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COMPARATIVE ARIMA MODELS FOR AGE-SPECIFIC FERTILITY RATES

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Abstract

Getting the forecasted values of the age-specific fertility rates (ASFR) is very important to prepare population projections. Either population projections are deterministic or probabilistic. The question is how to get the forecasted values of ASFR. One possible technique is obtaining ASFR through the United Nations model age patterns of fertility (UN age patterns). This technique is the common technique in most studies for Egypt. Another possible suggested technique is modeling ASFR themselves. This paper mainly examines the question of which one of the two previous techniques would provide us with more accurate results. This question is considered empirically based on integrated autoregressive moving average (ARIMA) models using

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annual data on Egypt during the period from 1966 to 2010. This comparative study is conducted between: (a) univariate ARIMA model for the total fertility rate then used the UN age patterns; and (b) multivariate ARIMA model to the vector of ASFR. The results of the accuracy measures indicate that multivariate ARIMA model has the ability to clearly represent ASFR. Consequently, multivariate ARIMA model is used for forecasting ASFR in Egypt until year 2030.

1. Introduction

Box-Jenkins methodology has been used in demographic field since 1970. It has shown high efficiency in modelling and forecasting many phenomena in different knowledge fields. Many studies relied upon the univariate integrated autoregressive moving average (ARIMA) models to model and forecast demographic variables. Saboia [26] used ARIMA models to forecast the total population size of Sweden. He found that both ARIMA $(1, 1, 0)$ and ARIMA $(0, 2, 1)$ models were able to represent the Swedish population size. The mean square error showed that ARIMA $(0, 2, 1)$ model was more accurate than the other one in representing population size. Saboia [27] applied the Box-Jenkins technique on birth time series data for Norway through 1919-1974. He estimated ARIMA $(4, 1, 1),$ based on this model, the forecasted values for the Norwegian births were obtained through 1975-2000. Pflumer [25] used the Box-Jenkins methodology to forecast the total population size of the USA until 2080 where the fitted model was ARIMA $(2, 2, 0).$

Lee-Carter model is used to get the age-specific rates by fitting and forecasting either fertility index or mortality index. This model is based on ARIMA models (for more details on this model see Lee and Carter [19], Carter [9], Šimpach [32].

Miller [21] applied the multivariate ARIMA model for fitting and predicating the total fertility rate (TFR) and the mean age of childbearing to white women in the USA. There are also a number of studies that applied multivariate ARIMA model for parameters of gamma function. Gamma function is a mathematical model used to get single-year specific fertility rates (Thompson et al. [33], Keilman and Pham [17]). Sarpong [29] depended on ARIMA models for modeling and forecasting maternal mortality ratios at hospital in city in Ghana. ARIMA $(1, 0, 2)$ model was fitted for modeling and forecasting maternal mortality ratios at that hospital. According to the Akaike's information criterion (AIC) the previous model is the best one among ARIMA $(1, 0, 0)$, ARIMA $(0, 0, 1)$, ARIMA $(1, 0, 1)$, ARIMA $(1, 0, 2)$, and ARIMA $(2, 0, 2)$.

There are many studies which used ARIMA models to get the future values of fertility rates to prepare probabilistic or stochastic population projections. Wilson and Bell [36] prepared probabilistic population projections for Australia through 2002 until 2050. They applied the Box-Jenkins methodology to predict the values of the total fertility rate which follows ARIMA $(1, 1, 0)$. Keilman et al. [18] showed stochastic population projections for Norway through 1996-2050. They fitted multivariate ARIMA $(1, 1, 0)$ to represent fertility rates during the period 1945 to 1995. Also, Dunstan [12] prepared probabilistic population projections for New Zealand until 2111. He suggested a random walk with drift model as an appropriate model for annual total fertility rate.

In Egypt, the application of univariate ARIMA models in the demographic field has been of limited use, while the multivariate ARIMA models have never been applied. Hussein [14] fitted ARIMA $(1, 1, 1)$ for the crude birth rate and ARIMA $(1, 1, 0)$ to the crude death rate for Egypt. He used time series from 1900 to 1990 annually. These models were relied upon, in order to forecast these rates as well as the natural increase rate. Mustafa [22] concluded that the more appropriate model to represent the Egyptian crude death rate is ARIMA $(1, 3, 2)$. Also, ARIMA $(2, 3, 1)$ to represent infant mortality rate of Egypt through the period from 1947 to 2008.

Similar to other developing countries, Egypt has not yet reached the stage of demographic stability. So, it is not reasonable to assume the fixity of the age distribution to get the fertility age pattern. Therefore, most of the studies for Egypt have relied upon the UN age patterns to get the forecasted values of the age-specific fertility rates. Particularly all the studies prepared by the official bodies such as Central Agency for Public Mobilization and Statistics (CAPMAS) and Cairo Demographic Center (CDC).

Accordingly, it is important to investigate the accuracy of obtaining the age-specific fertility rates through the UN age patterns. The current study is suggesting the use of univariate and multivariate ARIMA models for the age-specific fertility rates with a comparative study between these two models: (a) Univariate ARIMA model for the total fertility rate then used the UN age patterns; and (b) multivariate ARIMA model to the vector of the age-specific fertility rates. This objective will be done through the following sections. In Section 2, data preparation is achieved. In Section 3, a univariate ARIMA model for the total fertility rate is presented from 1966 to 2010. In Section 4, age-specific fertility rates are computed based on the UN age patterns. In Section 5, the multivariate ARIMA model is introduced for the ASFR during the same period of the time. Comparison between the results is illustrated in Section 6. Final conclusion and future work is given in Section 7.

2. Data Preparation

Central agency for public mobilization and statistics (CAPMAS) has published successive censuses from 1907 until 2006. Furthermore, Egypt is rich with vital statistics comparing with other developing countries and some of these statistics even go back to the beginning of the 20th century (Cairo Demographic Center [7]). Egypt (like several other countries) has encountered the problems resulted from errors that were found in its data. But, the quality of data has been improved, particularly in relation to successive censuses. Therefore, data should be modified and emendated before it can be ready for a proper use. This study uses annual data obtained from (CAPMAS) throughout the period from 1966 (which was the first year that has births according to mother’s age) until 2010. The collected data from (CAPMAS) contain the population size according to age and sex groups, the number of births according to mother's age. The age-specific fertility rates (ASFR) and the total fertility rate (TFR) are computed at the Egyptian national level. Prior to computing these rates, the following procedures have been followed yearly (United Nations [34]).

(1) Errors due to inaccurate age reports for each sex separately are adjusted by the formula:

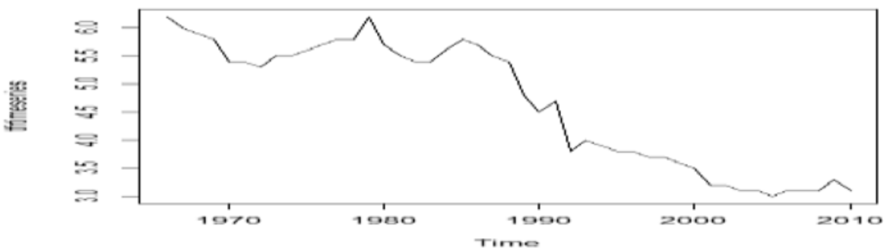
$$T_0' = 1/16\left(-T_{-2} + 4T_{-1} + 10T_0 + 4T_{+1} - T_{+2}\right), \tag{1}$$

where T_0' is the adjusted number of persons in a certain age group, T_0 is the reported number of persons in that age group. T_{-1} and T_{-2} are reported numbers in the first and second preceding groups, respectively. T_{+1} and T_{+2} are reported numbers in the first and second subsequent groups, respectively.

(2) Children under five years old are incompletely enumerated in the censuses of many countries. So, we have estimated from vital statistics the number of children under five years old by calculating the difference between number of births during the preceding five years and the deaths among these children.

(3) Pro-rating of the groups of unknown age has been done for births, deaths and the whole population for each sex separately.

It is also worth mentioning that, the ASFR and the TFR have not been previously computed (on an annual basis) for Egypt. What were available only rates for specific years of censuses and surveys but in the current study the ASFR and the TFR are computed annually from 1966 to 2010. Graph 1 presents the time series of TFR for Egypt through 1966 to 2010. It is clear that the TFR of 1966 was 6.2, it started to decrease generally to the lowermost in 2005 in which the TFR was 3. The ASFR are shown in Table A in the Appendix B.



Graph 1. TFR at the national level of Egypt.

3. Univariate ARIMA Models for TFR

A process Y_t is said to be *ARIMA* (p, d, q) if the differences of time series from order d (i.e., $\Delta^d Y_t$) has ARMA (p, q) which has the form (Bowerman and O'Connell [1], Shumway and Stoffer [30], Sharawey [28]):

$$\Phi_p(B)\Delta^d Y_t = \ominus_q(B)\varepsilon_t, \tag{2}$$

where

$$\Phi_p(B) = [1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p], \tag{3}$$

$$\ominus_q(B) = [1 + \ominus_1 B + \ominus_2 B^2 + \cdots + \ominus_q B^q], \tag{4}$$

ε_t are iid normally distributed $(0, \sigma^2)$ and B is the back shift operator.

3.1. Box-Jenkins methodology

The main idea of Box-Jenkins methodology is the appointment of the history of the time series and also the history of errors series to find suitable description to the manner of data change. Box-Jenkins methodology consists of a four-steps procedure, namely (1) model identification; (2) model parameter estimation; (3) model diagnostic checking (i.e. goodness of fit); (4) the last step that considers the main aim from time series analysis which is forecasting. In order to use the Box-Jenkins methodology, the time series should be stationary (weak stationary: the mean, variance, and autocorrelation are constant over time). In the model identification step, the determination of initial suitable model for the time series data, i.e., determination of three orders of ARIMA (p, d, q) model in the following matter:

(a) "d" is number of differences which makes time series to be stationary; (b) "p" is number of lagged values of time series and represents the order of autoregressive (AR); and "q" is number of lags of errors and represents the order of moving average (MA).

The auto correlation function (ACF) and the partial auto correlation function (PACF) provide more information in respect of the performance of the time series. The ACF provides information regarding the internal correlation between values or observations in a time series at different lags. Whilst, the PACF provides the same information with the effects of the intervening observations excluded. The plots of both ACF and PACF suggest that the model should build. After determination the initial model to the data of time series, the second step is the estimation of parameters of that model based on one of familiar methods in theory like least squares and maximum likelihood methods. The next step is diagnostic checking in which we know if the initial model is suitable for the time series or not. The residuals of adequate model should be white noise process (i.e., all the ACF are zero). In practice, if the residuals of the model are white noise, then the ACF of the residuals are not significant. Ljung-Box test can be used to determine whether the series of residuals is independent or not. If the model is adequate, it can be used for forecasting and this is the last step in Box-Jenkins methodology (Bowerman and O'Connell [1], Shumway and Stoffer [30], Sharawey [28]).

3.2. ARIMA model results

The graph of total fertility rate (TFR) of Egypt from 1966 to 2010 in Graph 1 indicates that the series is non-stationary. The first difference of logarithms successes to transfer it to a stationary time series as demonstrated in Figure 1 in Appendix A. Figures 2(a) and (b) show the ACF and the PACF of the difference series for the total fertility rate, respectively. It is clear that

ARIMA $(0, 1, 0)$ with drift which is random walk with drift is candidate to the TFR of Egypt. This model is adequate model as Figure 3 in Appendix A indicates that the residuals do not have any pattern.

The ACF and the PACF of residuals are presented in Figure 3(b) and (c) indicate that the residuals are independent. The *p*-value of Ljung-Box test was 0.429 which means that the null hypothesis said that correlation coefficients of residuals are non-significant is not rejected (i.e., the residuals are white noise series). Therefore, random walk model with drift is an adequate model to the TFR of Egypt. The fitted values of TFR are obtained based on this model as follows:

$$Y_t = Y_{t-1} - 0.0705. \tag{5}$$

4. The UN Pattern and ASFR

The United Nations population division has introduced regional patterns to indicate the age distribution of fertility. The United Nations model age patterns of fertility indicate the age-specific fertility rates corresponding to a certain value of the total fertility rate (i.e., the age distribution of fertility for certain value of the total fertility rate). These patterns are Sub-Saharan Africa, Arab countries, Asia, and average. Index of dissimilarity (ID) has estimated during the censuses years of Egypt. This index indicates the degree of dissimilarity between two percent distributions and it has the following form:

$$\text{index of dissimilarity} = 0.5 \sum |r_{1a} - r_{2a}|, \tag{6}$$

where r_{1a} is the percent distribution of ASFR for population of our interest (in this study Egypt), r_{2a} is the percent distribution of ASFR for United Nations model schedule. The value of ID is ranging from zero to 100, zero means completed similarity between the two percent distributions and 100 means completed dissimilarity between them.

Table 1. ID for United Nations model schedules of fertility and Egypt

Year	Sub-Saharan Africa	Arab Countries	Asia	Average
1966	21.11	12.07	13.92	15.42
1976	22.94	12.49	12.44	15.94
1986	20.92	10.87	10.42	13.92
1996	18.68	10.12	7.96	10.45
2006	16.77	7.27	4.13	7.17

As shown in the table above, Egypt follows United Nations model schedule of Asia in the different censuses years except for 1966 where Arab countries has appeared small advance of Asia. Using the fitted values of the TFR from ARIMA $(0, 1, 0)$ and the UN age patterns age-specific fertility rates are obtained and the residuals are shown in Table B of the Appendix B. Interpolation has been used to obtain the intervening values (United Nations [35], CAPMAS [8], Stover and Kirimeyer [31]).

5. Multivariate ARIMA Models

Many cases' observations are taken simultaneously on two or more time series one of these cases is age-specific fertility rates. The purpose of this section is fitting the adequate multivariate ARIMA model to a vector of age-specific fertility rates for Egypt through 1966 to 2010. Generally, vector ARIMA (VARIMA) model is more complicated than vector autoregressive (VAR) model (i.e., extension from univariate case to multivariate case is fairly easy with AR (or MA) but it is complicated and has many problems associated with ARMA). For instance, the number of parameters increases quadratically with the number of elements in vector and it becomes uncomfortably large when the lag length is more than one or two. As such, one of suggestions is to use external knowledge or a preliminary analysis of the data to known whether VAR model can be used instead of VARMA model or not (Griffiths et al. [15], Shumway and Stoffer [30], Chatfield [10]).

To determine the order of VAR model (p), we can use model selection criteria like Akaike's information criterion (AIC), Schwarz criterion (SC), and Hannan-Quinn criterion (HQ) and all of them have the same logic

(Luetkepohl [20], Onwukwe and Nwafor [23]):

$$\text{AIC}(P) = \ln\left|\sum \hat{u}\,\hat{u}(p)\right| + (k + pk^2)\frac{2}{T}, \tag{7}$$

$$\text{SC}(P) = \ln\left|\sum \hat{u}\,\hat{u}(p)\right| + (k + pk^2)\frac{\ln T}{T}, \tag{8}$$

$$HQ(P) = \ln\left|\sum \hat{u}\,\hat{u}(p)\right| + (k + pk^2)\frac{2\ln(\ln(T))}{T}, \tag{9}$$

where $\left|\sum \hat{u}\,\hat{u}(p)\right|$ is the determinant of variance covariance matrix for residuals, k is number of variables, T is number of observations and p is suggested number to be order of model.

The AIC criterion does not have a great meaning itself but whose importance is derived from its application to compare between models and the same data of time series. AIC is a function in variance of residuals and number of parameters and the best model which have less AIC. The logic of AIC in multivariate case does not differ from the univariate case. In this regard, it is the function in the determinant of variance covariance matrix of residuals instead of the variance of residuals (Shumway and Stoffer [30], Burre [2], Kandial [16]).

Multivariate results

In Subsection 3.2, the univariate ARIMA case for the total fertility rate concludes that AR model is an adequate model for TFR. Therefore, VAR model is suggested for the age-specific fertility rates in Egypt. The data is collected in five-year age groups, so vector of ASFR has seven rates from 15 to 45 years. Figure 4 indicates that ASFR where FR refers to fertility rate according to successive age groups.

It is clear that fertility rates at age groups from 25 to 45 decreased through the time from 1966 until 2010 as shown in Figure 4 in Appendix A. In general, fertility rate at age group 15 decreased until 1993 then increased to 2009 and decreased another time for the last year whereas fertility rate at age group 20 was fluctuating. The augmented Ducky-Fuller test (ADF unit root test) used to test the null hypothesis that the time series is not stationary (i.e. unit root exists). According to the ADF test, all the time series of ASFR are non-stationary and the first difference successes to transfer them to stationary time series. After transferring the time series to be stationary, determine the order of VAR model to get the initial model for ASFR time series.

Table 2. The values of different criteria

Criterion \ Order	1	2
AIC	2.077146e + 01	2.122621e + 01
SC	2.279874e + 01	2.528078e + 01
HQ	2.151454e + 01	2.271237e + 01

The best model which have less value and all the criteria candidate order 1. Therefore, VAR (1) is initial model to the differences of ASFR of Egypt. This model is checked by Portmanteau test Q to determine whether the residuals are multivariate white noise or not. The P-value was 0.99 which means that VAR (1) model is adequate model for the first difference of ASFR of Egypt and the fitted model is:

$$\begin{pmatrix} FR15_t \\ FR20_t \\ FR25_t \\ FR30_t \\ FR35_t \\ FR40_t \\ FR45_t \end{pmatrix} = \begin{pmatrix} -0.00301 & 0.17121 & -0.03781 & -0.15227 & -0.04234 & 0.44688 & -0.03298 \\ 1.23502 & 0.03192 & 0.17707 & 0.01082 & -0.81013 & 2.00881 & -1.34892 \\ -1.41186 & 0.03296 & 0.04874 & 0.03079 & -0.79146 & 1.03089 & 0.81008 \\ -0.77563 & 0.20007 & 0.30370 & -0.59233 & 0.14186 & 0.82510 & 0.41581 \\ -0.34198 & 0.08869 & 0.01575 & -0.15376 & 0.07512 & 0.58612 & 0.57851 \\ -0.02544 & 0.01290 & 0.04175 & -0.06325 & -0.03957 & 0.44421 & 0.27587 \\ 0.07219 & -0.02773 & -0.03237 & 0.03302 & 0.07445 & 0.07032 & -0.04424 \end{pmatrix}$$

$$\times \begin{pmatrix} FR15_{t-1} \\ FR20_{t-1} \\ FR25_{t-1} \\ FR30_{t-1} \\ FR35_{t-1} \\ FR40_{t-1} \\ FR45_{t-1} \end{pmatrix}. \qquad (10)$$

The residuals from this model are shown in Table C of the Appendix B.

6. Comparison between Univariate and Multivariate ARIMA and Forecasting

It is evident that how the fitted ASFR from multivariate ARIMA $(1, 1, 0)$ is closer to the actual ASFR than ASFR fitted from univariate ARIMA $(0, 1, 0)$ with drift and the UN age pattern. Particular consideration is given in Table 3 where the mean square error (MSE) and the mean absolute error (MAE) are extremely small in multivariate case compared with univariate case.

Table 3. Accuracy measures for ASFR fitted from univariate and multivariate ARIMA

ARIMA \ Measure	MSE	MAE
Univariate	1078.7	25.2
Multivariate	68.9	5.3

Multivariate ARIMA $(1, 1, 0)$ is used to obtain forecasts of the age-specific fertility rate for Egypt until 2030. This forecast is shown in Table 4 and a forecast for TFR which clearly shows that the TFR is still almost constant after 2010 to 2030 and does not decline in the future. This is a reasonable result considering that the TFR decreased to 3 in 2005 before it slightly started to increase to reach 3.5 in 2014 according to the latest Demographic Health Survey (DHS) of Egypt.

Table 4. Forecasts of age-specific fertility rates and total fertility rate of Egypt

Year	FR15	FR20	FR25	FR30	FR35	FR40	FR45	TFR
2011	34.520	207.873	173.221	132.099	71.421	17.290	1.902	3.192
2012	29.720	214.634	156.416	122.971	65.836	16.123	3.037	3.044
2013	32.620	206.497	166.454	127.067	68.769	16.225	2.198	3.099
2014	30.148	210.601	159.809	123.962	66.378	15.903	2.706	3.048
2015	31.536	207.075	164.987	126.128	67.886	16.030	2.303	3.080
2016	30.402	209.194	161.841	124.787	66.801	15.914	2.544	3.057

2017	31.083	207.609	164.252	125.780	67.524	15.981	2.361	3.073
2018	30.569	208.634	162.736	125.161	67.024	15.932	2.476	3.063
2019	30.896	207.909	163.840	125.607	67.362	15.964	2.393	3.070
2020	30.662	208.392	163.121	125.319	67.131	15.942	2.447	3.065
2021	30.816	208.059	163.627	125.521	67.287	15.957	2.409	3.068
2022	30.709	208.283	163.290	125.388	67.181	15.947	2.434	3.066
2023	30.781	208.130	163.522	125.479	67.253	15.954	2.417	3.068
2024	30.732	208.233	163.366	125.418	67.204	15.949	2.428	3.067
2025	30.765	208.163	163.473	125.460	67.237	15.952	2.420	3.067
2026	30.742	208.211	163.401	125.432	67.215	15.950	2.426	3.067
2027	30.757	208.178	163.450	125.451	67.230	15.951	2.422	3.067
2028	30.747	208.200	163.417	125.438	67.219	15.950	2.425	3.067
2029	30.754	208.185	163.439	125.447	67.226	15.951	2.423	3.067
2030	30.749	208.195	163.424	125.441	67.222	15.951	2.424	3.067

7. Conclusion and Future Work

Most of studies for Egypt have relied upon the UN age patterns to get the forecasted values of the age-specific fertility rates. Particularly all the studies were prepared by the official bodies such as Central Agency for Public Mobilization and Statistics (CAPMAS) and Cairo Demographic Center (CDC) beside other studies. The objective of the current paper was finding answer to the question of whether obtaining the age-specific fertility rates through the UN age patterns would provide us with more accurate results than these resulted from modeling those rates themselves. The United Nations model age patterns of fertility indicate the age-specific fertility rates corresponding to a certain value of the total fertility rate (i.e. the age distribution of fertility for certain value of the total fertility rate). A comparative study had been achieved to answer the previous question. This comparison was between univariate ARIMA for the total fertility rate, then the UN age patterns used to get the age-specific fertility rates and between multivariate ARIMA models to the vector of the age-specific fertility rates. The current paper is modeling and forecasting TFR and ASFR for Egypt using the univariate and multivariate ARIMA models, respectively. First, two time series data for TFR and ASFR from 1966 to 2010 are computed for the national level of Egypt. Then, the random walk with drift model is used to clearly represent the TFR. After that, the ASFR are computed based on the UN age patterns. The VAR (1) model is also used as a multivariate ARIMA model to clearly represent the vector of the age-specific fertility rates. The comparison between the univariate ARIMA $(0, 1, 0)$ with the UN age pattern and the multivariate ARIMA $(1, 1, 0)$ suggested obviously using the multivariate ARIMA $(1, 1, 0)$ to get the ASFR. Consequently, multivariate ARIMA model is used for forecasting age-specific fertility rates in Egypt until year 2030. In general, the forecasted values for ASFR and TFR will be almost constant after 2010 to 2030. Therefore, family planning polices and efforts should push the process of fertility decline to reach the replacement level of fertility. It is recommended to apply this comparison for modeling and forecasting fertility rates at the subnational level of Egypt. Also, it is recommended for the developing countries (which depending on UN age patterns) to obtain ASFR to achieve this comparison for modeling and forecasting fertility rates.

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Appendix A

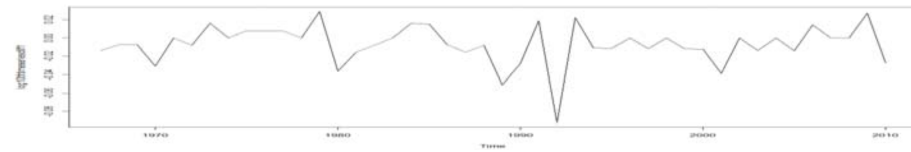
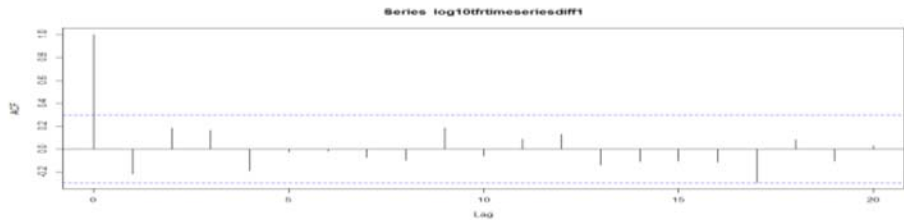
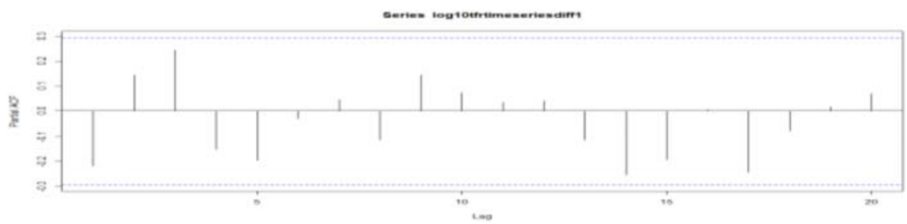


Figure 1. The time plot of difference series.



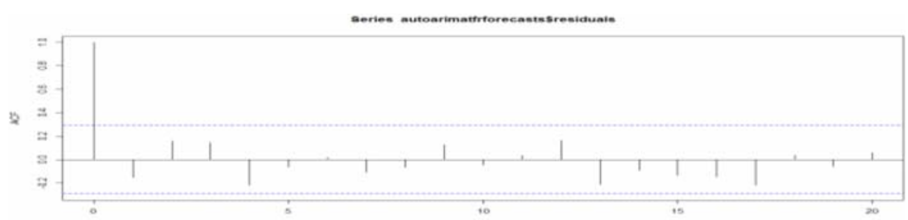
(a) The ACF of the difference series of TFR



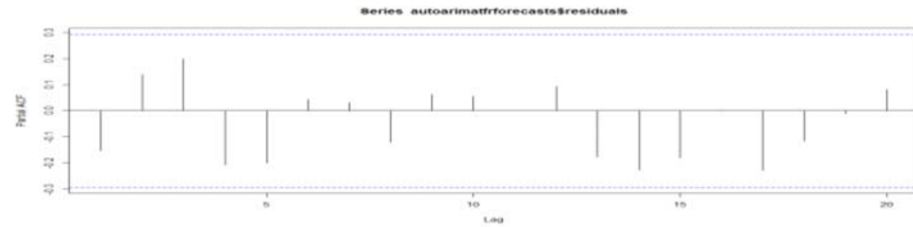
(b) The PACF of the difference series of TFR

Figure 2. The ACF and PACF of the difference series of TFR.

(a) The time plot of residuals of ARIMA $(0, 1, 0)$ with drift



(b) The ACF of residuals of ARIMA $(0, 1, 0)$ with drift



(c) The PACF of residuals of ARIMA $(0, 1, 0)$ with drift

Figure 3. Analysis of residuals of ARIMA $(0, 1, 0)$ with drift.

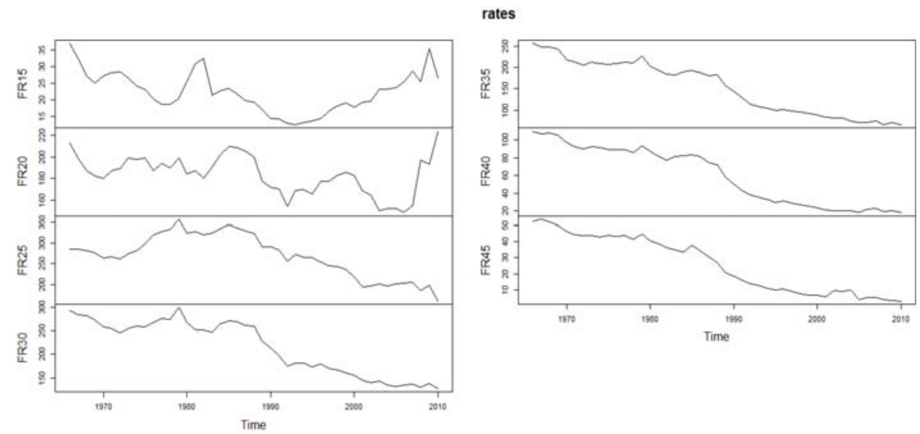


Figure 4. Age-specific fertility rates of Egypt 1966-2010.

Appendix B

Table A. The age-specific fertility rates of Egypt

Year	FR15	FR20	FR25	FR30	FR35	FR40	FR45
1966	37.08	213.00	285.83	294.06	256.24	109.65	52.36
1967	32.40	198.78	284.83	284.07	248.31	106.36	53.76
1968	26.97	187.03	282.29	281.66	248.31	107.79	51.75
1969	24.96	182.57	276.05	273.03	241.58	105.10	50.09
1970	27.23	180.09	263.62	258.32	218.08	96.72	46.41
1971	28.14	187.07	267.08	254.26	212.83	92.70	44.45
1972	28.44	189.51	261.52	245.46	204.63	89.78	43.73
1973	26.58	198.96	275.27	254.93	212.27	92.71	43.86
1974	24.17	197.54	282.42	259.10	209.42	91.11	42.96

1975	23.23	199.56	298.23	258.77	206.44	89.15	43.80
1976	20.09	186.94	319.58	268.47	209.50	88.88	43.43
1977	18.61	194.11	327.36	275.22	213.32	88.76	43.95
1978	18.62	189.91	331.89	273.98	211.37	85.68	41.67
1979	20.45	199.09	356.51	299.25	226.23	93.37	44.63
1980	25.39	184.39	324.55	268.43	203.01	87.44	40.70
1981	30.85	187.54	326.69	252.01	193.89	81.34	38.55
1982	32.56	180.17	320.14	251.71	184.94	77.34	36.31
1983	21.35	190.52	323.77	246.32	182.64	80.91	35.05
1984	22.73	202.07	334.55	265.37	189.67	82.38	33.62
1985	23.47	210.11	344.58	271.13	193.22	83.21	37.95
1986	21.71	208.92	336.37	270.21	187.54	80.97	34.40
1987	19.83	205.45	329.93	260.94	180.30	74.27	30.55
1988	19.21	200.12	323.57	260.08	182.54	71.95	27.02
1989	17.04	177.62	291.59	228.28	158.50	58.58	20.74
1990	14.18	172.01	291.57	213.59	143.67	50.31	18.63
1991	14.23	170.19	282.89	198.19	128.75	42.88	16.19
1992	12.95	153.86	255.74	175.39	113.54	37.89	13.62
1993	12.40	168.70	273.10	181.97	108.28	35.17	12.57
1994	13.18	169.68	265.62	180.86	105.15	33.14	11.03
1995	13.43	165.67	265.69	173.98	99.40	29.84	9.92
1996	14.29	177.35	253.87	179.23	101.36	31.27	10.20
1997	16.51	177.49	245.60	169.58	98.59	28.45	9.15
1998	18.08	182.85	242.87	167.15	95.96	26.82	7.60
1999	19.04	186.11	236.67	160.03	91.92	25.38	6.75
2000	17.55	183.32	218.00	155.27	88.67	23.80	6.53
2001	19.19	168.51	194.45	144.34	83.19	20.99	5.48
2002	19.45	165.09	196.77	140.05	82.25	20.59	9.25
2003	23.25	150.01	201.18	142.77	81.99	20.59	8.81
2004	23.21	152.28	197.07	135.35	75.43	20.15	9.81
2005	23.60	151.99	201.76	131.03	72.25	18.93	3.82
2006	25.45	148.32	203.28	134.40	72.20	22.37	5.15
2007	28.59	155.17	205.73	135.97	74.59	22.91	5.19
2008	25.54	197.39	185.99	129.47	66.56	19.28	3.66
2009	35.50	193.43	198.65	137.67	71.26	20.22	3.25
2010	26.70	224.15	160.21	126.70	66.25	18.30	3.00

Table B. The residuals according to univariate ARIMA and the UN age patterns

Year	FR15	FR20	FR25	FR30	FR35	FR40	FR45
1966	-69.56	-58.064	-17.474	30.436	72.224	17.394	33.016
1967	-76.72	-72.78	-16.49	22.43	64.79	13.36	33.92
1968	-76.516	-72.806	-5.158	32.798	74.378	19.998	33.106
1969	-61.344	-78.662	-28.798	14.814	74.076	32.948	39.418
1970	-72.064	-73.674	-36.428	18.122	51.412	13.272	28.854
1971	-62.914	-53.974	5.048	31.342	60.402	17.652	28.974
1972	-62.614	-51.534	-0.512	22.542	52.202	14.732	28.254
1973	-36.444	-26.512	-0.538	15.314	54.606	24.278	33.668
1974	-68.926	-46.756	16.092	31.868	53.468	13.998	26.976
1975	-69.866	-44.736	31.902	31.538	50.488	12.038	27.816
1976	-75.06	-60.56	48.98	36.92	50	9.68	26.93
1977	-59.566	-54.978	32.128	22.996	48.904	17.752	33.422
1978	-63.574	-65.222	31.842	18.734	45.386	14.088	31.068
1979	-61.744	-56.042	56.462	44.004	60.246	21.778	34.028
1980	-83.73	-87.17	23.23	6.79	19.49	-5.56	20.86
1981	-47.326	-61.548	31.458	-0.214	29.474	10.332	28.022
1982	-60.536	-64.126	53.812	24.478	28.988	0.228	20.326
1983	-45.324	-40.772	43.082	3.474	23.216	11.798	24.768
1984	-43.944	-29.222	53.862	22.524	30.246	13.268	23.338
1985	-71.68	-37.39	73.98	39.58	33.72	4.01	21.45
1986	-60.484	-46.212	36.322	14.964	21.556	9.378	23.798
1087	-58.346	-43.638	34.698	8.716	15.884	3.262	20.022
1988	-73.886	-44.176	57.242	32.848	26.588	-5.162	11.036
1989	-49.634	-53.672	10.902	-14.566	-0.924	-10.532	10.458
1990	-36.124	-32.278	33.714	-12.586	-4.746	-13.242	9.414
1991	-25.546	-14.962	41.946	-13.714	-8.354	-13.968	8.446
1992	-37.354	-50.428	-2.116	-50.786	-34.876	-25.662	4.404
1993	-12.612	8.786	53.764	-2.29	0.536	-3.68	8.204
1994	-15.368	5.334	43.476	-11.8	-14.346	-13.27	5.336
1995	-13.322	3.486	44.834	-14.5	-14.144	-12.72	4.904
1996	-10.722	17.436	34.534	-5.03	-6.384	-7.58	5.834
1997	-8.502	17.576	26.264	-14.68	-9.154	-10.4	4.784
1998	-5.248	25.314	25.286	-12.85	-6.136	-8.46	3.856
1999	-4.288	28.574	19.086	-19.97	-10.176	-9.9	3.006

2000	-4.15	28.27	2.4	-20.43	-7.93	-8.05	3.38
2001	-0.938	16.054	-18.934	-27.02	-8.066	-7.57	2.896
2002	3.702	21.064	-8.574	-18.05	6.114	1.06	8.196
2003	7.502	5.984	-4.164	-15.33	5.854	1.06	7.756
2004	8.81	11.28	-5.13	-18.25	4.03	3.35	9.21
2005	9.2	10.99	-0.44	-22.57	0.85	2.13	3.22
2006	11.05	7.32	1.08	-19.2	0.8	5.57	4.55
2007	14.19	14.17	3.53	-17.63	3.19	6.11	4.59
2008	11.14	56.39	-16.21	-24.13	-4.84	2.48	3.06
2009	21.1	52.43	-3.55	-15.93	-0.14	3.42	2.65
2010	9.548	77.206	-48.046	-35.86	-14.774	-4.1	1.464

Table C. The residuals according to multivariate ARIMA

Year	FR15	FR20	FR25	FR30	FR35	FR40	FR45
1968	-3.41481	-3.15773	-12.3284	-5.54803	-0.15464	1.66368	-0.88624
1969	-1.21523	-2.49765	-13.172	-11.4854	-7.54692	-2.81129	-1.78647
1970	2.337114	-0.92515	-15.7559	-14.7237	-21.9741	-7.26748	-2.95727
1971	1.271238	-0.55067	0.82479	5.03169	2.488241	-0.53264	0.06407
1972	0.125124	1.697716	-2.97291	-8.07048	-5.30462	-1.2692	0.239643
1973	-2.88954	8.334733	11.73003	9.560968	8.998429	3.75167	1.068949
1974	-3.0551	-1.49143	6.171422	-1.27904	-5.44651	-2.78167	-1.13282
1975	0.747963	3.432645	12.11743	0.488264	-1.47522	-1.18589	1.344757
1976	-2.16704	-11.674	18.16321	5.250501	3.143362	-0.47949	0.677853
1977	3.17948	10.08639	5.429951	6.040752	5.159115	0.019849	0.545339
1978	0.32661	-0.00969	4.350846	-2.84578	-2.68274	-3.04428	-2.20187
1979	3.744506	10.04653	28.07322	27.77276	18.23695	9.402341	3.29002
1980	5.454265	-21.3117	-30.2376	-33.4275	-27.244	-9.07524	-5.3555
1981	3.651823	-8.68969	3.026116	-8.3791	-2.8678	-3.60944	-0.96477
1982	1.0643	-12.4265	2.278457	0.156143	-4.41183	-2.07826	-0.92524
1983	-8.8896	7.386716	5.457228	4.721366	3.30542	6.000725	-0.93955
1984	-2.90902	13.74264	-9.86269	1.895443	0.196674	-0.77022	-0.17548
1985	1.674948	4.662487	15.66943	10.91028	5.196121	1.486257	3.58733
1986	-1.9438	2.876177	-9.64358	-4.5742	-8.67212	-3.80068	-3.37911
1087	-1.50179	-4.70027	-7.76502	-4.31469	-3.95546	-4.68948	-3.5705
1988	0.859646	0.730761	-4.0108	2.987754	7.279089	1.110583	-2.54928
1989	-0.62067	-18.7104	-25.3237	-26.7329	-20.5796	-11.0167	-6.72433
1990	-0.31783	2.709967	0.047428	-3.93238	-4.68408	-1.98549	-0.11041

1991	1.742642	3.790997	-13.5734	-15.3882	-10.4817	-4.69027	-0.31536
1992	-1.03596	-15.0887	-28.3038	-19.6273	-10.3704	-2.19821	-0.86327
1993	-0.75777	16.25895	13.30019	10.93489	-1.77354	-0.53183	-0.16561
1994	0.852814	-2.16954	-10.2878	-2.44215	-1.29768	-1.2495	-0.21064
1995	0.361061	-4.20845	2.40113	-2.09362	-3.31279	-1.84748	-1.04013
1996	1.699157	12.02282	-11.3878	6.140948	4.352979	2.590453	0.990834
1997	0.030838	-0.15966	-7.16764	-6.18572	-3.66077	-2.74961	-1.5737
1998	0.88913	6.189899	2.660431	-0.79502	-0.77732	-0.41781	-1.30067
1999	0.151005	0.711238	-3.09993	-5.21924	-2.25234	-0.45083	-0.57751
2000	-2.91398	-4.44317	-17.9214	-4.88672	-2.56915	-1.08476	0.19388
2001	1.240599	-9.27087	-25.2567	-6.81781	-4.8785	-1.69066	-1.12034
2002	1.234489	-0.90647	6.004129	4.150021	2.285728	1.444086	3.398935
2003	4.09307	-10.5218	1.514046	-0.74762	-2.44305	-1.22506	-0.06978
2004	3.130164	-3.55102	1.62834	-0.94558	-3.29325	-0.04455	0.36335
2005	-1.3315	-2.58125	-0.56754	-7.08489	-4.30461	-1.89233	-5.25745
2006	1.633121	-13.1452	5.578122	3.697799	3.801619	5.063242	1.649981
2007	2.85701	-0.78242	0.333953	1.890199	1.051862	-1.11892	-0.43342
2008	-4.0039	38.59297	-14.3929	-6.0513	-7.87782	-3.79689	-1.75608
2009	2.199676	0.743163	6.744967	4.296246	2.83786	2.446479	1.344681
2010	-6.54506	17.58377	-22.0307	-2.70702	-0.85999	-1.74023	-1.37341