solution* (I) *is considered as an optimal solution to model* (7), *then solution* (II) *is also an optimal solution to model* (6):

Solution (I):

$$(\lambda_{k_1}^{(o_1)*},\ \lambda_{k_2}^{(o_1)*},\ \lambda_{k_1}^{(o_2)*},\ \lambda_{k_2}^{(o_2)*},\ q_{io_2o_1}^{*},\ d_{ro_1}^{*},\ \eta_{ik_2k_1}^{(o_1)*},\ \eta_{ik_2k_1}^{(o_2)*},$$

$$\gamma_{rk_1}^{(o_1)*},\ \gamma_{rk_1}^{(o_2)*},\ a_{io_1}^{+*},\ a_{io_1}^{-*},\ b_{ro_1}^{+*},\ b_{ro_1}^{-*}).$$

Solution (II):

$$\begin{cases} (\hat{\lambda}_{o_1}^{(o_1)} = 1;\ \hat{\lambda}_{k_1}^{(o_1)} = 0,\ k_1 \neq o_1;\ \hat{\lambda}_{k_2}^{(o_1)} = 0;\ \hat{\lambda}_{k_1}^{(o_2)} = 0;\ \hat{\lambda}_{o_2}^{(o_2)} = 1; \\ \hat{\lambda}_{k_2}^{(o_2)} = 0,\ k_2 \neq o_2;\ \hat{q}_{io_2o_1} = q_{io_2o_1}^{*}; \\ \hat{d}_{ro_1} = d_{ro_1}^{*};\ \hat{a}_{io_1}^{+} = a_{io_1}^{+*};\ \hat{a}_{io_1}^{-} = a_{io_1}^{-*};\ \hat{b}_{ro_1}^{+} = b_{ro_1}^{+*};\ \hat{b}_{ro_1}^{-} = b_{ro_1}^{-*}). \end{cases}$$

Proof. We assume that (I) is an optimal solution to model (7). Now, we prove that (II) is an optimal solution to model (6). Obviously, this solution is a feasible solution to model (6). Contrary to what we have mentioned so far, we assume that (II) is not an optimal solution to model (6); therefore, there is a feasible solution to this problem, such that its objective function value is lower than the objective function value with (I).

If $(\bar{\lambda}_{o_1}^{(o_1)} = 1;\ \bar{\lambda}_{k_1}^{(o_1)} = 0,\ k_1 \neq o_1;\ \bar{\lambda}_{k_2}^{(o_1)} = 0;\ \bar{\lambda}_{k_1}^{(o_2)} = 0;\ \bar{\lambda}_{o_2}^{(o_2)} = 1;\ \bar{\lambda}_{k_2}^{(o_2)} = 0,\ k_2 \neq o_2;\ \bar{q}_{io_2o_1} = \tilde{q}_{io_2o_1},\ \bar{d}_{rk_1} = \tilde{d}_{ro_1},\ \bar{a}_{io_1}^{+} = \tilde{a}_{io_1}^{+},\ \bar{a}_{io_1}^{-} = \tilde{a}_{io_1}^{-},\ \bar{b}_{ro_1}^{+} = \tilde{b}_{ro_1}^{+},\ \bar{b}_{ro_1}^{-} = \tilde{b}_{ro_1}^{-})$ is the feasible solution to model (6) which $(\tilde{\lambda}_{k_1}^{(o_1)},\ \tilde{\lambda}_{k_2}^{(o_1)},\ \tilde{\lambda}_{k_1}^{(o_2)},\ \tilde{\lambda}_{k_2}^{(o_2)},\ \tilde{q}_{io_2o_1},\ \tilde{d}_{ro_1},\ \tilde{a}_{io_1}^{+},\ \tilde{a}_{io_1}^{-},\ \tilde{b}_{ro_1}^{+},\ \tilde{b}_{ro_1}^{-})$ is also a feasible solution to model (7), with these two solutions, the values of the objective function of both models are equal; therefore, we have

$$w_1 \sum_{o_1} \sum_{o_2} \sum_{i} \overline{q}_{io_2o_1} c_{io_2o_1} + w_2 \sum_{o_1} \sum_{i} (\overline{a}^{+}_{io_1} + \overline{a}^{-}_{io_1}) + w_3 \sum_{o_1} \sum_{i} (\overline{b}^{+}_{ro_1} + \overline{b}^{-}_{ro_1})$$

$$< w_1 \sum_{o_1} \sum_{o_2} \sum_{i} \hat{q}_{io_2o_1} c_{io_2o_1} + w_2 \sum_{o_1} \sum_{i} (\hat{a}^{+}_{io_1} + \hat{a}^{-}_{io_1}) + w_3 \sum_{o_1} \sum_{i} (\hat{b}^{+}_{ro_1} + \hat{b}^{-}_{ro_1}).$$

Next,

$$w_1 \sum_{o_1} \sum_{o_2} \sum_{i} \tilde{q}_{io_2o_1} c_{io_2o_1} + w_2 \sum_{o_1} \sum_{i} (\tilde{a}^{+}_{io_1} + \tilde{a}^{-}_{io_1}) + w_3 \sum_{o_1} \sum_{i} (\tilde{b}^{+}_{ro_1} + \tilde{b}^{-}_{ro_1})$$

$$< w_1 \sum_{o_1} \sum_{o_2} \sum_{i} q^{*}_{io_2o_1} c_{io_2o_1} + w_2 \sum_{o_1} \sum_{i} (a^{+*}_{io_1} + a^{-*}_{io_1}) + w_3 \sum_{o_1} \sum_{i} (b^{+*}_{ro_1} + b^{-*}_{ro_1}).$$

This is a contradiction with the assumption, and the proof is completed. □

4. Application

This section describes the application of the proposed approach to commercial bank branches in Iran. Operating data from 2009 are collected from a sample of the branches supervised by the Central Bank (CB) in various regions of the Gulian province [1]. The original case in [1] is simplified by considering only 10 branches, two input variables (checking accounts (I1) and operational costs (I2)), and one output variable (deposits (O)). All the models are run using GAMS 24.1.3 software.

The original data and the current efficiency (E) of branches determined by LP (3) are depicted in Table 1. DMUs 7 and 9 are efficient. Moreover, the checking accounts and the excess operational costs of all the inefficient branches are calculated as $\sum_{k_2} rem_1^{(k_2)} = 2.083$ and $\sum_{k_2} rem_2^{(k_2)} = 2.506,$ respectively, and with these resources allocated to the efficient units, demand changes for deposits is predicted as $D = 3.$ In this application, it is assumed that the transformation costs are equal to 1 for all branches, and it is considered that $w_2 = 0.6$ and $w_3 = 0.4.$ The magnitude sizes of branches 7 and 9 on the input side are 1 and 0.624, respectively, demonstrating that the input size of branch 7 is greater than branch 9. The proportions of input consumption, therefore, are 0.616 and 0.384, respectively.

The resources transferred from inefficient branches to efficient branches 7 and 9 (TI1 and TI2) during the application of the proposed model (6) are listed in Table 2. No resources are transferred from branch 5 to branch 7 or from branches 1, 2, and 3 to branch 9. Branches 4, 6, 8, and 10 transfer some or all their excess resources to both branches 7 and 9. As stated, branch 7 has a greater proportion of input consumption than branch 9, and it is seen in practice that branch 7 receives more inputs than branch 9. The last two columns of Table 2 confirm that the excess resources of all the inefficient branches are transferred; in other words, all the amounts in these two columns are nonzero, and all the excess resources are allocated to branches 7 and 9 in full.

The changed inputs of the efficient branches and the limited excess resources of the inefficient branches raise interest in changing their outputs, in particular, in increasing the outputs of the efficient branches and keeping the outputs of the inefficient branches unchanged. The amounts added to the present outputs of the efficient branches are listed in Table 3. For example, the amounts added to the checking accounts and the operational costs of branch 7 are 1.302 and 1.567, respectively; therefore, it is expected that its output also increases. As seen in Table 3, the amount added to deposits is 1.889.

New input-output plans for all branches are indicated in Table 4. The last column of the table presents the new efficiency scores after reallocation. The results show that after the reallocation, the efficiency of all branches is increased, and the output of the efficient branches 7 and 9 is improved.

The approach proposed by Gimenez-Garcia et al. [6] is run, and the results are depicted in Tables 5 and 6. As seen in Table 5, no inputs are transferred to DMU 7, while all the inputs are transferred to DMU 9, which is an unfair reallocation. However, the definitions proposed by Amirteimoori and Kordrostami [1] for input usage and output target can prevent such unfair reallocations. Moreover, unlike the results from the proposed approach in the fourth column of Table 4, the output of efficient branches remains unchanged. However, the proposed approach intends that more output is produced by the higher input consumption. In most cases, it is expected that producing more outputs by consuming more inputs is harmful for efficient branches.

As the last column of Table 6 shows, the efficiency scores of some branches decrease; for example, the score of branch 9 decreases from 1.000 to 0.250, indicating that this efficient branch become inefficient after obtaining new inputs. Certainly, the increased inputs and the unchanged output gave rise to this inefficiency. As seen in Table 6, therefore, approach of Gimenez-Garcia et al. [6] worsens the performance of some branches.

For more details, Tables 1, 4 and 6 compare the proposed approach and the approach of Gimenez-Garcia et al. [6]. For example, inefficient branch 1 is observed to produce the output amount of 0.199 by consuming the input amounts of 0.355 and 0.350 (0.355, 0.350, and 0.199). The proposed approach suggests that the observed amounts are converted into 0.229, 0.225, and 0.199. In other words, this branch produces the same level of output by consuming fewer inputs, increasing its efficiency score from 0.644 to 0.790. The approach of Gimenez-Garcia et al. [6] suggests that the observed amounts are converted into 0.229, 0.225, and 0.190. In other words, this branch produces fewer outputs by consuming fewer inputs, decreasing its efficiency score from 0.644 to 0.330.

Regarding the other example, efficient branch 7 is observed to produce the output amount of 0.672 by consuming the input amounts of 0.984 and 0.745 (0.984, 0.745, and 0.672). The proposed approach suggests that these observed amounts are converted into 2.286, 2.305, and 2.561. In other words, this branch produces more output by consuming more inputs, leaving the efficiency score unchanged (equal to 1).

Table 1. Input/output data for bank branches

Bank branches	I1	I2	O	E
1	0.355	0.350	0.199	0.644
2	0.422	0.714	0.170	0.278
3	0.536	0.774	0.282	0.423
4	0.430	0.674	0.387	0.669
5	0.451	0.648	0.440	0.789
6	0.254	0.521	0.226	0.604
7	0.984	0.745	0.672	1.000
8	0.565	0.597	0.145	0.276
9	0.285	0.492	0.420	1.000
10	1.000	0.728	0.278	0.423

Table 2. The transferred resources under proposed model (7)

From/To	7		9			
	TI1	TI2	TI1	TI2	Total TI1	Total TI2
1	0.126	0.125	0.000	0.000	0.126	0.125
2	0.304	0.516	0.000	0.000	0.304	0.516
3	0.309	0.447	0.000	0.000	0.309	0.447
4	0.053	0.000	0.089	0.223	0.142	0.223
5	0.000	0.000	0.095	0.137	0.095	0.137
6	0.101	0.000	0.000	0.206	0.101	0.206
8	0.409	0.059	0.000	0.373	0.409	0.432
10	0.000	0.420	0.597	0.000	0.597	0.420
Total	1.302	1.567	0.781	0.939	2.083	2.506

Table 3. Amount added to the current output of the efficient units under proposed model (7)

Efficient units	7	9	Total
Added amount	1.889	1.111	3.000

Table 4. New input, output, and efficiency of DMUs under proposed model (7)

Bank branches	I1	I2	O	E
1	0.229	0.225	0.199	0.790
2	0.118	0.198	0.170	0.960
3	0.227	0.327	0.282	0.840
4	0.288	0.451	0.387	0.910
5	0.356	0.511	0.440	0.860
6	0.153	0.315	0.226	1.000
7	2.286	2.305	2.561	1.000
8	0.156	0.224	0.145	0.830
9	1.066	1.431	1.531	1.000
10	0.403	0.131	0.278	0.810

Table 5. Transferred resources under Gimenez-Garcia et al.'s [6] approach

From/To	7		9			
	TI1	TI2	TI1	TI2	Total TI1	Total TI2
1	0.000	0.000	0.126	0.125	0.126	0.125
2	0.000	0.000	0.304	0.516	0.304	0.516
3	0.000	0.000	0.309	0.447	0.309	0.447
4	0.000	0.000	0.142	0.223	0.142	0.223
5	0.000	0.000	0.095	0.137	0.095	0.137
6	0.000	0.000	0.101	0.206	0.101	0.206
8	0.000	0.000	0.409	0.432	0.409	0.432
10	0.000	0.000	0.597	0.420	0.597	0.420

Table 6. New input/output and new efficiency of DMUs under Gimenez-Garcia et al.'s [6] approach

	I1	I2	O	E
1	0.229	0.225	0.190	0.330
2	0.118	0.198	0.167	0.530
3	0.227	0.327	0.277	0.450
4	0.288	0.451	0.387	0.500
5	0.356	0.511	0.432	1.000
6	0.153	0.315	0.229	0.340
7	0.984	0.745	0.672	1.000
8	0.156	0.165	0.146	0.350
9	2.368	2.548	0.420	0.250
10	0.403	0.308	0.264	0.250

5. Conclusion

The present research is concerned with the optimization problem of production planning and resources reallocation in a centralized management system. This study uses DEA, which is a powerful tool based on mathematical programming for the optimal transformation of resources from the inefficient units to the efficient units of an organization. The proposed method is applied in two steps. First, the inefficient DMUs and their excess resource are identified, and second, the allocated resources and the output modifications for the efficient units are calculated at the same time. Three areas neglected in the approach of Gimenez-Garcia et al. [6] are addressed by this transformation and constitute the contribution of this research. First, the efficiency of all the units is preserved or improved after the reallocation. Second, changing the inputs of efficient units simultaneously changes their outputs, according to the allowable amounts determined by the DM in the management system and labeled change demands. Third, the amounts added to the current outputs and inputs become proportional to the operational units' potential to produce more output.

The real-world example of Iranian commercial bank branches illustrates the applicability of the proposed approach. The transformation is accomplished in a way that exhibits advantages of the proposed approach. To establish its effectiveness, comparisons are made with the approach of Gimenez-Garcia et al. [6] to show that the plans obtained in the proposed approach are more reasonable. As seen in the results, the approach of Gimenez-Garcia et al. [6] leads to unfair reallocation and the efficiency scores of some branches increase and decrease unreasonably. In the approach of Gimenez-Garcia et al. [6], an efficient branch may become inefficient, resources might be transferred to only some efficient branches even in the same cost conditions, and the output level might remain fixed despite higher consumption of inputs. The drawbacks observed in the approach of Gimenez-Garcia et al. [6] do not appear in performing proposed approach in the case study.

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