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THEORETICAL DETERMINATION OF THE MASS DEFECT OUTSIDE THE SPACETIME GEOMETRY PROPERTIES

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Abstract

The article describes a theoretical approach to determining the mass defect of a tangible object and cohesive energy of its constituent parts. It is shown that the mass defect emergence at the interacting objects' fusion into a unified whole is a common phenomenon regardless of the fusion process spatial scale. The mass defect emergence regularities are true for all cases of exchange interactions regardless of their intensity. In particular, for strong interaction, for the verification purposes, the derived equations are used to calculate a tangible object's mass defect in approximation of pairwise and triple interactions of particles comprising the object as a whole. The determined values are: the cohesive energy of protons in nucleus ${}_2He^3$ and neutrons in nucleus ${}_1H^3$, as well as the distance between protons in nucleus ${}_2He^3$.

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Introduction

Mass is a physical scalar and is not localizable as such. However, the concept of mass density is rather widely used in physics. Nevertheless, physics does not use the concept of “mass”, but the concepts of “body mass”, “particle mass” and “volume element mass”. In this case, particles are considered as volume elements. This is what assigns a specific meaning to the concept of “mass density”. That is physics operates on geometrical objects that have the property of mass and are eventually localized in space. In this sense, mass is localizable. In the same sense, we can treat energy as a localizable concept. These considerations are rather obvious in the macrocosm, but not obvious and are not acceptable in both the microcosm and the megacosm. The capability of mass localization is based in the specified sense on the mass property, which we call “*mass additivity*”. Let us take a certain body and measure its mass m_1. Next, we take another body and independently measure its mass m_2. Then we join these two bodies into one and measure the mass of the resulting body as single entity m_{12}. The mass additivity property consists in the following: $m + m_2 = m_{12}$. In modern physics, the mass additivity property in the macrocosm is absolute (if we do not consider the question where the macrocosm “ends” and the microcosm “starts”). It is also well known that instead of the mass additivity in the microcosm for interacting particles, the so-called “mass defect” takes place, that is: $m_1 + m_2 > m_{12}$. The concept of “mass defect” is firmly rooted in nuclear physics and widely used in the analysis of the elementary particle merger (synthesis) and core fission processes. At the same time, no one has tried to demonstrate theoretically the causes of the mass defect emergence phenomenon based on the “first principles”. The modern physics’ understanding of mass additivity in the megacosm is unclear. Today, the mass additivity property is extrapolated to any space objects, up to the entire universe. At the same time, the problem of “hidden” mass or, as is now commonly said “*dark matter*” has been already formulated. The problem is as follows. The mass of galactic clusters can be defined as the sum of its constituent galaxies' masses. We denote it as $m\Sigma$. On the other hand, assuming that a cluster is gravitationally stable (which usually can hardly be doubted, judging by its appearance), we can determine its mass as a whole by the virial theorem (Gaite [2]). This mass is called *virial*. We denote it as M. The problem is that M is always more than $\mathrm{m}\Sigma$, and for certain clusters, it is more by tens and even hundreds of times. The only explanation for this contradiction is that the galaxy cluster in addition to the observed luminous matter also contains unobservable entities - extinct stars and galaxies, intergalactic gas, or even black holes. However, it has not been possible to locate such substances so far. Thus, the problem persists. It is even conceivable that the property of mass additivity can be formulated in various ways:

in the microcosm: $m_1 + m_2 > m_{12}$,

in the macrocosm: $m_1 + m_2 = m_{12}$,

in the megacosm: $m_1 + m_2 \, ? \, m_{12}$.

Intuition tells us that in the megacosm $m_1 + m_2 < m_{12}$. If it is so, how can we explain this inequation? If m_1 is the mass of one galaxy and m_2 is the mass of another, then if we consider them as a single entity under the assumption that they only interact gravitationally, their total mass m_{12} is greater than the sum of masses m_1 and m_2. We can even explain this. Indeed, the mass defect is explained by the fact that the interaction energy is due to the interacting particles' masses, and, therefore, the effective mass of the two particles system is less. Strictly speaking, we should talk about "mass + energy", but it does not change the essence of the matter, since for the stability of the system of two interacting particles, part of the energy is emitted from the system during its formation. And this part is significant. If we consider two interacting galaxies as a single entity, the mass equivalent to the energy of their gravitational interaction is added to their masses, that is the effective mass of the system of two interacting galaxies increases. If it is so, the problem of "dark matter" is excluded, at least partially, or maybe even completely. The first evidence of the so-called gravitational mass defect existence were the results of classical work (Gershtein et al. [3]). These results prove that if there is spherically symmetric distribution of matter of "radius" *a* (in terms of coordinates), the total body mass *m* can be expressed in terms of its energy-momentum tensor T_0^0 by the following formula:

$$m = \frac{4\pi}{c^2}\int_0^a T_0^0 r^2 dr.$$

For static distribution of matter, $T_0^0 = \varepsilon,$ where ε is the energy density in space. Given that the volume of a spherical shell in spherical coordinates is equal to: $dV = 4\pi r^2 \sqrt{g_{11}} dr,$ where g_{11} is the component of the metric tensor of the part of Schwarzschild's spacetime that is the most important one in physical terms, we find that

$$m = \int_0^a \frac{\varepsilon}{c^2} 4\pi r^2 dr < \int_V \frac{\varepsilon}{c^2} dV.$$

This difference indicates the gravitational body mass defect existence. We can say that part of the total energy of the system is contained in the energy of the gravitational field, though it is impossible to localize this energy in space.

Certain efforts to explain and describe formally the gravitational mass defect are being made. For example, Logunov et al. [4] suggest a covariant approach to determining the gravitational mass defect. The system of gravitational field equations is supplemented with the body mass equation. The gravitational mass defect is identified with the effect of the substance's total mass reduction due to its own gravitational interaction. In this case, the gravitational field, being one of the matter's forms, is massless. In Ryabov et al. [7] and Lattimer [6], the origin of the total mass defect was explained through the fact that when combining elements of a certain mass, which already have specified density in an integral unit, we must consider the energy of the gravitational interaction between these elements. The incomplete mass defect was identified with the energy that is displayed in the form of either quanta, or neutrinos, or gravity waves during the formation from the originally rarefied diffuse matter of the single dense material object.

The theory of nuclear reactions is based on the fundamental assumption that the mass of reacting particles always changes in the course of building a certain atomic nucleus. The comparison of different viewpoints on the physical origin of the mass defect reveals the existence of contradictions in this issue. Most probably, all of these contradictions come from the same source. The theory of relativity, having presented its relativistic relations, provided certain tools for quantitative microprocess calculation; however, it has not explained the physical aspect of the mass defect emergence process. The mass defect emergence remains metric in many terms. It is a property of the space.

The aim of the study was to build a mathematical model of the mass defect emergence process at the merger of any number of interacting objects into a single entity and to identify the possible cause for the mass defect emergence. The problem formulation and its solution method presented in the paper can be used to solve the problem of hypothetical diproton Stability on the Universe (Bradford [1]) and in the theoretical study of the possibility of two-proton radioactivity (Grigorenko and Zhukov [5]). When solving the hypothetical diproton stability problem and when studying the double proton radioactivity, the proposed method will allow defining and accounting for the correction to the short-range three-particle potential determined by the pair interference of the interacting nucleons.

Problem Statement

Let us consider the function $F(m_1, m_2, m_3, \dots, m_N)$, the numerical value of which is equal to the mass of object M formed by N interacting particles with masses $m_1, m_2, m_3, \dots, m_N$.

In this case, the value of function $F(0, 0, 0, \dots, m_i, \dots, 0)$ is equal to the mass of ith particle not interacting with other particles:

$$F(0,\, 0,\, 0,\, \ldots,\, m_i,\, \ldots,\, 0) = m_i.$$

And the value of function $F(0,\, 0,\, 0,\, \ldots m_i,\, \ldots,\, m_j,\, \ldots,\, 0)$ is equal to the mass of the object formed by two interacting *i*th and *j*th particles with masses m_i and m_j, respectively:

$$F(0,\, 0,\, 0,\, \ldots m_i,\, \ldots,\, m_j,\, \ldots,\, 0) = m_{i,\, j}.$$

In this case:

$$F(0,\, 0,\, 0,\, \ldots m_i,\, \ldots,\, m_j,\, \ldots,\, 0) \neq m_i + m_j.$$

The problem is set as follows: to find the value of function $F(m_1,\, m_2,\, m_3,\, \ldots,\, m_N)$ using the values of functions $F(m_1,\, 0,\, 0,\, \ldots,\, 0)$; $F(0,\, m_2,\, 0,\, \ldots,\, 0)$; $F(0,\, 0,\, m_3,\, \ldots,\, 0),\, \ldots,\, F(0,\, 0,\, 0,\, \ldots m_N)$, i.e., to find a value of the mass of object M formed by N interacting particles with masses $m_1,\, m_2,\, m_3,\, \ldots,\, m_N$ using known mass values of the particles not interacting against each other.

Problem Solution

Let us expand the function $F(m_1,\, m_2,\, m_3,\, \ldots,\, m_N)$ by Maclaurin's series. The expansion will be as follows:

$$\begin{aligned} F(m_1,\, m_2,\, m_3,\, \ldots,\, m_N) \approx\ & F(0,\, 0,\, 0,\, \ldots,\, 0) \\ & + \sum_i \frac{\partial F}{\partial m_i}\Big|_0 m_i + \frac{1}{2!}\sum_i \frac{\partial^2 F}{\partial m_i^2}\Big|_0 m_i^2 \\ & + \frac{1}{2!}\sum_{i,\, j} \frac{\partial^2 F}{\partial m_i \partial m_j}\Big|_0 m_i m_j + \frac{1}{3!}\sum_i \frac{\partial^3 F}{\partial m_i^3}\Big|_0 m_i^3 \\ & + \frac{1}{3!}\sum_{i,\, j,\, k} \frac{\partial^3 F}{\partial m_i \partial m_j \partial m_k}\Big|_0 m_i m_j m_k + \cdots. \end{aligned}$$

In this case,

$$\frac{\partial F}{\partial m_i}\Big|_0 \approx \frac{F(0, 0, 0, \ldots, m_i, \ldots, 0) - F(0, 0, 0, \ldots, 0, \ldots, 0)}{m_i} = 1.$$

Thus, $\frac{\partial^2 F}{\partial m_i^2}\Big|_0 = 0$ and all derivatives of higher orders $\frac{\partial^{(n)} F}{\partial m_i^{(n)}}\Big|_0 = 0$, then the expansion will be as follows:

$$F(m_1, m_2, m_3, \ldots, m_N)$$

$$\approx \sum_i m_i + \frac{1}{2!}\sum_{i,j} \frac{\partial^2 F}{\partial m_i \partial m_j}\Big|_0 m_i m_j + \frac{1}{3!}\sum_{i,j,k} \frac{\partial^3 F}{\partial m_i \partial m_j \partial m_k}\Big|_0 m_i m_j m_k + \ldots. \quad (1)$$

Using the definition of the derivative of function of several variables, we can draw algebraic expressions for mixed derivatives of function F at zero point $F(0, 0, 0, \ldots, 0, \ldots, 0)$:

$$\frac{\partial^2 F}{\partial m_i \partial m_j}\Big|_0 \approx \frac{m_{i,j} - m_i - m_j}{m_i m_j},$$

$$\frac{\partial^3 F}{\partial m_i \partial m_j \partial m_k}\Big|_0 \approx \frac{m_{i,j,k} - m_{i,j} - m_{j,k} - m_{i,k} + m_i + m_j + m_k}{m_i m_j m_k},$$

$$\frac{\partial^4 F}{\partial m_i \partial m_j \partial m_k \partial m_l}\Big|_0 \approx \frac{m_{i,j,k,l} - m_{i,j,l} - m_{i,k,l} - m_{j,k,l} - m_{i,j,k} + m_{i,l}}{m_i m_j m_k m_l}$$

$$+ \frac{m_{j,l} + m_{k,l} + m_{i,j} + m_{i,k} + m_{j,k} - m_i - m_j - m_k - m_l}{m_i m_j m_k m_l}. \quad (2)$$

Here, $m_{i,j}$ is the mass of the object formed by two interacting ith and jth particles with masses m_i and m_j, respectively; $m_{i,j,k}$ is the mass of the object formed by three interacting ith, jth and kth particles with masses m_i, m_j and m_k, respectively; $m_{i,j,k,l}$ is the mass of object formed by four interacting particles with masses m_i, m_j, m_k and m_l, respectively.

Equation (1) shows that the mass of object M is different from the arithmetical sum of the masses of the interacting particles which form the object:

$$F(m_1, m_2, m_3, \ldots, m_N) = M \neq \sum_i m_i.$$

This difference called *mass defect* is defined by the following equation:

$$\Delta m = M - \sum_i m_i = \frac{1}{2!}\sum_{i,j} \frac{\partial^2 F}{\partial m_i \partial m_j}\Big|_0 m_i m_j + \frac{1}{3!}\sum_{i,j,k} \frac{\partial^3 F}{\partial m_i \partial m_j \partial m_k}\Big|_0 m_i m_j m_k + \cdots. \tag{3}$$

To determine the sign of Δm, let us consider the object formed by K interacting particles, and consider pairwise interactions only. In this case, subject to equations for mixed derivatives (2), equation (1) can be written as:

$$M \approx \sum_{i=1}^{K} m_i + \sum_{j=i+1}^{K} \sum_{i=1}^{K} \Delta_{i,j}. \tag{4}$$

Mass defects caused by pairwise interactions $\Delta_{i,j}$ are defined as follows:

$$\Delta_{i,j} = (\eta_{i,j} - 1)(m_i + m_j),$$

where the pairwise interaction coefficients are: $\eta_{i,j} = \dfrac{m_{i,j}}{m_i + m_j}$.

If all pairwise interaction coefficients $\eta_{i,j} < 1$, i.e., $m_{i,j} < m_i + m_j$, then all values $\Delta_{i,j} < 0$ and the resulting mass defect is:

$$\Delta m = \sum_{j=i+1}^{N} \sum_{i=1}^{N} \Delta_{i,j} < 0.$$

And the mass of the object formed by two interacting particles is less than the sum of the masses of individual particles.

Mass Defect and Cohesive Energy of Nucleons in Nucleus

If an atomic nucleus is considered as the object with mass M, and equation (4) is multiplied by the speed of light squared, then the second term in:

$$c^2 \sum_{j=i+1}^{K} \sum_{i=1}^{K} \Delta_{i,\,j}$$

is nothing but cohesive energy of nucleons in nucleus. Specific cohesive energy per nucleon is:

$$\varepsilon = \frac{c^2}{K} \sum_{j=i+1}^{K} \sum_{i=1}^{K} \Delta_{i,\,j},$$

where $K = Z + N$, Z is the number of protons, and N is the number of neutrons in nucleus.

Equation (4) can be revised as:

$$E_\Sigma \approx \sum_{i=1}^{K} E_i + c^2 \sum_{j=i+1}^{K} \sum_{i=1}^{K} \Delta_{i,\,j}, \tag{5}$$

where E_Σ is the nucleus rest energy, E_i is the rest energy of *i*th nucleon being a part of nucleus, $c^2\Delta_{i,\,j}$ is the cohesive energy of *i*th and *j*th nucleons.

To show where equation (5) may be used, let us consider nucleus ${}_2He^3$. It comprises 3 nucleons: nucleon no. 1 – proton, nucleon no. 2 – proton, and nucleon no. 3 – neutron. Three combinations are possible: proton-neutron; proton-neutron, and proton-proton. The proton-neutron combination forms a deuteron with known cohesive energy of nucleons $c^2\Delta_{p,\,n}$. The rest energies of all nucleons in nucleus and the nucleus rest energy are also known. Next, we determine the cohesive energy of nucleons no. 1 and no. 2 in nucleus, i.e., the cohesive energy of two protons $c^2\Delta_{p,\,p}$ in nucleus ${}_2He^3$.

Equation (5) in the context of the case considered should be written as:

$$E_{{}_2He^3} = 2E_p + E_n + 2c^2\Delta_{p,n} + c^2\Delta_{p,p}.$$

Using this equation and the values of the terms in the right part of the equation from (Grigorenko and Zhukov [5]), we derive that the cohesive energy of two protons in nucleus ${}_2He^3$ is approximately –4.49MeV (the negative value – attraction due to exchange interaction).

Next, we consider nucleus ${}_1H^3$. It comprises three nucleons: nucleon no. 1 – proton, nucleon no. 2 – neutron, and nucleon no. 3 – neutron. Three combinations are possible: proton-neutron, proton-neutron, and neutron-neutron. Next, we determine the cohesive energy of nucleons no. 2 and no. 3 in nucleus, i.e., cohesive energy of two neutrons $c^2\Delta_{n,n}$ in nucleus ${}_1H^3$.

In the context of the case considered, equation (5) should be revised as follows:

$$E_{{}_1H^3} = 2E_n + E_p + 2c^2\Delta_{p,n} + c^2\Delta_{n,n}.$$

Using this equation, we derive that the cohesive energy of two neutrons in nucleus ${}_1H^3$ is –5.80MeV approximately (the negative value – attraction due to exchange interaction). The cohesive energy's absolute value in a neutron-neutron system is more than the absolute value for a proton-proton system, what can be explained by the Coulomb repulsion of protons. The difference of cohesive energies is about 1.31MeV (a positive value), which corresponds to the Coulomb repulsion energy of two protons with the difference between centers $1.12 \cdot 10^{-15}$m, approximately. The obtained values are in very good agreement with the experimental ones, which shows the correctness of the source assumptions in the general problem formulation (Spyrou et al. [8]).

Conclusion

The problem of determining the mass of a material object formed by a merger of any number of interacting material objects into a single entity is formulated and solved outside the properties of spacetime geometry, which gives grounds to assume the mass to be the invariant. The mass defect formation in interacting objects' fusion into a single entity is a common phenomenon regardless of the fusion process' spatial scale. Mass defect Δm arising in the formation of an object with rest mass M due to the fusion into a single entity of K interacting objects with masses m_i $(i = 1, \ldots, K)$ is defined as follows:

$$\Delta m = M - \sum_{i=1}^{K} m_i \approx \sum_{j=i+1}^{K} \sum_{i=1}^{K} \Delta_{i,j} + \sum_{k=j+1}^{K} \sum_{j=i+1}^{K} \sum_{i=1}^{K} \Delta_{i,j,k} + \ldots,$$

where $\Delta_{i,j}$ and $\Delta_{i,j,k}$ are mass defects caused by pairwise and triple interactions of objects forming a single entity in fusion $(\Delta m < 0)$. The mass defect value is determined by the intensity of pairwise, triple interactions, and interactions of higher order between parts (particles) forming a single entity in fusion. When determining the mass defect of a tangible object formed by fusion of three other objects, it is sufficient to take into account pairwise interactions between them. Mass defects caused by pairwise interactions $\Delta_{i,j}$ are determined as follows:

$$\Delta_{i,j} = (\eta_{i,j} - 1)(m_i + m_j),$$

where the coefficients of pairwise interactions are:

$$\eta_{i,j} = \frac{m_{i,j}}{m_i + m_j}.$$

The mass defects caused by triple interactions and interactions of higher orders are determined by the same formula.

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