Far East Journal of Mathematical Sciences (FJMS)

http://www.pphmj.com
http://dx.doi.org/10.17654/MS103020503
Volume 103, Number 2, 2018, Pages 503-514 **ISSN: 0972-0871**

ALTERNATIVE APPROACH TO OPTIMIZE A MULTIPRODUCT INTRA-SUPPLY CHAIN SYSTEM WITH FAILURE IN REWORK

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Abstract

Optimization of a multiproduct intra-supply chain system with failure in rework was studied [1] using the mathematical modeling and Hessian matrix differential equations. The present paper is intended to reexamine the problem with an alternative algebraic method and show that optimization can also be achieved without the need of applying

Received: September 23, 2017; Accepted: November 5, 2017

2010 Mathematics Subject Classification: 93E20, 90B05, 90B30, 90B25.

Keywords and phrases: operations research, multiproduct system, failure in rework, multi-delivery, algebraic approach, common cycle length.

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differential calculus. The proposed simplified solution procedure enables production managers to have a handle on optimization of a multiproduct intra-supply chain system with ease.

1. Introduction

Optimization of a single product fabrication system featuring perfect production condition and continuous inventory issuing policy was first studied by Taft [2], with the objective of minimizing total system costs consisting of the setup, variable fabrication, and holding costs.

With the intention of increase utilization, production managers would often consider to fabricate multiple products on a single machine. During past decades, researchers have also paid extensively attentions in diverse aspects of multiproduct manufacturing systems [3-17]. In addition, in real-life fabrication processes, owing to diverse uncontrollable/unexpected factors, equipment failures and/or nonconforming items being produced are inevitable. Therefore, different features of unreliable production systems with subsequent corrected actions have been broadly investigated [18-29]. Also, since the intra-supply chain system exists in today's transnational firms, so optimization of the so-called vendor-buyer integrated inventory system becomes one of important operating goals to managers. Hence, studies on various aspects of supply chain systems have been extensively carried out in past years [30-38].

In a recent study, Chiu et al. [1] optimized a multiproduct intra-supply chain system with failure in rework. They employed mathematical modeling and Hessian matrix equations [39] to simultaneously derive the optimal decision on fabrication cycle length and number of deliveries per cycle. This paper uses an alternative algebraic method (as proposed by Grubbstrom and Erdem [40] and others [41-43]) to reexamine their problem, and demonstrate that optimization can also be accomplished without the need of applying differential calculus. Such a simplified solution process enables production managers, who may only have basic knowledge of algebra, to understand and have a good handle of optimizing a real multiproduct intra-supply chain system.

2. The Problem and the Alternative Approach

Optimization of a multiproduct intra-supply chain system with failure in rework was studied [1] using the mathematical modeling and Hessian matrix differential equations. The present paper is intended to reexamine the problem with an alternative algebraic method and show that optimization can also be done without using the differential calculus. The multiproduct production-inventory problem [1] is described as follows. Annual fabrication rate of L products are P_{1i} (where $i = 1, 2, ..., L$) in order to meet the demand rate λ_i per year, in a multiproduct intra-supply chain system with multi-delivery policy (see Figure 1).

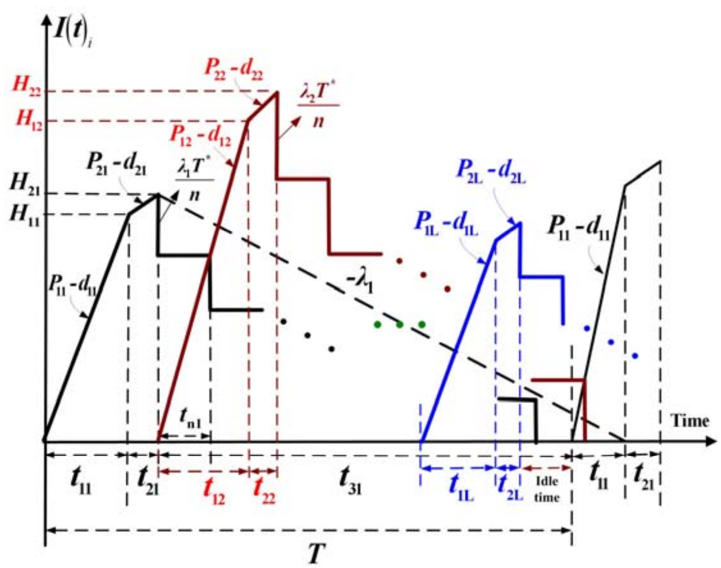

Figure 1. On-hand inventory level of perfect quality products in the proposed multiproduct intra-supply chain system with failure in rework [1].

All items produced are screened and random nonconforming portion x_i exists in the fabrication processes, with a production rate of d_{1i}, thus, $d_{1i} = x_i P_{1i}$. In the end of fabrication, all nonconforming products entered rework process with rework rate of P_{2i}. In the rework process, a portion φ_i of the reworked items fails and becomes scrap. So, production rate of scraps is d_{2i} $(\text{i.e., } \varphi_i P_{2i})$. Product screening cost is included in unit fabrication cost C_i, and unit rework and disposal costs are C_{Ri} and C_{Si}, respectively. No shortages are permitted in the proposed system, hence, $(P_{1i} - d_{1i} - \lambda_i)$ must greater than zero. Upon completion of rework process, *n* installments of end products are delivered to sales location using a multi-shipment policy, at fixed time interval t_{ni} in distribution time t_{3i} (see Figure 1). Additional notation utilized in the proposed study also includes the following:

T = common fabrication cycle length, a decision variable,

t_{1i} = uptime for product *i* in the proposed system,

t_{2i} = rework time for product *i* in the proposed system,

t_{3i} = product distribution time in the proposed system,

t_{ni} = fixed interval of time between two consecutive deliveries of *i*,

H_{1i} = on-hand inventory level of product *i* in the end of uptime,

H_{2i} = on-hand inventory level of product *i* in the end of rework time,

Q_i = lot size of product *i* in the proposed multiproduct intra-supply chain system,

n = number of shipments of end items to be distributed to sales location, the other decision variable,

h_i = unit holding cost for product *i*,

h_{1i} = unit holding cost for reworked items of product *i*,

K_i = setup cost of product *i*,

$I(t)_i$ = on-hand inventory level of perfect quality product *i* at time *t*,

K_{1i} = fixed distribution cost per shipment for product *i*,

C_{Ti} = unit distribution cost for product i,

D_i = fixed quantity of end product i distributed per shipment,

I_i = left over quantity of product i per shipment after depletion during t_{ni},

h_{2i} = unit holding cost for product i at the sales location,

$TC(Q_i, n)$ = total fabrication-inventory-distribution cost per cycle for product i,

$E[TCU(T, n)]$ = total expected fabrication-inventory-distribution cost per unit time for producing L products in the proposed system.

From analytical result of mathematical modeling and system cost analysis [1], total fabrication-inventory-distribution costs per cycle $TC(Q_i, n)$ (for $i = 1, 2, ..., L$) for L products are obtained as shown in equation (1). Each $TC(Q_i, n)$ consists of the fabrication setup and variable costs, rework and disposal costs, fixed and variable distribution costs, holding costs in production units' uptime, rework time, and distribution time, and stock holding costs in sales location:

$$\sum_{i=1}^{L} TC(Q_i, n)$$

$$= \sum_{i=1}^{L} \left\{ \begin{array}{l} K_i + C_i Q_i + C_{Ri}(x_i Q_i) + C_{Si}\varphi_i(x_i Q_i) + nK_{1i} + C_{Ti}[Q_i(1-\varphi_i x_i)] \\ + h_i \left[\dfrac{H_{1i} + d_{1i}t_{1i}}{2}(t_{1i}) + \dfrac{H_{1i} + H_{2i}}{2}(t_{2i}) + \dfrac{n-1}{2n}(H_{2i}t_{3i}) \right] \\ + h_{1i}\dfrac{d_{1i}t_{1i}}{2}(t_{2i}) \\ + h_{2i}\left[n\dfrac{(D_i - I_i)}{2}t_{ni} + \dfrac{nI_i}{2}(t_{1i} + t_{2i}) + \dfrac{n(n+1)}{2}I_i t_{ni} \right] \end{array} \right\}. \quad (1)$$

Further derivation by incorporating the expected values of x to cope with its randomness and using common cycle length to replace lot sizes, total expected system costs $E[TCU(T, n)]$ can be derived [1] as shown in equation (2):

$$E[TCU(T,\ n)]$$

$$= \sum_{i=1}^{L} \left\{ \begin{array}{l} \left[C_i \lambda_i E_{0i} + \dfrac{K_i}{T} + C_{Ri} \lambda_i E_{1i} + C_{Si} \varphi_i \lambda_i E_{1i} + C_{Ti} \lambda_i + \dfrac{nK_{1i}}{T} \right] \\ + \dfrac{h_i \lambda_i^2}{2} T \left[\dfrac{1}{\lambda_i} + \dfrac{\varphi_i E_{0i} E_{1i}}{P_{1i}} + \dfrac{E_{0i} E_{1i} - E_{1i}^2}{P_{2i}} \right] + \dfrac{h_{1i} \lambda_i^2 E_{1i}^2}{2P_{2i}} T \\ + \dfrac{h_{2i} \lambda_i^2}{2} T \left(\dfrac{E_{0i}}{P_{1i}} + \dfrac{E_{1i}}{P_{2i}} \right) + \dfrac{\lambda_i^2}{2n} T \left[\dfrac{1}{\lambda_i} - \dfrac{E_{0i}}{P_{1i}} - \dfrac{E_{1i}}{P_{2i}} \right] (h_{2i} - h_i) \end{array} \right\}, \quad (2)$$

where $E_{0i} = \dfrac{1}{1 - \varphi_i E[x_i]}$ and $E_{1i} = \dfrac{E[x_i]}{1 - \varphi_i E[x_i]}$.

The proposed alternative optimization method

It can be seen the resulting $E[TCU(T, n)]$ consists of terms of constant, T^{-1}, nT^{-1}, T, and Tn^{-1}. Suppose that we assume the following notations:

$$\pi_1 = \sum_{i=1}^{L} [C_i \lambda_i E_{0i} + C_{Ri} \lambda_i E_{1i} + C_{Si} \varphi_i \lambda_i E_{1i} + C_{Ti} \lambda_i], \quad (3)$$

$$\pi_2 = \sum_{i=1}^{L} (K_i), \quad (4)$$

$$\pi_3 = \sum_{i=1}^{L} (K_{1i}), \quad (5)$$

$$\pi_4 = \sum_{i=1}^{L} \left\{ \frac{h_i \lambda_i^2}{2} \left[\frac{1}{\lambda_i} + \frac{\varphi_i E_{0i} E_{1i}}{P_{1i}} + \frac{E_{0i} E_{1i} - E_{1i}^2}{P_{2i}} \right] + \frac{h_{1i} \lambda_i^2 E_{1i}^2}{2P_{2i}} + \frac{h_{2i} \lambda_i^2}{2} \left(\frac{E_{0i}}{P_{1i}} + \frac{E_{1i}}{P_{2i}} \right) \right\}, \quad (6)$$

$$\pi_5 = \sum_{i=1}^{L}\left\{\frac{\lambda_i^2}{2}\left[\frac{1}{\lambda_i} - \frac{E_{0i}}{P_{1i}} - \frac{E_{1i}}{P_{2i}}\right](h_{2i} - h_i)\right\}. \tag{7}$$

Substituting equations (3) to (7) in equation (2), one can obtain $E[TCU(T, n)]$ as follows:

$$E[TCU(T, n)] = \pi_1 + \pi_2 T^{-1} + \pi_3(nT^{-1}) + \pi_4 T + \pi_5(Tn^{-1}). \tag{8}$$

One can further rearrange equation (8) as follows:

$$E[TCU(T, n)] = \pi_1 + (\sqrt{\pi_2} - \sqrt{\pi_4}T)^2 T^{-1} + (\sqrt{\pi_3} - \sqrt{\pi_5}Tn^{-1})^2(nT^{-1}) + 2\sqrt{\pi_2\pi_4} + 2\sqrt{\pi_3\pi_5}. \tag{9}$$

If both the second and third terms of $E[TCU(T, n)]$ (i.e., equation (9)) equal to zeros, then equation (9) can be minimized. That is

$$T = \sqrt{\frac{\pi_2}{\pi_4}} \quad \text{and} \quad n = \frac{\sqrt{\pi_5}T}{\sqrt{\pi_3}} = \sqrt{\frac{\pi_5}{\pi_3}}\sqrt{\frac{\pi_2}{\pi_4}}. \tag{10}$$

Substituting equations (3) to (7) in equation (10), one can obtain n^* as shown in equation (11)

$$n^* = \sqrt{\frac{\sum_{i=1}^{L}(K_i)\cdot\sum_{i=1}^{L}\left[\lambda_i^2(h_{2i} - h_i)\left(\frac{1}{\lambda_i} - \frac{E_{0i}}{P_{1i}} - \frac{E_{1i}}{P_{2i}}\right)\right]}{\left(\sum_{i=1}^{L}K_{1i}\right)\sum_{i=1}^{L}\left\{+\frac{h_i\lambda_i^2}{2}\left[\frac{1}{\lambda_i} + \frac{\varphi_i E_{0i}E_{1i}}{P_{1i}} + \frac{E_{0i}E_{1i} - E_{1i}^2}{P_{2i}}\right] + \frac{h_{1i}\lambda_i^2 E_{1i}^2}{2P_{2i}} + \frac{h_{2i}\lambda_i^2}{2}\left(\frac{E_{0i}}{P_{1i}} + \frac{E_{1i}}{P_{2i}}\right)\right\}}}. \tag{11}$$

Once n^* has been obtained, $E[TCU(T, n)]$ can now be reconsidered as a function with single decision variable as shown in equation (12):

$$E[TCU(T, n)] = \pi_1 + \pi_6 T^{-1} + \pi_7(T), \tag{12}$$

where $\pi_6 = (\pi_2 + \pi_3 n)$ and $\pi_7 = (\pi_4 + \pi_5 n^{-1})$.

Equation (12) can be rearranged as follows:

$$E[TCU(T,\ n)] = \pi_1 + (\sqrt{\pi_6} - \sqrt{\pi_7}T)^2 T^{-1} + 2\sqrt{\pi_6}\sqrt{\pi_7}. \qquad (13)$$

Similarly, if the second of $E[TCU(T,\ n)]$ (i.e., equation (13)) equals to zero, then equation (13) can be minimized. That is

$$T^* = \frac{\sqrt{\pi_6}}{\sqrt{\pi_7}}. \qquad (14)$$

Substituting equations (3) to (7) and (12) in equation (14), one can obtain T^* as shown in equation (15):

$$T^* = \sqrt{\frac{2\sum_{i=1}^{L}(K_i + nK_{1i})}{\sum_{i=1}^{L}\left\{\begin{array}{l} h_i\lambda_i^2\left[\dfrac{1}{\lambda_i} + \dfrac{\varphi_i E_{0i}E_{1i}}{P_{1i}} + \dfrac{E_{0i}E_{1i} - E_{1i}^2}{P_{2i}}\right] + \dfrac{h_{1i}\lambda_i^2 E_{1i}^2}{P_{2i}} \\ + h_{2i}\lambda_i^2\left(\dfrac{E_{0i}}{P_{1i}} + \dfrac{E_{1i}}{P_{2i}}\right) + \dfrac{\lambda_i^2}{n}\left[\dfrac{1}{\lambda_i} - \dfrac{E_{0i}}{P_{1i}} - \dfrac{E_{1i}}{P_{2i}}\right](h_{2i} - h_i) \end{array}\right\}}}. \qquad (15)$$

The aforementioned equations (11) and (15) are same as what were obtained by the use of Hessian matrix equations [1]. Similarly, one can apply the same procedure [1] to find the integer value of n^*, its corresponding T^*, and $E[TCU(T^*,\ n^*)]$.

3. Concluding Remarks

The present paper is intended to resolve the problem in [1] using an alternative algebraic method and demonstrate that the optimization of a specific multiproduct intra-supply chain system can be accomplished without the need of applying differential calculus. Such a simplified solution process enables production managers, who may only have basic knowledge of algebra, to understand and have a good handle of optimizing a real multiproduct intra-supply chain system.

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