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MODELING OF PORTFOLIO OPTIMIZATION UNDER RISK AVERSION BASED ON VALUE-AT-RISK

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Abstract

We discuss modeling of portfolio optimization under risk aversion, wherein the portfolio risk is measured based on value-at-risk (VaR), assuming that the process of portfolio selection is affected by the level of risk aversion investor. Precisely, a review is presented on

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(i) the association of utility function in forming risk aversion and its association with the investment portfolio selection, and (ii) the model formulation of mean-variance Markowitz optimal portfolio and risk aversion. In this paper, we conduct mean-VaR optimal portfolio modeling with risk aversion, and as numeric illustrations, those optimal portfolio models are utilized to analyze some stocks traded capital market in Indonesia. The results of five stocks show that investment portfolio of global optimal mean-VaR obtains the value-at-risk as 0.0245 and the mean as 0.0033, when the risk aversion score is 0.132. This work helps the investors in deciding their investments.

1. Introduction

In portfolio forming, the investors always tend to maximize their return expectation with a particular risk level that they are willing to bear, or seek for a portfolio that offers the lowest risk with a particular return level [20]. Those kinds of portfolio characteristics are called as *efficient portfolio* [1]. In order to be an efficient portfolio, an assumption on how the investors' behavior in making a decision on the investment that they are about to take necessarily holds [4]. One of the essential assumptions is that all investors do not like risk (risk aversion). This kind of investors, once they are faced to two investments which offer the same return with different risks will likely select the investments with lower risk [9].

Meanwhile, optimal portfolio refers to a portfolio chosen by an investor from a series of alternative options which are contained in a set of efficient portfolios [3]. Obviously, the portfolio chosen by the investor is the portfolio suitable with the corresponding investor's preference, either according to the return or the risk he is willing to bear [14]. The nature of portfolio forming is allocating the wealth on various investment alternatives (investment diversification) so that the overall investment risk can be minimized [2]. According to Tandelilin [22], in 1952, Markowitz developed a periodical portfolio selection model to maximize return expectation and minimize its risk. The objective function of Markowitz portfolio model is to maximize mean-variance, since investment portfolio risk is measured by variance

[5, 19, 23]. Recently, measurement model for risk in investment has been widely developed so that the investor can identify and anticipate the risk much earlier. One of the forms of the most utilized risk measurement is value-at-risk [21]. Value-at-risk is a statistical risk measurement tool that measures maximum loss expected from an investment at a particular confidence level and particular time period in a normal market condition [6, 13].

Therefore, it is important to formulate an optimal portfolio model according to the risk measurement of value-at-risk (VaR). Hence, according to the previous description, this paper exhibits a research on portfolio optimization modeling under risk aversion based on value-at-risk. This study aims to produce a value-at-risk based optimal portfolio model which can be utilized to determine weight proportion of funding allocation in some stocks asset in investment portfolio forming. The generated model, moreover, will be used to analyze some stocks asset traded on capital market in Indonesia.

2. Mathematical Models

This section discusses risk aversion model, mean-variance portfolio optimization model and mean-VaR portfolio optimization modeling. This aims to comprehend the association between those models and the portfolio optimization modeling being observed in this paper.

2.1. Model of risk aversion

Referring to Omisore et al. [16] and Tandelilin [22], in 1952, Markowitz popularized a method of efficient portfolio selection. Suppose portfolio p and weight vector $\mathbf{w}$ are given. Then the investor has two objectives, which are [17]:

(i) maximizing expected value μ_p portfolio return, and

(ii) minimize portfolio risk measured using σ_p^2.

Based on individual preference, an investor may put the weight over those two different objectives and maximize:

$$\mu_p - \frac{1}{2}\rho\sigma_p^2, \text{ with } \rho > 0.$$

Parameter ρ is called *risk aversion* [7, 12]. The factor $\frac{1}{2}\rho$ is obtained this way. Suppose an initial capital is given as W_0, under a portfolio of weight vector **w**. By the end of the period, the capital will turn into $W_0(1 + R_p)$ [7], where R_p is a random variable of return portfolio of weight vector **w**. The utility equation of $W_0(1 + R_p)$ is $u\{W_0(1 + R_p)\}$. If it is described using the approximation of second-order Taylor series, then we will obtain a utility equation as:

$$u\{W_0(1 + R_p)\} \approx u(W_0) + W_0 \cdot u'W_0 \cdot R_p + \tfrac{1}{2}W_0^2 \cdot u''(W_0) \cdot R_p^2.$$

Taking expectation, we have

$$\begin{aligned} E[u\{W_0(1 + R_p)\}] &\approx u(W_0) + W_0 \cdot u'W_0 \cdot E[R_p] + \tfrac{1}{2}W_0^2 \cdot u''(W_0) \cdot E[R_p^2] \\ &\approx u(W_0) + W_0 \cdot u'W_0 \cdot \mu_p + \tfrac{1}{2}W_0^2 \cdot u''(W_0) \cdot \sigma_p^2 \\ &\approx u(W_0) + W_0 \cdot u'W_0 \left\{\mu_p + \tfrac{1}{2}\frac{W_0 \cdot u''(W_0)}{u'(W_0)} \cdot \sigma_p^2\right\}. \end{aligned}$$

According to the approximations above, in maximizing using expectation utility approaches, equivalent can be done by maximizing the form of:

$$\left\{\mu_p - \frac{1}{2}\rho\sigma_p^2\right\}.$$

When $\rho = -W_0 \cdot u''(W_0)/u'(W_0)$, the relative measurement is called the *risk aversion* [12].

2.2. Model of mean-variance portfolio optimization

In this subsection, we will discuss mean-variance Markowitz investment portfolio optimization model. Suppose there is risking asset *N* (regular capital or capital index, and those of same kinds) with the return defined as

$r_1, ..., r_N$. It is assumed that the first and the second moments of $r_1, ..., r_N$ exist. Then the expectation value vector of return is given by [17, 18]:

$$\boldsymbol{\mu}^T = (\mu_1, ..., \mu_N), \text{ with } \mu_i = E[r_i], \quad i = 1, ..., N,$$

where $\mathbf{A}^T$ is a transpose matrix of **A**. Then covariant matrix inter-return stock asset is given by

$$\boldsymbol{\Sigma} = (\sigma_{ij})_{i,\, j=1, ..., N}, \text{ with } \sigma_{ij} = Cov(r_i, r_j), \quad i, j = 1, ..., N.$$

As what has been explained previously, portfolio return of which the weight vector is defined as $\mathbf{w}^T = (w_1, ..., w_N)$, where $\mathbf{w}^T\mathbf{e} = \mathbf{e}^T\mathbf{w} = 1$ is required $\mathbf{e}^T = (1, 1, ..., 1)$. The return expectation of portfolio can be defined using vector equation as follows [11, 18, 20]:

$$\mu_p = E[r_p] = \boldsymbol{\mu}^T\mathbf{w},$$

and return variance of the portfolio can be defined using matrix-vector equation as follows [11, 18, 20]:

$$\sigma_p^2 = Var(r_p) = \mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w}.$$

In the optimization of mean-variance, an efficient portfolio is defined as follows:

Definition 2.1. A *portfolio* p^* is called (*mean-variance*) *efficient* if there is no portfolio *p* with $\mu_p \geq \mu_{p^*}$ and $\sigma_p^2 < \sigma_{p^*}^2$ [16].

In order to obtain an efficient portfolio, an objective function is commonly used by maximizing:

$$\left\{\mu_p - \frac{1}{2}\rho\sigma_p^2\right\}, \text{ with } \rho > 0,$$

where parameter ρ is called *risk aversion* from investor [7, 12]. It means, for an investor with risk aversion defined as should complete portfolio problem:

$$\text{Maximum}\left\{\mu_p - \frac{1}{2}\rho\sigma_p^2\right\}; \quad \rho > 0,$$

$$\text{Subject to } \mathbf{w}^T\mathbf{e} = 1$$

or

$$\text{Maximum}\left\{\mathbf{w}^T\boldsymbol{\mu} - \frac{1}{2}\rho(\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w})\right\}; \quad \rho > 0,$$

$$\text{Subject to } \mathbf{w}^T\mathbf{e} = 1.$$

Lagrangian multiplier function is given as [3]:

$$L(\mathbf{w}, \lambda) = \mathbf{w}^T\boldsymbol{\mu} - \frac{1}{2}\rho(\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w}) + \lambda(\mathbf{w}^T\mathbf{e} - 1).$$

According to Kuhn-Tucker method, an equation system is obtained as [3]:

$$\frac{\partial L}{\partial \mathbf{w}} = \boldsymbol{\mu} - \rho\boldsymbol{\Sigma}\mathbf{w} + \lambda\mathbf{e} = 0$$

and

$$\frac{\partial L}{\partial \lambda} = \mathbf{w}^T\mathbf{e} - 1 = 0.$$

Since $\boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma} = \boldsymbol{\Sigma}\boldsymbol{\Sigma}^{-1} = \mathbf{I}$ and $\mathbf{I}\mathbf{w} = \mathbf{w}^T\mathbf{I} = \mathbf{w}$, we obtain $\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} - \rho\mathbf{w} + \lambda\boldsymbol{\Sigma}^{-1}\mathbf{e} = 0$, or:

$$\rho\mathbf{w} = \boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \lambda\boldsymbol{\Sigma}^{-1}\mathbf{e}.$$

Multiplying above by $\mathbf{e}^T$, and using $\mathbf{e}^T\mathbf{w} = \mathbf{w}^T\mathbf{e} = 1$, we obtain $\rho = \mathbf{e}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \lambda(\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\mathbf{e})$, or:

$$\lambda = \frac{\rho - \mathbf{e}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\mathbf{e}}.$$

Thus,

$$\rho\mathbf{w} = \mathbf{\Sigma}^{-1}\boldsymbol{\mu} + \left(\frac{\rho - \mathbf{e}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu}}{\mathbf{e}^T\mathbf{\Sigma}^{-1}\mathbf{e}}\right)\mathbf{\Sigma}^{-1}\mathbf{e} = \frac{\rho\mathbf{\Sigma}^{-1}\mathbf{e}}{\mathbf{e}^T\mathbf{\Sigma}^{-1}\mathbf{e}}$$

$$+\left(\mathbf{\Sigma}^{-1}\boldsymbol{\mu} - \frac{\mathbf{e}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu}}{\mathbf{e}^T\mathbf{\Sigma}^{-1}\mathbf{e}}\mathbf{\Sigma}^{-1}\mathbf{e}\right),$$

so that an optimum weight vector equation is given as

$$\mathbf{w}^* = \frac{\mathbf{\Sigma}^{-1}\mathbf{e}}{\mathbf{e}^T\mathbf{\Sigma}^{-1}\mathbf{e}} + \frac{1}{\rho}\left(\mathbf{\Sigma}^{-1}\boldsymbol{\mu} - \frac{\mathbf{e}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu}}{\mathbf{e}^T\mathbf{\Sigma}^{-1}\mathbf{e}}\mathbf{\Sigma}^{-1}\mathbf{e}\right); \quad \rho > 0.$$

Above equation is utilized in determining investment portfolio weight for every investor who has different risk aversion levels [3].

2.3. Modeling of mean-VaR portfolio optimization

This subsection discusses mean-VaR investment portfolio optimization model with risk aversion. It is assumed that return asset has a particular distribution and portfolio risk is measured using value-at-risk (VaR). According to Dowd [6], risk measurement model of value-at-risk for portfolio p is formulated as $VaR_p = -W_0\{\mu_p + z_\alpha\sigma_p\}$. The value-at-risk for the portfolio is defined as:

$$VaR_p = -W_0\{\mathbf{w}^T\boldsymbol{\mu} + z_\alpha(\mathbf{w}^T\mathbf{\Sigma}\mathbf{w})^{1/2}\},$$

where the sign (–) refers to the loss, W_0 is the initial invested capital, and z_α is a percentile from standard normal distribution when it is given significance level as $(1-\alpha)\%$ [8].

Therefore, for a single initial wealth $W_0 = 1$, an objection function of investment portfolio model is by maximizing $\{\mu_p - \rho VaR_p\}$ or maximizing $\{\mathbf{w}^T\boldsymbol{\mu} + \rho[\mathbf{w}^T\boldsymbol{\mu} + z_\alpha(\mathbf{w}^T\mathbf{\Sigma}\mathbf{w})^{1/2}]\}$. Hence, investment portfolio optimization model that needs to be solved is [10, 15]:

$$\text{Maximum}\{(1+\rho)\mathbf{w}^T\boldsymbol{\mu} + \rho z_\alpha(\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w})^{1/2}\}$$

$$\text{Subject to } \mathbf{w}^T\mathbf{e} = 1.$$

Its Lagrangian multiplier function is given as:

$$L(\mathbf{w}, \lambda) = (1+\rho)\mathbf{w}^T\boldsymbol{\mu} + \rho z_\alpha(\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w})^{1/2} + \lambda(\mathbf{w}^T\mathbf{e} - 1).$$

According to Kuhn-Tucker method, an equation is obtained as follows:

$$\frac{\partial L}{\partial \mathbf{w}} = (1+\rho)\boldsymbol{\mu} + \frac{\rho z_\alpha \boldsymbol{\Sigma}\mathbf{w}}{(\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w})^{1/2}} + \lambda\mathbf{e} = 0,$$

$$\frac{\partial L}{\partial \lambda} = \mathbf{w}^T\mathbf{e} - 1 = 0 \text{ or } \mathbf{w}^T\mathbf{e} = 1.$$

We have

$$\frac{\rho z_\alpha \boldsymbol{\Sigma}\mathbf{w}}{(\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w})^{1/2}} = -\{(1+\rho)\boldsymbol{\mu} + \lambda\mathbf{e}\}.$$

Multiplying above equation by $\boldsymbol{\Sigma}^{-1}/\rho z_\alpha$, and using $\boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma} = \boldsymbol{\Sigma}\boldsymbol{\Sigma}^{-1} = \mathbf{I}$ and $\mathbf{I}\mathbf{w} = \mathbf{w}^T\mathbf{I} = \mathbf{w}$, we have

$$\frac{\mathbf{w}}{(\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w})^{1/2}} = -\frac{(1+\rho)\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \lambda\boldsymbol{\Sigma}^{-1}\mathbf{e}}{\rho z_\alpha}.$$

Multiplying above equation by $\mathbf{e}^T$, and using $\mathbf{e}^T\mathbf{w} = \mathbf{w}^T\mathbf{e} = 1$, we have

$$\frac{1}{(\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w})^{1/2}} = -\frac{\{(1+\rho)\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \lambda\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\mathbf{e}\}}{\rho z_\alpha}$$

or

$$(\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w})^{1/2} = -\frac{\rho z_\alpha}{\{(1+\rho)\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \lambda\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\mathbf{e}\}}.$$

Thus, the optimal portfolio weight vector equation is as follows:

$$\mathbf{w}^* = \frac{(1+\rho)\mathbf{\Sigma}^{-1}\boldsymbol{\mu} + \lambda\mathbf{\Sigma}^{-1}\mathbf{e}}{\{(1+\rho)\mathbf{e}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu} + \lambda\mathbf{e}^T\mathbf{\Sigma}^{-1}\mathbf{e}\}}.$$

Furthermore, we have

$$\rho z_\alpha(\mathbf{w}^T\mathbf{\Sigma}\mathbf{w})^{1/2} = -\{(1+\rho)\boldsymbol{\mu}^T\mathbf{w} + \lambda\}.$$

Therefore,

$$(\mathbf{e}^T\mathbf{\Sigma}^{-1}\mathbf{e})\lambda^2 + \{(1+\rho)\mathbf{e}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu} + \boldsymbol{\mu}^T\mathbf{\Sigma}^{-1}\mathbf{e}\}\lambda + \{(1+\rho)^2\boldsymbol{\mu}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu} - (\rho z_\alpha)^2\} = 0.$$

Above equation is a quadratic in λ, so that the constant λ can be computed as

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad \lambda > 0,$$

where $a = \mathbf{e}^T\mathbf{\Sigma}^{-1}\mathbf{e}$; $b = (1+\rho)\{\mathbf{e}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu} + \boldsymbol{\mu}^T\mathbf{\Sigma}^{-1}\mathbf{e}\}$; and $c = (1+\rho)^2\boldsymbol{\mu}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu} - (\rho \cdot z_\alpha)^2$, with $\mathbf{\Sigma}^{-1}$ is the inverse of a matrix $\mathbf{\Sigma}$.

The optimal portfolio weight vector equation and values of λ are used together in determining weight proportion of funding allocation on some stocks asset in forming an optimal investment portfolio forming.

3. Numerical Illustration

The analyzed capital data is accessed through website http://www.finance.go.id//. The data consists of 5 (five) selected stocks, during the period from January 2, 2013 up to June 4, 2016. The data includes stocks of INDF, DEWA, AALI, LSIP and ASII. The mean return of those five stocks is given in the vectors $\boldsymbol{\mu}^T =$ (0.0154, 0.0390, 0.0033, 0.0088, 0.0003), respectively. Meanwhile, the value of variance return altogether with covariance value inter-return of the five stocks is given in the form of the covariance matrix $\mathbf{\Sigma}$ and its inverse $\mathbf{\Sigma}^{-1}$, as follows:

$$\mathbf{\Sigma} = \begin{bmatrix} 0.0026 & 0.0001 & -0.0002 & 0.0002 & 0.0001 \\ 0.0001 & 0.0028 & 0.0003 & 0.0000 & 0.0001 \\ -0.0002 & 0.0003 & 0.0013 & 0.0006 & 0.0004 \\ 0.0002 & 0.0000 & 0.0006 & 0.0019 & 0.0003 \\ 0.0001 & 0.0001 & 0.0004 & 0.0003 & 0.0002 \end{bmatrix},$$

$$\mathbf{\Sigma}^{-1} = 10^4 \begin{bmatrix} 0.0450 & -0.0022 & 0.0365 & -0.0018 & -0.0916 \\ -0.0022 & 0.0370 & -0.0092 & 0.0039 & -0.0048 \\ 0.0365 & -0.0092 & 0.2310 & -0.0022 & -0.4723 \\ -0.0018 & 0.0039 & -0.0022 & 0.0694 & -0.1008 \\ -0.0916 & -0.0048 & -0.4723 & -0.1008 & 1.6441 \end{bmatrix}.$$

Since there are many analyzed stocks which are consisted of five, the unity vector is determined as $\mathbf{e}^T = (1, 1, 1, 1, 1)$.

Moreover, vector $\boldsymbol{\mu}^T$, vector $\mathbf{e}^T$ and covariance inverse matrix $\mathbf{\Sigma}^{-1}$ are used together for the investment portfolio optimization calculation process. It is assumed that within short sales stocks trading transaction is not allowed. In this study, the optimization is conducted using two approaches. They are mean-variance model and mean-VaR model. In this optimization process, the values of risk aversion $\rho > 0$ are determined, respectively, simulated from the smallest to biggest values. The investment portfolio optimization process is done using the help from Matlab 7.0 software.

The mean-variance investment portfolio optimization process is conducted by optimum weight vector equation. The value of risk aversion which compiles the assumption that short sales are not allowed is $6.2 \le \rho \le 13.8$. The change of value from 6.2 up to 13.8 over here is conducted using $\Delta\rho$ as many as 0.2. From the process of mean-variance investment portfolio optimization, we obtain the efficient portfolio graphic as presented in Figure 1.

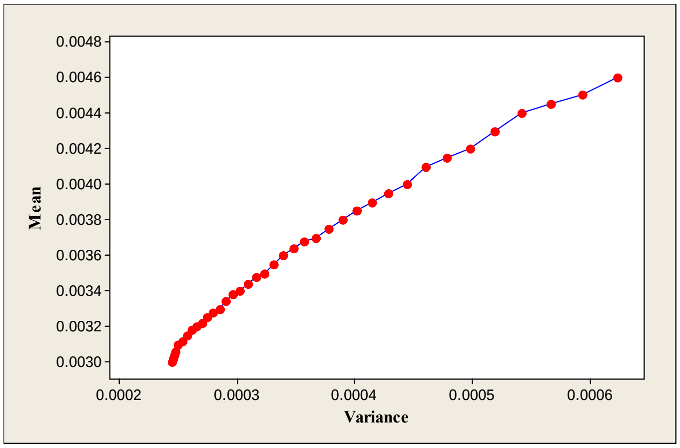

Figure 1. Mean-variance efficient portfolio graphic.

In Figure 1, the minimum portfolio is located on risk value $\hat{\sigma}_p^2 = 0.00023712$ and mean value $\hat{\mu}_p = 0.003$, which happens when the value of risk aversion is $\rho = 13.6$. Minimum portfolio is generated for combination of portfolio weights as $\mathbf{w}^{Min} = (0.1238, 0.078, 0.0033, 0.0661, 0.7288)$. Meanwhile, maximum portfolio is located on risk value $\hat{\sigma}_p^2 = 0.000623$ and mean value $\hat{\mu}_p = 0.0046$, which happens when the value of risk aversion is $\rho = 6.2$. Maximum portfolio is generated for combination of portfolio weights as $\mathbf{w}^{Max} = (0.2945, 0.1314, 0.3575, 0.1958, 0.0209)$.

Mean-VaR investment portfolio optimization process is performed via optimal portfolio weight vector equation and values of λ, and in this process, the score $\alpha = 5\%$ is determined so that we get a score $z_{5\%} = -1.645$. Risk aversion value which is compiled to the assumption stating that short sales are not allowed, is $0.125 \leq \rho \leq 0.140$. The change of value ρ from 0.125 to 0.140 is processed using $\Delta\rho$ as many as 0.001. From the process of mean-VaR investment portfolio optimization, we obtain an efficient portfolio gra phic as presented in Figure 2.

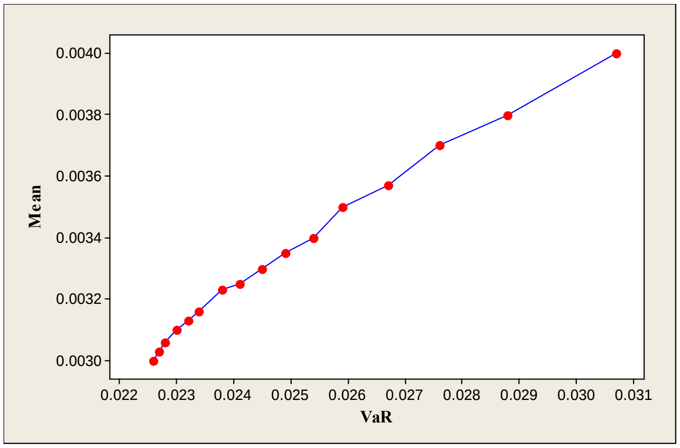

Figure 2. Mean-VaR efficient portfolio graphic.

According to Figure 2, the minimum portfolio is located on risk value of $VaR_p = 0.0224$ and mean value of $\hat{\mu}_p = 0.003,$ which happens when the value of risk aversion is $\rho = 0.140.$ Minimum portfolio is generated for combination of portfolio weights as

$$\mathbf{w}^{Min} = (0.1246,\ 0.0783,\ 0.0051,\ 0.0667,\ 0.7254).$$

Meanwhile, the maximum portfolio is located on risk value of $VaR_p =$ 0.0307 and mean value of $\hat{\mu}_p = 0.004,$ which happens when the value of risk aversion is $\rho = 0.125.$ Maximum portfolio is generated for combination of portfolio weights as

$$\mathbf{w}^{Max} = (0.2308,\ 0.1115,\ 0.2254,\ 0.1474,\ 0.2848).$$

In the process of mean-VaR investment portfolio optimization, we obtain global optimal portfolio at the value of $VaR_p = 0.0245$ and mean value of $\hat{\mu}_p = 0.0033,$ which happens when the value of risk aversion is $\rho = 0.132.$ In this global optimal portfolio, the biggest ratio that it has is $\hat{\mu}_p / Var_p =$ $0.1346939.$ Global optimal portfolio is obtained for combination of portfolio weights as $\mathbf{w}^{Glob} = (0.1538,\ 0.0874,\ 0.0657,\ 0.0889,\ 0.6041).$

According to the analysis, whether at the investment portfolio optimization using mean-variance or mean-VaR, it is clearly shown that the bigger risk aversion will make the investors investing in the portfolio with the smaller risk size.

4. Conclusion

This paper exhibits a research conducted on the modeling of portfolio optimization under risk aversion based on value-at-risk. According to the conducted research, some conclusions are drawn as follows. First, the equation for risk aversion may be derived from the equation of utility, and by using an approximation of second-order Taylor series, the equation of utility equivalent to the objective function of Markowitz model optimization portfolio may be obtained. Second, the optimal solution of mean-variance investment portfolio model from Markowitz using risk aversion is determined in the form of weight vector equation of $\mathbf{w}^*$. Third, the optimal solution of mean-VaR investment portfolio model using risk aversion is stated in the form of weight vector equation of $\mathbf{w}^*$ via optimal portfolio weight vector equation and values of λ. Fourth, according to the analysis of five stocks asset for mean-VaR investment portfolio, we obtained the weight combination of global optimal portfolios of

$$\mathbf{w}^{Glob} = (0.1538,\ 0.0874,\ 0.0657,\ 0.0889,\ 0.6041),$$

of which the mean of portfolio return expectation has the value of 0.0033 and value-at-risk at the score of 0.0245 is achieved when the risk aversion value reaches 0.132. This global optimal portfolio has the biggest ratio between its means toward the risk.

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