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A MODIFIED FUZZY TOPSIS METHOD USING COSINE SIMILARITIES AND OCHIAI COEFFICIENTS

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Abstract

A modified technique for order preference by similarity to ideal solution (TOPSIS) approach has been proposed in this paper using cosine similarities and Ochiai coefficients to deal with different criteria which are of fuzzy in nature. The distances between the ideal solution and the positive/negative ideal solutions are obtained using the proposed cosine similarities and Ochiai coefficient techniques. The closeness coefficients and TOPSIS grades are compared and the

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study validates that Ochiai measure approach is better than the cosine similarity measure. A numerical example is illustrated to prove the effectiveness of the proposed method in determining the best alternative.

1. Introduction

A multi-criteria decision making (MCDM) problem shall have *m* alternatives and *n* criteria. Each criterion may consider different ratings and weights. TOPSIS is one of the classical methods of MCDM, introduced by Hwang and Yoon [1] in 1981 by defining the positive and negative ideal solutions. The ideal solution is the solution set that consists of all the possible best values of the criteria which maximizes the benefit criteria and minimizes the cost criteria, similarly the negative ideal solution is the solution set that consists of all the possible worst values of the criteria which maximizes the cost criteria and minimizes the benefit criteria [2-5]. The alternative which has the shortest distance from the ideal solution and the farthest distance from the negative ideal solution is said to be *optimal*.

Chen [6] extended TOPSIS in group decision making under fuzzy environment by introducing vertex method to find the distance between two trapezoidal fuzzy numbers. Chu and Lin [7] proposed an improved fuzzy TOPSIS model using mean of relative areas instead of vertex method, for the reason that the weighted normalized fuzzy ratings [8, 9] are not exactly trapezoidal fuzzy numbers. Hwang and Yoon [1] used Euclidean distance measure to obtain the distance between the positive and negative ideal solutions. Ye [10] ranked the alternatives using expected weights and weighted cosine similarity measures for the trapezoidal fuzzy numbers. In this paper, the traditional TOPSIS method has been modified by using the cosine similarity and Ochiai coefficients [11] to calculate the distance between the positive and negative ideal solutions for trapezoidal fuzzy numbers.

2. Definitions and Preliminaries

2.1. Trapezoidal fuzzy number

A trapezoidal fuzzy number can be represented as $\tilde{a} = (a_1, a_2, a_3, a_4)$ whose membership function is given by

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, & x \leq a_1, \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2, \\ 1, & a_2 \leq x \leq a_3, \\ \dfrac{a_4 - x}{a_4 - a_{.3}}, & a_3 \leq x \leq a_4, \\ 0, & a_4 \leq x. \end{cases}$$

2.2. Euclidean distance

Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be any two triangular fuzzy numbers. Then the distance between them by vertex method is shown in Figure 1 and given by

$$d(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{6}((a_1 - b_1)^2 + 2(a_2 - b_2)^2 + 2(a_3 - b_3)^2 + (a_4 - b_4)^2)}.$$

2.3. Cosine similarity measure

Let $\tilde{a} = (a_1, a_2, a_3, a_4)$ and $\tilde{b} = (b_1, b_2, b_3, b_4)$ be any two trapezoidal fuzzy numbers. Then the cosine similarity measure between $\tilde{a}$ and $\tilde{b}$ is shown in Figure 2 and given by

$$d(\tilde{a}, \tilde{b}) = \frac{\sum_{p=1}^{4} a_p b_p}{\sqrt{\sum_{p=1}^{4} (a_p)^2} \sqrt{\sum_{p=1}^{4} (b_p)^2}}.$$

It can be easily verified that the cosine similarity measures any two trapezoidal fuzzy numbers satisfying the following properties:

(i) $d(\tilde{a}, \tilde{b}) \geq 0$; (ii) $d(\tilde{a}, \tilde{b}) = d(\tilde{b}, \tilde{a})$; (iii) $d(\tilde{a}, \tilde{b}) = 1$ iff $\tilde{a} = \tilde{b}$, i.e., $\tilde{a} = \tilde{b}$ for $p = 1, 2, 3, 4$.

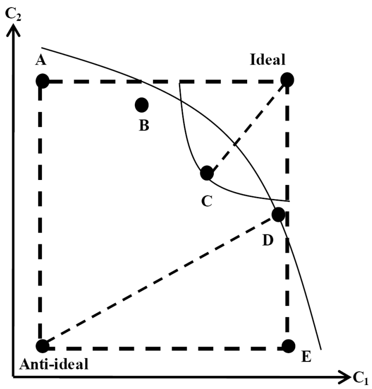

Figure 1. Euclidean distance measure.

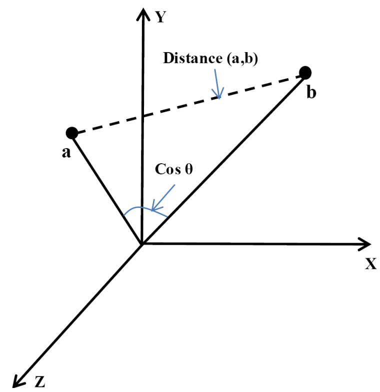

Figure 2. Cosine similarity measure.

2.4. Ochiai coefficient

Let $\tilde{a} = (a_1, a_2, a_3, a_4)$ and $\tilde{b} = (b_1, b_2, b_3, b_4)$ be any two trapezoidal fuzzy numbers. Then the Ochiai coefficient between $\tilde{a}$ and $\tilde{b}$ is given by

$$Ochiai(\tilde{a}, \tilde{b}) = \frac{\sum_{p=1}^{4} \min(a_p b_p)}{\sqrt{\sum_{p=1}^{4} a_p}\sqrt{\sum_{p=1}^{4} b_p}}.$$

3. Optimization Steps using Modified Fuzzy TOPSIS Method

(a) Construct fuzzy decision matrix such that each x_{ij} is trapezoidal.

(b) Normalize the decision matrix using

$$n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, \quad i = 1, \ldots, m; \quad j = 1, \ldots, n. \tag{1}$$

(c) Obtain weighted normalized decision matrix by

$$v_{ij} = w_j n_{ij}, \tag{2}$$

$i = 1, \ldots, m;$ $j = 1, \ldots, n,$ where w_j is the weight of the jth criterion and $\sum_{j=1}^{n} w_j = 1.$

(d) Determine the positive and negative ideal solutions using

$$A^{+} = \{v_1^{+}, \ldots, v_n^{+}\} = \{\max_i v_{ij},\ j = 1, \ldots, n\};$$

$$A^{-} = \{v_1^{-}, \ldots, v_n^{-}\} = \{\min_i v_{ij},\ j = 1, \ldots, n\}.$$

(e) The separation measures are given by

Cosine similarity	**Ochiai coefficient**
$d_i^{+} = \dfrac{\sum_{j=1}^{n} a_{ij} b_{ij}^{+}}{\sqrt{\sum_{j=1}^{n} (a_{ij})^2} \sqrt{\sum_{j=1}^{n} (b_{ij}^{+})^2}},$ $i = 1, \ldots, m,$	$d_i^{+} = \dfrac{\sum_{j=1}^{n} \min(a_{ij}, b_{ij}^{+})}{\sqrt{\sum_{j=1}^{n} a_{ij}} \sqrt{\sum_{j=1}^{n} b_{ij}^{+}}},$ $i = 1, \ldots, m$
$d_i^{-} = \dfrac{\sum_{j=1}^{n} a_{ij} b_{ij}^{-}}{\sqrt{\sum_{j=1}^{n} (a_{ij})^2} \sqrt{\sum_{j=1}^{n} (b_{ij}^{-})^2}},$ $i = 1, \ldots, m.$ (3)	$d_i^{-} = \dfrac{\sum_{j=1}^{n} \min(a_{ij}, b_{ij}^{-})}{\sqrt{\sum_{j=1}^{n} a_{ij}} \sqrt{\sum_{j=1}^{n} b_{ij}^{-}}},$ $i = 1, \ldots, m.$ (4)

(f) Relative closeness to ideal solution is calculated by

$$C_i = \frac{d_i^{-}}{d_i^{+} + d_i^{-}}, \quad i = 1, \ldots, m. \tag{5}$$

(g) Rank preference is given according to ascending order of the TOPSIS grade C_i.

4. Numerical Example

A manufacturing company desires to select a suitable material supplier to purchase the key components of new products. After preliminary screening, five candidates (A1, A2, A3, A4 and A5) remain for further evaluation. A committee of three decision makers, D1, D2 and D3 has been formed to select the most suitable supplier. Five benefit criteria are considered: (1) profitability of supplier (C1), (2) relationship closeness (C2), (3) technological capability (C3), (4) conformance quality (C4) and (5) conflict resolution (C5). The linguistic rating variables to the suppliers of 3 decision makers with respect to each criterion are evaluated and shown as fuzzy decision matrix in Table 1.

Table 1. Fuzzy decision matrix

	C1	C2	C3	C4	C5
A1	(5, 6, 7, 8)	(5, 7, 8, 10)	(7, 8, 8, 9)	(7, 8, 8, 9)	(7, 8, 8, 9)
A2	(7, 8, 8, 9)	(8, 9, 10, 10)	(8, 9, 10, 10	(7, 8.7, 9.3, 10)	(8, 9, 10, 10)
A3	(7, 8.7, 9.3, 10)	(7, 8.3, 8.7, 10)	(7, 8.7, 9.3, 10)	(8, 9, 10, 10)	(7, 8.3, 8.7, 10)
A4	(7, 8, 8, 9)	(5, 7.3, 7.7, 9)	(5, 6.7, 7.3, 9)	(7, 8, 8, 9)	(7, 8.3, 8.7, 10)
A5	(5, 6, 7, 8)	(5, 7.3, 7.7, 9)	(5, 6, 7, 8)	(5, 6.7, 7.3, 9)	(5, 6, 7, 8)
Weight	(0.7, 0.8, 0.8, 0.9)	(0.8, 0.9, 1.0, 1.0)	(0.7, 0.87, 0.93, 1.0)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)

Using equations (1) and (2), the weighted normalized matrix is obtained and shown in Table 2.

Table 2. Weighted normalized matrix

	C1	C2	C3	C4	C5
A1	(0.35, 0.48, 0.56, 0.72)	(0.40, 0.63, 0.80, 1)	(0.49, 0.70, 0.74, 0.90)	(0.49, 0.64, 0.64, 0.81)	(0.49, 0.64, 0.64, 0.81)
A2	(0.49, 0.64, 0.64, 0.81)	(0.64, 0.81, 1, 1)	(0.56, 0.78, 0.93, 1)	(0.49, 0.70, 0.74, 0.90)	(0.56, 0.72, 0.80, 0.90)
A3	(0.49, 0.70, 0.74, 0.90)	(0.56, 0.75, 0.87, 1)	(0.49, 0.76, 0.86, 1)	(0.56, 0.72, 0.80, 0.90)	(0.49, 0.66, 0.70, 0.90)
A4	(0.49, 0.64, 0.64, 0.81)	(0.40, 0.66, 0.77, 0.90)	(0.35, 0.58, 0.68, 0.90)	(0.49, 0.64, 0.64, 0.81)	(0.49, 0.66, 0.70, 0.90)
A5	(0.35, 0.48, 0.56, 0.72)	(0.40, 0.66, 0.77, 0.90)	(0.35, 0.52, 0.65, 0.80)	(0.35, 0.54, 0.58, 0.81)	(0.35, 0.48, 0.56, 0.72)

The positive ideal solution (A^*) and negative ideal solution (A^-) are the best and worst values of each criterion, as shown in Table 3.

Table 3. Positive and negative ideal solutions

	C1	C2	C3	C4	C5
A^*	(0.90, 0.90, 0.90, 0.90)	(1, 1, 1, 1)	(1, 1, 1, 1)	(0.90, 0.90, 0.90, 0.90)	(0.90, 0.90, 0.90, 0.90)
A^-	(0.35, 0.35, 0.35, 0.35)	(0.40, 0.40, 0.40, 0.40)	(0.35, 0.35, 0.35, 0.35)	(0.35, 0.35, 0.35, 0.35)	(0.35, 0.35, 0.35, 0.35)

Table 4. Cosine similarity grade and rank

	d_i^+	d_i^-	Grade	Rank
A1	0.9717	0.9702	0.4996	1
A2	0.9812	0.9802	0.4997	2
A3	0.9787	0.9780	0.4998	3
A4	0.9714	0.9723	0.5003	5
A5	0.9630	0.9638	0.5002	4

Table 5. Ochiai grade and rank

d_i^+	d_i^-	Grade	Rank
0.8293	0.7462	0.4736	4
0.8966	0.6902	0.4350	1
0.8887	0.6963	0.4393	2
0.8513	0.7400	0.4650	3
0.8123	0.7895	0.4929	5

Table 6. Traditional TOPSIS

Grade	**Rank**
0.50	4
0.64	1
0.62	2
0.51	3
0.41	5

The closeness coefficient grade and ranking of alternatives are calculated separately for cosine similarity and Ochiai coefficient and shown in Table 4 and Table 5, respectively. Table 6 shows the closeness coefficient grade and ranking of alternatives using traditional fuzzy TOPSIS method.

5. Results and Discussion

The rank preferences of TOPSIS grade are given according to ascending order for both cosine similarity and Ochiai coefficient. The traditional TOPSIS rank takes descending order. The rankings of Ochiai measure and traditional TOPSIS are the same but the cosine similarity ranking differs a lot and this rank reversal is noted for all problems. It is observed from Figure 3 that the distance between the positive and negative solutions is very negligible in cosine similarity measure, hence the measures of closeness coefficient and TOPSIS grade that are obtained from this difference do not assure ideal ranking. On the other hand, the positive and negative ideal solutions are determined perfectly by Ochiai measure as shown in Figure 4. Hence, the separation in the TOPSIS grade that is obtained from the closeness coefficient using Ochiai measure ranks the alternative to perfection. The TOPSIS grade of cosine, Ochiai and traditional approaches are compared and shown in Figure 5.

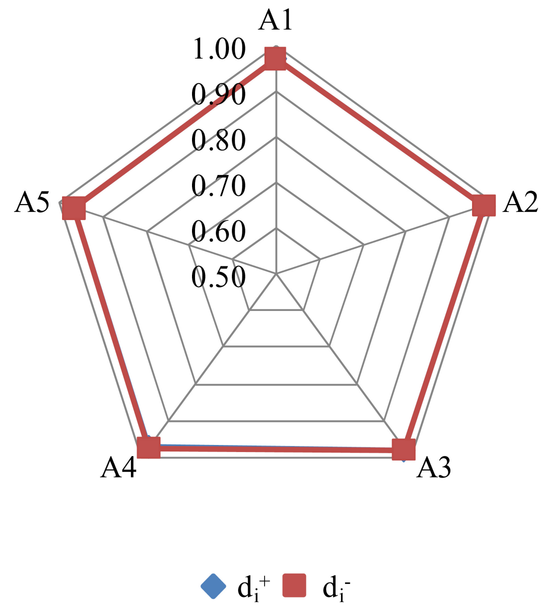

Figure 3. Cosine TOPSIS.

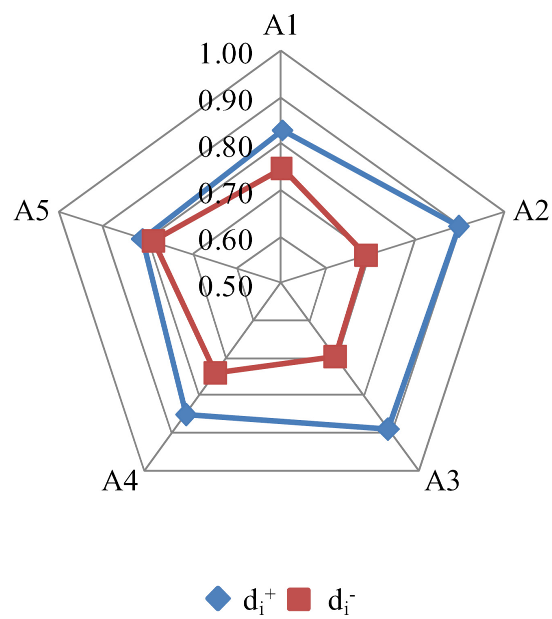

Figure 4. Ochiai TOPSIS.

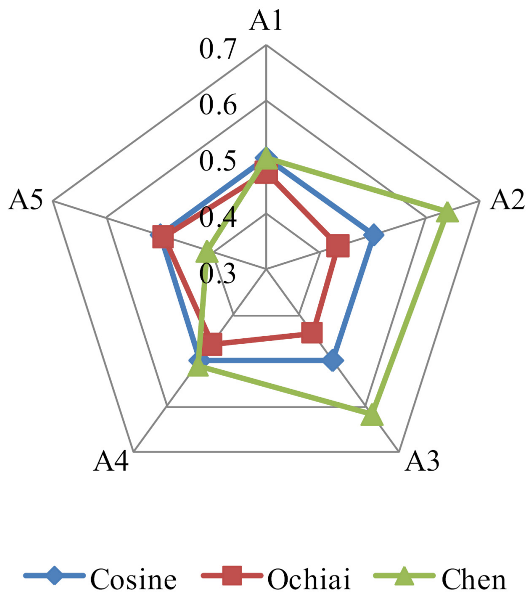

Figure 5. Comparison of relative closeness.

6. Conclusion

In this paper, a modified fuzzy TOPSIS method has been proposed to rank and select the best alternative from the set of alternatives using cosine similarity measure and Ochiai coefficients for multi-criteria decision making problems. The alternative assessments of the decision makers and criteria weights are taken as trapezoidal fuzzy numbers.

The separation measures, closeness coefficients and TOPSIS grade are determined for cosine similarity measure and Ochiai coefficients separately and compared. The Ochiai measure precisely differentiates the ideal solution which has the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution to choose the best alternative. The cosine similarity measure fails to extricate the positive and negative ideal solutions. The comparison of relative closeness between cosine similarity and Ochiai coefficient with traditional TOPSIS indicates that the proposed method using Ochiai measure is very effective for multi-criteria problems under fuzzy environment.

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