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EFFECT OF RADIATION ON CONVECTIVE MHD FLOW PAST A MOVING VERTICAL POROUS PLATE

S. H. Islam and N. Ahmed

Department of Mathematics
Gauhati University
Guwahati - 781014
Assam, India

Abstract

A theoretical investigation of the problem of the flow past a continuously moving infinite vertical porous plate with uniform suction is obtained. A uniform magnetic field is applied normal to the plate. The equations governing the flow are solved analytically. The results are obtained for velocity field, temperature field and the coefficient of skin friction at the plate in the direction of flow. The influence of various parameters entering into the problem on the above mentioned field is discussed graphically.

Nomenclature

$\vec{q}$

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ρ	fluid density
$\vec{J}$	the current density
p	fluid pressure
B_0	strength of uniform magnetic field
C_p	specific heat at constant pressure
M	Hartmann number
Pr	Prandtl number
Gr	Grashof number
σ^*	Stefan Boltzmann constant
β	coefficient of volume expansion
g	acceleration due to gravity
κ	thermal conductivity
κ^*	mean absorption coefficient
μ	coefficient of viscosity
υ	kinematic viscosity
σ	electrical conductivity
u	component of fluid velocity
T'	fluid temperature
T'_∞	fluid temperature far distance from the plate
v_0	suction velocity
u_w	uniform velocity of the plate

1. Introduction

Theoretical investigation of problems of MHD free convective flow past a moving vertical porous plate has been carried out by several researchers because of its varied and wide applications in MHD pumps, MHD generators, MHD flow meters, MHD flow control (reduction of turbulent drag), astrophysics (planetary magnetic field), geophysics (stars, galaxies), jet printers, etc. Boundary layer behaviour on continuous solid surface was studied by Sakiadis [7]. Prasad et al. [6] analyzed the effect of radiation and mass transfer on two dimensional flow past an infinite vertical plate. The exact solution for MHD boundary layer flow and heat transfer over a continuous moving, horizontal flat surface with uniform suction and internal heat generation/absorption has been studied by Vajravelu [9]. Vajravelu [10] extended his work [9] to consider the flow past a vertical surface instead of a horizontal surface. Narahari and Ishak [3] investigated the radiation effect on free convective flow near a moving vertical plate with Newtonian heating. Chauhan and Rastogi [1] also studied the radiation effect on natural convection on MHD flow in a rotating vertical porous channel partially filled with a porous medium. Perdikis and Rapti [5] studied the MHD unsteady free convection flow in the presence of radiation. An exact solution to the problem of a hydro-magnetic convective flow past a continuously moving vertical surface with uniform suction was presented by Kumar et al. [2].

The present study deals with an exact solution to the problem of MHD steady free convection flow of a viscous, incompressible, electrically conducting and radiating fluid past a moving vertical infinite plate in the presence of uniform magnetic field and appropriate thermal radiation. This work is a generalization of the work done by Kumar et al. [2] to include the effect of radiation. In the absence of the radiation, the present solution is consistent to the solution of Kumar et al. [2].

2. Basic Equations

Equation of continuity:

$$\vec{\nabla} \cdot \vec{q} = 0. \tag{1}$$

Gauss's law of magnetism:

$$\vec{\nabla} \cdot \vec{B} = 0. \tag{2}$$

Ohm's law:

$$\vec{J} = \sigma[\vec{q} \times \vec{B}]. \tag{3}$$

MHD momentum equation:

$$\rho\left[\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \vec{\nabla})\vec{q}\right] = -\vec{\nabla}p + \vec{J} \times \vec{B} - \rho\vec{g} + \mu\nabla^2\vec{q}. \tag{4}$$

Energy equation:

$$\rho C_p\left[\frac{\partial T'}{\partial t} + (\vec{q} \cdot \vec{\nabla})T'\right] = \kappa\nabla^2 T' + \vec{\nabla} \cdot \vec{q}_r. \tag{5}$$

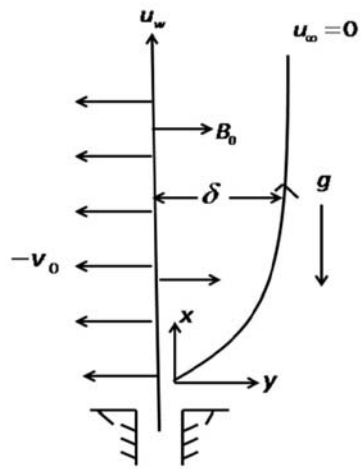

Figure 1. Physical model of the problem.

Consider the steady MHD two dimensional free convective flow of an incompressible viscous electrically conducting fluid past a vertical uniformly moving porous plate. A uniform magnetic field is assumed to be applied normal to the plate and directed into the fluid region and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. The free convective flow from a slot and the plate moving vertically with a uniform velocity u_w and heat is supplied from the plate to the fluid at a uniform rate, in the presence of a uniform magnetic field of strength B_0. We introduce a Cartesian coordinate system (x, y, z) with

x-axis taken vertically upward along the plate, y-axis normal to the plate and z-axis along the width of the plate. The induced magnetic field is assumed to be negligible. Under the above assumptions together with usual boundary layer approximations, the governing equations reduce to:

Momentum equation:

$$-\rho v_0 \frac{du}{dy} = \beta g \rho (T' - T'_\infty) + \mu \frac{d^2 u}{dy^2} - \sigma B_0^2 u. \tag{6}$$

Energy equation:

$$-\rho v_0 C_p \frac{dT'}{dy} = \kappa \frac{d^2 T'}{dy^2} + \left(\frac{16\sigma^* T_\infty'^3}{3\kappa^*} \right) \frac{d^2 T'}{dy^2} \tag{7}$$

subject to the relevant boundary conditions:

$$u = u_w, \ \frac{dT'}{dy} = -\frac{q}{\kappa} \ \text{ at } \ y = 0,$$

$$u \to 0, \ T' \to T'_\infty \ \text{ as } \ y \to \infty.$$

To get the mathematical model normalized, the following dimensionless quantities and similarity parameters are introduced:

$$Y = \frac{y v_0}{\upsilon}, \quad U = \frac{u}{u_w}, \quad T = \frac{T' - T'_\infty}{\left(\frac{q\upsilon}{k v_0} \right)},$$

$$Gr = \frac{\upsilon g \rho \left(\frac{q\upsilon}{k v_0} \right)}{u_w v_0^2}, \quad Pr = \frac{\mu C_p}{\kappa}, \quad M = \frac{\sigma B_0^2 \upsilon}{\rho v_0^2}.$$

Equations (6) and (7) get reduced to the following non-dimensional forms:

$$\frac{d^2 U}{dY^2} + \frac{dU}{dY} + GrT - MU = 0, \tag{8}$$

$$\frac{d^2T}{dY^2} + L\frac{dT}{dY} = 0, \tag{9}$$

where

$$L = \frac{NPr}{1+N}, \quad N = \frac{3\kappa\kappa^*}{16\sigma^* T_\infty'^3}.$$

The corresponding initial and boundary conditions become

$$\left.\begin{array}{l} U = 1, \dfrac{dT}{dY} = -1 \text{ at } Y = 0 \\ U \to 0, T \to 0 \text{ as } Y \to \infty \end{array}\right\}. \tag{A}$$

Solving equations (8) and (9) subject to conditions (A), we derive:

$$T(Y) = \frac{1}{L}e^{-LY},$$

$$U(Y) = (1 + B_2)e^{-B_1 Y} - B_2 e^{-LY},$$

where

$$B_1 = \frac{1}{2}\left[1 + \sqrt{1+4M}\right], \quad B_2 = \frac{Gr}{L(L^2 - L - M)}.$$

3. Skin Friction

The non-dimensional skin friction at the plate in the direction of flow is given by

$$\begin{aligned} \tau &= \left(\frac{dU}{dY}\right)_{Y=0} \\ &= B_2 L - B_1(1 + B_2) \\ &= \frac{Gr}{(L^2 - L - M)} - \frac{1}{2}\left(1 + \sqrt{1+4M}\right)\left[1 + \frac{Gr}{L(L^2 - L - M)}\right]. \end{aligned}$$

4. Results and Discussion

In order to get the physical insight into the problem, we have computed the numerical calculations for dimensionless velocity field, temperature field and skin friction at the plate for different values of the physical parameters involved in the problem which have been demonstrated graphically. In the present investigation, the values for Prandtl number are chosen as .71, 7.0 and .025 which represent air, water and mercury, respectively. The value of Grashof number *Gr* is considered as positive which corresponds for heated plate.

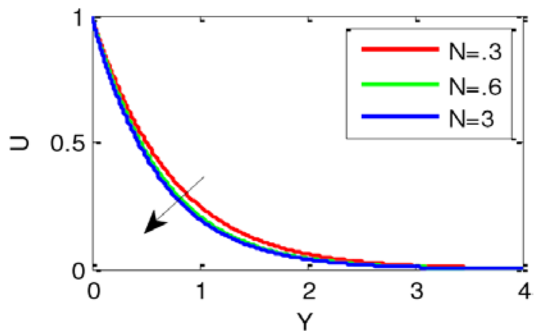

Figure 2. Velocity profile for various values of the radiation parameter *N*, when $Pr = 7$; $Gr = 1$; $M = 1$.

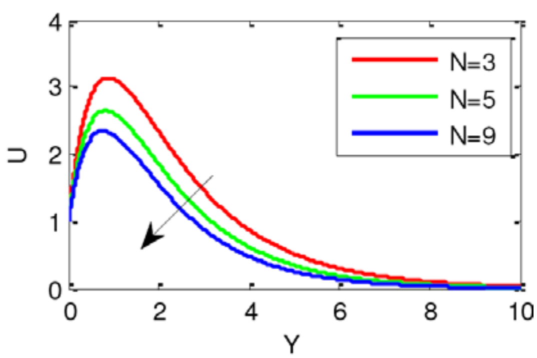

Figure 3. Velocity profile for various values of the radiation parameter *N*, when $Pr = .71$; $Gr = 5$; $M = 1$.

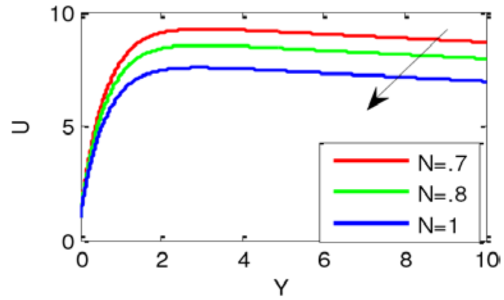

Figure 4. Velocity profile for various values of the radiation parameter *N*, when $Pr = .025$; $Gr = .1$; $M = 1$.

Figure 2-4 present the variation of fluid velocity versus normal coordinate Y under the influence of radiation. All these figures clearly indicate that there is retardation in the fluid velocity under the effect of radiation, irrespective of the fluid being water, air and mercury.

Figure 3 (in case of air) simulates that fluid velocity first increases to its peak value in a thin layer close to the plate and thereafter it falls asymptotically to its minimum values as $Y \to \infty$. In case of mercury, that is from Figure 4, it is seen that fluid velocity first increases in a thin layer adjacent to the plate and thereafter it slowly and steadily decreases as we move far away from the plate. It is inferred from Figure 2 that, in case of water, fluid velocity directly falls asymptotically from its maximum values at $Y = 0$ to its minimum values as $Y \to \infty$. That is to say that the buoyancy force has significant effect in case of air and mercury while the effect of buoyancy force on water vapor is not so pronounced near the plate.

Changes of behaviour of temperature against Y under the effect of radiation are demonstrated in Figures 5-7. All the figures uniquely establish the fact that the effect of radiation causes a comprehensive fall in fluid temperature. In this observation, it is immaterial whether the fluid is water, air or mercury.

Further, all the figures suggest that the fluid temperature falls asymptotically as the value of Y increases. Moreover, the same figures indicate that there is a substantial fall in plate temperature under radiation effect.

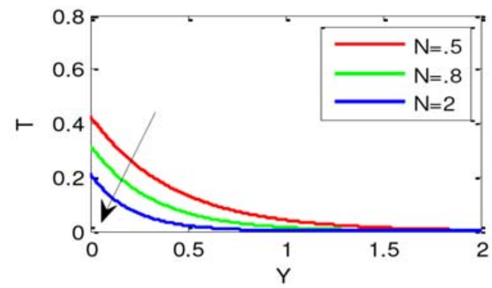

Figure 5. Temperature profile for various values of the radiation parameter N, when $Pr = 7$; $Gr = 1$; $M = 1$.

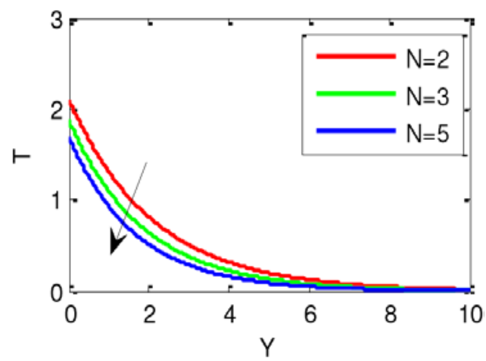

Figure 6. Temperature profile for various values of the radiation parameter N, when $Pr = .71$; $Gr = 1$; $M = 1$.

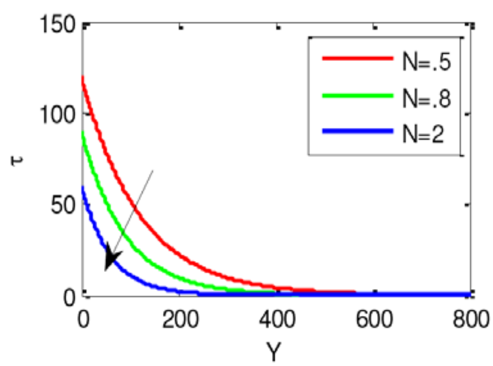

Figure 7. Temperature profile for various values of the radiation parameter N, when $Pr = .025$; $Gr = 1$; $M = 1$.

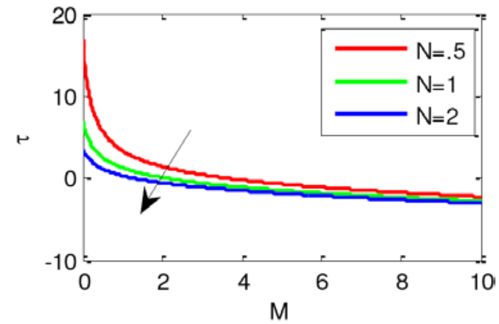

Figure 8. Skin friction profile for various values of the radiation parameter N, when $Pr = .71$; $Gr = 1$.

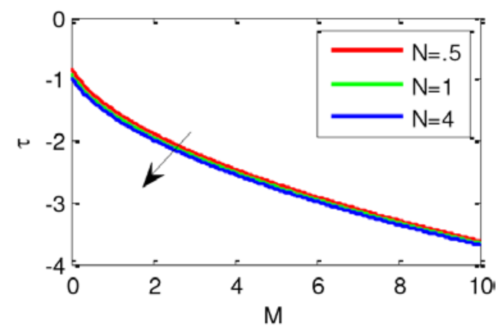

Figure 9. Skin friction profile for various values of the radiation parameter N, when $Pr = 7$; $Gr = 1$.

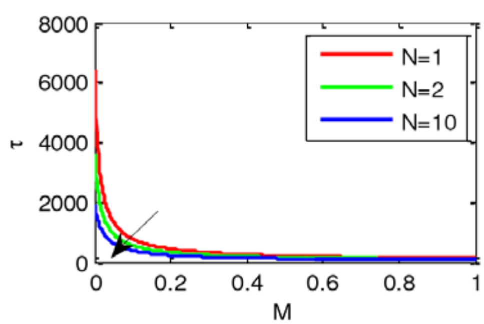

Figure 10. Skin friction profile for various values of the radiation parameter N, when $Pr = .025$; $Gr = 1$.

The influence of radiation on the skin friction at the plate under the magnetic field (M) has been demonstrated in Figures 8-10 for air, water and mercury, respectively.

Figure 8 (in case of air) exhibits the effect of thermal radiation on the coefficient of skin friction against the Hartmann number (M). It is clearly evident from this figure that the maximum drag force in magnitude is attained in the vicinity of the plate and thereafter it decreases to zero for high intensity magnetic field. Again, Figures 9 and 10 (in case of water and mercury, respectively) illustrate the influence of radiation effect on the skin friction at the plate versus M. It is inferred from these two figures that thermal radiation has a tendency to reduce the skin friction at the plate comprehensively.

Figures 8-10 uniquely establish the fact that the frictional resistance of the fluid on the plate gets reduced under the influence of the magnetic field as well as the thermal radiation.

5. Conclusions

- There is retardation in the fluid velocity under the effect of radiation, irrespective of the fluid being air, water and mercury.

- The effect of radiation causes a comprehensive fall in the fluid temperature.

- The skin friction shows a decelerating trend with the rise of the radiation parameter against magnetic effect.

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