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DYNAMICS OF KNOWLEDGE DISSEMINATION IN A FOUR-TYPE POPULATION SOCIETY

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Abstract

Knowledge is an important part for organizations, nations and societies to exist, and to grow. Lack of knowledge may make organizations, nations and societies lose their competitiveness. This paper discusses modeling of knowledge dissemination in a society that consists of four types of populations; disseminant, solariant, antusiant and ignorant. The model is in the form of a dynamical system that has two equilibrium points; a society of merely disseminant population and a society of merely antusiant and ignorant populations. The former is a stable point that requires no worry because it attracts the society to be disseminant. The latter is unstable. Considering a starting point of a society, however, there is a possibility that the current condition will be attracted to this unstable point, which drives the society to lose their competitiveness.

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1. Introduction

This paper is motivated by mathematical model of extreme ideology in [1]. The model is to understand the spread of extreme ideology in a closed population. The population is divided into five sub-populations, i.e., virgin sub-population, semi fanatic sub-population, fanatic sub-population, aware sub-population and recovered sub-population. The topic of this paper, however, is the opposite point of view that is knowledge dissemination. This paper is a continuation of preliminary work discussed in [4].

A knowledge is “facts, information, and skills acquired through experience or education; the theoretical or practical understanding of a subject” [13]. Knowledge is an important part for organizations, nations and society to exist, and to grow. Lack of knowledge, more often than not, makes organizations, nations and societies lose their competitiveness, and in some cases becomes ‘extinct’. To avoid that to happen, knowledge management is needed. A knowledge is started and invented because the presence of knowledge originator. “Knowledge dissemination is a crucial part of knowledge management because it ensures knowledge is available to those who need it” [6]. Knowledge dissemination schemes will need to consider aspects of knowledge, the choice of dissemination agents who carry out knowledge dissemination, and their own choices of the mechanisms they use to disseminate knowledge in its various forms, [5]. Dissemination knowledge among a society should consider knowledge transfer person to person. Paulin and Suneson in [9] discussed knowledge transfer and knowledge sharing. Knowledge transfer has been studied broadly especially for business organizations [3].

One may also compare the knowledge dissemination with the spread of rumor. A rumor is “an unverified account or explanation of events circulating from person to person and pertaining to an object, event, or issue in public concern” [10]. A rumor is “usually created to defame someone or to spread false information about public events; it does not only infringe upon others’ interests, but also poses a threat to social stability, and it appears when a group tries to make sense of an ambiguous, uncertain, or chaotic situation”

[2]. The spread of rumors often results in financial loss, social instability, and, in extreme cases, even injuries and loss of lives [12].

2. Mathematical Model

A society of four populations is considered, namely disseminant (*D*), solariant (*S*), antusiant (*A*) and ignorant (*I*). The characteristics of these populations are described below.

■ Disseminant: Population of ones who have knowledge and are willing to disseminate their knowledge.

■ Solariant: Population of ones who have knowledge, but are not willing to disseminate their knowledge.

■ Antusiant: Population of ones without knowledge, but they are willing to learn.

■ Ignorant: Population of ones without knowledge and are not willing to learn.

Assumption:

The presence of disseminants will transfer ignorants to antusiants, and antusiants to solariants. The rate of transfer from ignorants to antusiants is β_1. This parameter represents the rate of awareness of the ignorants to learn becoming antusiants. The transfer of antusiants into solariants with the rate of β_2. This parameter is related how fast the antusiants to learn to become solariants. Finally, solariants transfer themselves to disseminants with the rate of α. This parameter may represent rate of awareness of solariants to contribute in making the society better by disseminating their knowledge. In general, it is assumed that $\alpha, \beta_1, \beta_2 > 0$. Otherwise, it will be mentioned.

Figure 1 shows a schematic plot of the model. The dotted arrows show the influence of disseminants to ignorants to become antusiants, and to antusiants to become solariants. The solid arrows represent path of transfers, from ignorants to antusiants, from antusiants to solariants and from solariants to disseminants. Note that there is no transfer back from disseminants to ignorants.

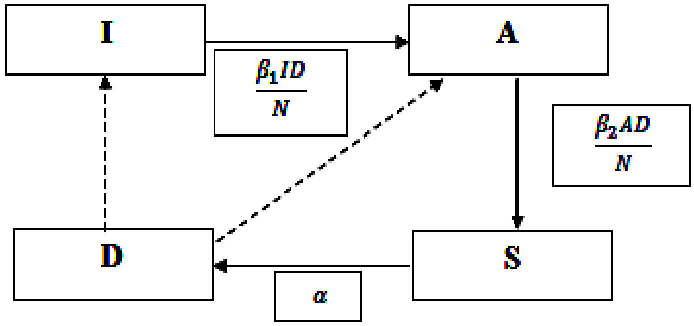


Figure 1. Schematic plot of the knowledge transfer among the populations.

Hence, the model is a system of equations

$$\frac{dI}{dt} = -\frac{\beta_1 ID}{N},$$

$$\frac{dA}{dt} = \frac{\beta_1 ID}{N} - \frac{\beta_2 AD}{N},$$

$$\frac{dS}{dt} = \frac{\beta_2 AD}{N} - \alpha S,$$

$$\frac{dD}{dt} = \alpha S, \tag{1a}$$

where $N = I + A + S + D$. Observe that

$$\frac{dN}{dt} = \frac{dI}{dt} + \frac{dA}{dt} + \frac{dS}{dt} + \frac{dD}{dt} = 0,$$

meaning that N is constant for all time. In short, (1) will be written as

$$\frac{d\boldsymbol{x}}{dt} = f(\boldsymbol{x}), \tag{1b}$$

where $\boldsymbol{x} = (I, A, S, D)$.

One may compare this model with the related models often used for epidemic disease such as SEIZ (susceptible, exposed, infected and skeptic) for four population groups or SEFRA (susceptible, exposed, fanatic, recovered, aware) for five population groups.

3. Analytical Results

In this section, some well-known definitions and theorems are recalled to provide the results. The theorems are not accompanied by the proofs, instead the readers are advised to refer to the related references.

Definition 1. An equilibrium point of the system (1) is $\mathbf{x}^0 = (I^*, A^*, S^*, D^*)$ that satisfies $\frac{dA}{dt} = 0$, $\frac{dI}{dt} = 0$, $\frac{dS}{dt} = 0$ and $\frac{dD}{dt} = 0$.

The model (1) yields two equilibrium points, i.e., $E_1 = (I^*, A^*, 0, 0)$, where

$$I^* + A^* = N,$$

and $E_2 = (0, 0, 0, D^*)$, where

$$D^* = N.$$

The equilibrium E_1 is the society without the presence of disseminants and solariants. On the other hand, the equilibrium E_2 is a society that merely consists of disseminants.

Definition 2. An equilibrium point $\mathbf{x}^0 = (I^*, A^*, S^*, D^*)$ of the system (1) is *stable* if there exists a positive number ε such that if $P = (I(0), A(0), S(0)D(0))$ is in V_ε, where

$$V_\varepsilon = \{(x_1, x_2, x_3, x_4) : | x_1 - I^* | < \varepsilon, | x_2 - A^* | < \varepsilon,$$

$$| x_3 - S^* | < \varepsilon, | x_4 - D^* | < \varepsilon\},$$

then $(I(t), A(t), S(t), D(t))$ remains in V_ε for $t > 0$.

If an equilibrium point is not stable, then it is called *unstable*.

It is more convenient to apply theorem below rather than Definition 2 to investigate the stability of the equilibrium points E_1 and E_2.

Theorem 1. *Consider a dynamical system defined by* (1). *Let* $\boldsymbol{x}^0$ *be an equilibrium point and denote by A the Jacobian matrix of* $f(\boldsymbol{x})$ *evaluated at the equilibrium,* $A = D(f(\boldsymbol{x}^0))$. *Then* $\boldsymbol{x}^0$ *is stable if all eigenvalues* $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ *of A satisfy* $\mathrm{Re}(\lambda) < 0$.

The proof may be found in some standard text books such as [7, 11] or in the original form [8].

Jacobian matrix of system (1) evaluated at point E_1 is

$$A = D(f(E_1)) = \begin{bmatrix} 0 & 0 & 0 & -\frac{\beta_1 I^*}{N} \\ 0 & 0 & 0 & \frac{\beta_1 I^*}{N} - \frac{\beta_2 A^*}{N} \\ 0 & 0 & -\alpha & \frac{\beta_2 A^*}{N} \\ 0 & 0 & \alpha & 0 \end{bmatrix}. \tag{2}$$

Eigenvalues of A defined by (2) are

$$\lambda_1 = 0, \tag{3a}$$

$$\lambda_2 = 0, \tag{3b}$$

$$\lambda_3 = -\frac{1}{2}\alpha + \frac{1}{2}\sqrt{\alpha^2 + \beta_2 4A\alpha/N}, \tag{3c}$$

$$\lambda_4 = -\frac{1}{2}\alpha - \frac{1}{2}\sqrt{\alpha^2 + \beta_2 4A\alpha/N}. \tag{3d}$$

Observe that

$$0 < \frac{1}{2}\alpha < \frac{1}{2}\sqrt{\alpha^2 + \beta_2 4A\alpha/N}. \tag{4}$$

This implies $\lambda_3 > 0$ and $\lambda_4 < 0$. The value of λ_3 shows that E_1 is not a stable point. On the other hand, there exists another eigenvalue which is less than zero, namely $\lambda_4 < 0$. Since $\lambda_4 < 0$, geometrically there is a manifold $W^S(E_1)$, where all points on it which are close enough to E_1 will be attracted to E_1.

On the other hand, Jacobian matrix of system (1) evaluated at point E_2 is

$$A = D(f(E_2)) = \begin{bmatrix} -\frac{\beta_1 D^*}{N} & 0 & 0 & 0 \\ \frac{\beta_1 D^*}{N} & -\frac{\beta_2 D^*}{N} & 0 & 0 \\ 0 & \beta_2 D^* & -\alpha & 0 \\ 0 & 0 & \alpha & 0 \end{bmatrix} \tag{5}$$

which gives eigenvalues

$$\lambda_5 = -\beta_1 D, \tag{6a}$$

$$\lambda_6 = -\beta_2 D, \tag{6b}$$

$$\lambda_7 = 0, \tag{6c}$$

$$\lambda_8 = -\alpha. \tag{6d}$$

Based on Theorem 1, three eigenvalues guarantee the stability of E_2, except $\lambda_7 = 0$. Therefore, the focus is on investigating this eigenvalue and the corresponding eigenvector and eigenspace. Seeking the corresponding eigenvector *u* from

$$A\boldsymbol{u} = \lambda_7 \boldsymbol{u} = \boldsymbol{0}, \tag{7}$$

one has

$$\boldsymbol{u} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}. \tag{8}$$

Let E^c be the linear generalized eigenspace of A which corresponds to $\lambda_7 = 0$, hence E^c is a hyperline in the form

$$E^c = span(\boldsymbol{u}^T) = \{(0,\ 0,\ 0,\ D) : D \in R\}. \tag{9}$$

E^c is the tangent of the corresponding one-dimensional center manifold W^c at point E_2. It will be shown that W^c is not in the state space X of the system (1), where

$$X = \{(I,\ A,\ S,\ D) : I \geq 0,\ S \geq 0,\ D \geq 0,\ I + A + S + D = N\}.$$

Let $(x_1,\ x_2,\ x_3,\ x_4) \in X$ such that $\boldsymbol{v} = (x_1,\ x_2,\ x_3,\ x_4 - N)^T$ be a unit vector. Hence,

$$(x_1^2 + x_2^2 + x_3^2 + (x_4 - N)^2)^{1/2} = 1.$$

Suppose that W^c is in X. Then dot product

$$(0,\ 0,\ 0,\ 1)^T \cdot (x_1,\ x_2,\ x_3,\ x_4 - N)^T = 1. \tag{10}$$

Observe that

$$(0,\ 0,\ 0,\ 1)^T \cdot (x_1,\ x_2,\ x_3,\ x_4 - N)^T = x_4 - N. \tag{11}$$

Equations (10) and (11) imply that

$$x_1 + x_2 + x_3 + x_4 = N + 1. \tag{12}$$

Equation (12) is contradictory to $(x_1,\ x_2,\ x_3,\ x_4) \in X$.

This shows that the stability of E_2 in X is not determined by W^c, but merely by eigenvalues (6a), (6b) and (6d). Therefore, equilibrium E_2 is stable with respect to the state space X. Geometrically, all points on X which are close enough to E_2 never leave this small 'hyper-region'. Moreover, because some eigenvalues are less than zero, it means that some points in this hyper-region go to E_2 as t tends to infinity.

An interpretation for the equilibrium E_2 can be explained as follows. When a society is dominated by disseminants, then it will remain forever as disseminant society. The society is full with knowledgeable or educated people, which is good for them at present time and the future. Equilibrium E_1, however, is the opposite. When a society is dominated merely by ignorants and antusiants, two things may happen. By the presence of disseminants in ‘enough’ number, the society may leave this condition to increase population of antusiants, solariants and disseminants. In absence of disseminants, in the future, the society will go to a condition where only ignorants and solariants remain to exist.

4. Numerical Results

For the numerical simulation, a closed population N is assumed. This population consists of 40% ignorants, 30% antusiants, 20% solariants and 10% disseminants. Moreover, 4 cases are considered based on the values of β_1 and β_2. These parameters are considered to be important because the value β_1 shows how ignorant the society is, and the value of β_2 shows how fast the society to learn. These for cases are

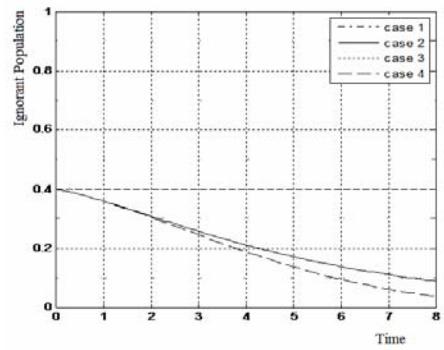
(1) $\beta_1 = 0, \beta_2 > 0,$

(2) $\beta_1 > 0, \beta_2 = 0,$

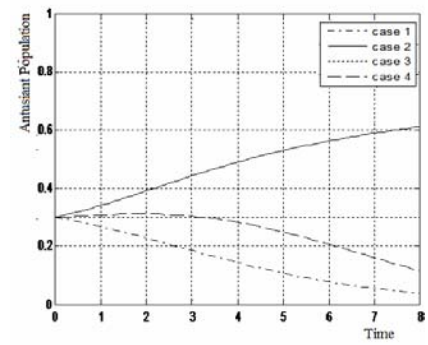
(3) $\beta_1 = 0, \beta_2 = 0,$ and

(4) $\beta_1 > 0, \beta_2 > 0.$

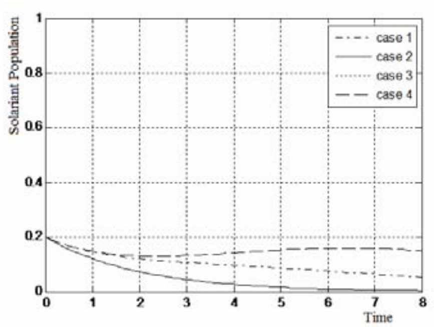
For $\alpha = 0,$ all eigenvalues of Jacobian matrices of both E_1 and E_2 in (2) and (5) are less or equal to zero. Hence, both E_1 and E_2 are stable points. Hence, the transfer rate of solariants to disseminants α is assumed to be nonnegative, in this case, $\alpha = 0.5.$



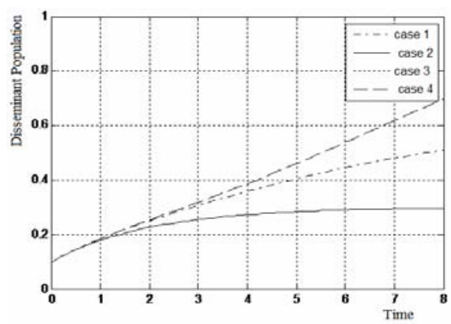
(a) Ignorant population



(b) Antusiant population



(c) Solariant population



(d) Disseminant population

Figure 2. Dynamics of ignorant, antusiant, solariant and disseminant populations.

Simulation of the dynamics of population ignorants, antusiants, solariants and disseminants are given in Figure 2. Since $\beta_1 = 0$ for the cases (1) and (3), the dynamics of ignorant population is constant, Figure 2(a). For case (2) and case (4), the ignorant population, however, decreases. It is because $\beta_1 > 0$.

Figure 2(b) shows the dynamics of antusiant population. Since $\beta_1 = 0$ and $\beta_2 = 0$ for the case (3), the antusiant population for this case is constant. There is no individual transfer from ignorant to antusiant, or from antusiant to solariant. Since $\beta_1 > 0$ and $\beta_2 = 0$ for the case (2), the number of antusiants increases as the effect of the decrease of the ignorant population. For cases (1) and (4), the number of antusiants decreases, meaning that the rate of change from ignorant to antusiant is less than the rate of change from antusiant to solariant.

Figure 2(c) shows that the growth of solariant for the cases (2) and (3) coincides. In all cases, the number of solariants decreases, meaning that the rate of change of solariants to disseminants is larger than the rate of change from antusiants to solariants.

Observe that for all cases, the population of disseminants grows over time, see Figure 2(d). It is because value of α is greater than 0, meaning that the solariants are aware to transform becoming disseminants.

5. Conclusion

Knowledge is important for organizations, nations and society to exist, and to grow. Lack of knowledge may make organizations, nations and societies lose their competitiveness, or may extinct. Mathematical model of knowledge dissemination in a society that consists of four populations has been discussed. The model yields two equilibrium points. One equilibrium consists of merely disseminants, i.e., ones who have knowledge and are willing to transfer their knowledge to others, and the other equilibrium consists of merely ignorants and antusiants, i.e., ones without knowledge. The former equilibrium is stable. If a society is built by a lot of disseminants, then it will turn all members to be disseminants. It is good for the society, since all have knowledge and share to each other. The latter equilibrium, however, is a saddle point. It means that there is possibility a current condition may be attracted to the point where all members of society are ones without knowledge. This condition should be anticipated by top managers of organizations, and leaders of nations and societies.

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