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ANALYSIS OF LOCATION AREA RESIDENCE TIME IN MOBILE CELLULAR NETWORKS

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Abstract

In order to accurately analyze the performance of location registration schemes, a proper mobility model of user equipment (UE) is essential. In previous work by Wang et al. [10], it is asserted that if the cell residence time of UE follows an exponential distribution and a fluid-flow model is adopted, then location area (LA) residence time of UE also follows an exponential distribution. However, this result is incorrect, because the LA residence time of UE, which is composed of various combinations of cell residence times, cannot follow an exponential distribution in general.

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In this study, to prove that Wang et al.'s result is incorrect, we first obtain accurate analytical results for LA residence time for a small LA that is composed of 7 hexagonal cells, and then show that analyzed LA residence time does not follow an exponential distribution. Furthermore, we perform computer simulations for various situations, and then show that, through a goodness-of-fit test, simulated LA residence time cannot follow an exponential distribution.

1. Introduction

In mobile cellular networks, location management (LM) of user equipment (UE) is necessary to connect an incoming call to UE [1, 7]. LM is composed of location registration and paging. *Location registration* is the process that UE uses to register its new LA when it enters a new LA, and *paging* is the process by which the network pages UE over all cells of the registered LA to find it and connect an incoming call to it when an incoming call arrives.

In general, there is a trade-off relation between location registration cost and paging cost; many effective LM schemes have been proposed to optimize these costs on radio channels [1, 5, 7, 11]. To analyze the performance of these LM schemes, it is essential to adopt an adequate model for UE's mobility such as a random-walk model [5, 7, 10] or a fluid-flow model [4, 6].

In Wang et al.'s study [10], it was asserted that if cell residence time follows an exponential distribution and a fluid-flow model is adopted, then LA residence time also follows an exponential distribution.

However, this assertion is intuitively incorrect, since the LA residence time of UE, which is composed of various combinations of cell residence times, cannot follow an exponential distribution in general. Nevertheless, Wang et al. analyzed the performance of movement-based registration (MBR) considering the LA architecture, by using their assertion.

In this paper, we investigate which distribution LA residence time of UE follows, assuming that the cell residence time of UE follows an exponential distribution, by both of analytical method and computer simulations.

Assuming a random-walk mobility model as in Wang et al.'s study, we obtain analytical and simulation results for LA residence time and, through a goodness-of-fit test, show that the characteristics of LA residence time are different from those of an exponential distribution, to prevent other researchers from using this false assertion.

Section 2 introduces some assumptions about mobile cellular networks such as random walk mobility model and LA architecture, and then presents analytical equations for LA residence time. Section 3 verifies the accuracy of the analytical results by computer simulations and presents some numerical results to prove our claim. Finally, Section 4 concludes the paper.

2. Mobility Model and Performance Analysis

Wang et al. [10, pp. 1889-1890] asserted that if cell residence time follows an exponential distribution with mean $\frac{1}{\theta}$ and a fluid-flow model is adopted, then LA residence time also follows an exponential distribution, with mean $\frac{3n^2 - 3n + 1}{\theta(2n - 1)}$, where n is the radius of LA in terms of cells.

However, Wang et al.'s assertion is apparently false, noting that sum of n independent identically distributed (i.i.d.) exponential random variables each having mean $\frac{1}{\lambda}$ follows a gamma distribution with parameters n and λ [8, pp. 35-36] and the LA residence time is composed of various combinations of cell residence times. Nevertheless, Wang et al. ([10], Figure 3 and Figure 5) analyzed the performance of MBR considering the LA architecture, by using embedded Markov chain, which is possible from the exponentially distributed LA residence time.

In this section, we derive analytical equations for LA residence time, given that cell residence time follows an exponential distribution, to show that the distribution of LA residence time is not an exponential distribution.

First, let us introduce some assumptions about mobile cellular networks to obtain an analytical equation for LA residence time. Assume that a mobile cellular network is composed of hexagonal cells of the same size, and the LA is composed of several cells. In this study, as in Wang et al.'s study, it is assumed that a VLR (visitor location register) controls only one LA [10]. In other words, it is assumed that a VLR area is composed of only one LA, but it does not matter in our study. Figure 1 shows a small LA with radius = 2 that is made up of two rings. Cell 1 is included in ring 0, and cells 2 to 7 are included in ring 1. In general, an LA with radius = n is made up of ring 0 to ring $(n-1)$ and the total number of cells is $1+\sum_{i=1}^{n-1} 6i$ $(n \geq 2)$.

In Figure 1, a UE that entered a ring 1 cell from another LA stays in the cell for a time that follows an exponential distribution, and then ① enters another LA or ② enters a ring 0 cell or a ring 1 cell in the current LA. In the case of ②, UE stays in the cell for a time that follows an exponential distribution, and then repeat ① or ②. LA residence time is defined as the time from the instant when UE enters a ring 1 cell from another LA until the instant when it enters another LA [10].

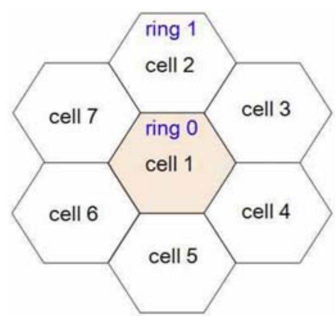

Figure 1. An LA of radius = 2.

The following are defined to analyze the mobility of UE:

n: radius of LA, $n = 1, 2, 3, \ldots,$

K: number of cell crossings of UE that entered a new LA until it enters another new LA,

t_c: cell residence time (follows an exponential distribution with mean $\frac{1}{\theta}$),

t_{LA}: LA residence time of UE that entered a new LA until it enters another new LA,

t_K: LA residence time of UE, given that it resides in an LA with K cell crossings,

$p(K)$: probability that a UE that entered a new LA will enter another new LA with K cell crossings.

Let us obtain $p(K)$ for $n = 2$. The possible events of UE with 1 cell crossing are as follows:

A: UE in a ring 1 cell moves into another LA,

B: UE in a ring 1 cell moves into another ring 1 cell in the same LA,

C: UE in a ring 1 cell moves into a ring 0 cell in the same LA,

D: UE in the ring 0 cell moves into a ring 1 cell in the same LA.

The probability for each event is as follows:

$$P[A] = \frac{3}{6} = \frac{1}{2},$$

$$P[B] = \frac{2}{6} = \frac{1}{3},$$

$$P[C] = \frac{1}{6},$$

$$P[D] = 1.$$

Since in the event, with 1 cell crossing, UE in a ring 1 cell moves into another LA, is made up of only one case (A), $p(1)$ can be obtained easily as follows:

$$p(1) = P(A) = \frac{1}{2}. \tag{1}$$

Since in the event, with 2 cell crossings, UE in a ring 1 cell moves into another LA, is made up of only one case $(B \to A)$, $p(2)$ can be obtained as follows:

$$p(2) = P(BA) = P(B)P(A) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}, \tag{2}$$

where $P(B \to A)$ is expressed by $P(BA)$ for convenience.

Since in the event, with 3 cell crossings, UE in a ring 1 cell moves into another LA, is made up of two cases $(B \to B \to A$ and $C \to D \to A)$, $p(3)$ can be obtained as follows:

$$p(3) = P(BBA) + P(CDA) = \left(\frac{1}{3}\right)^2 \times \frac{1}{2} + \frac{1}{6} \times 1 \times \frac{1}{2} = \frac{5}{36}. \tag{3}$$

In a similar way, $p(4)$ and $p(5)$ can be obtained as:

$$p(4) = P(BBBA) + \binom{2}{1} P(CDBA) = \left(\frac{1}{3}\right)^3 \times \frac{1}{2} + \binom{2}{1} \times \frac{1}{6} \times 1 \times \frac{1}{3} \times \frac{1}{2}, \tag{4}$$

$$\begin{aligned} p(5) &= P(BBBBA) + \binom{3}{1} P(CDBBA) + \binom{2}{2} P(CDCDA) \\ &= \left(\frac{1}{3}\right)^4 \times \frac{1}{2} + \binom{3}{1} \times \frac{1}{6} \times 1 \times \left(\frac{1}{3}\right)^2 \times \frac{1}{2} + \binom{2}{2} \times \left(\frac{1}{6} \times 1\right)^2 \times \frac{1}{2}. \end{aligned} \tag{5}$$

Now, let us obtain a general equation of $p(K)$ for $K = 1, 2, \ldots$.

Defining π_i as the probability that, among K cell crossings, i cell crossings $(K > 2i)$ occur from a ring 1 cell to the ring 0 cell, π_i can be obtained as

$$\pi_i = \binom{K - i - 1}{i} \left(\frac{1}{6} \times 1\right)^i \times \frac{1}{2} \times \left(\frac{1}{3}\right)^{K-2i-1}. \tag{6}$$

In the above, the range of i depends on K, because, to move into a new LA, UE must go through a ring 1 cell in the current LA; so the range of i is $0 \le i \le \frac{K}{2} - 1$ if K is even, and $0 \le i \le \frac{K-1}{2}$ if K is odd. In other words, the range of i is $0 \le i \le \left\lfloor \frac{K-1}{2} \right\rfloor$ for $K = 1, 2, ..., \lfloor x \rfloor$, where $\lfloor x \rfloor$ means the largest integer that is less than or equal to x.

Finally, the general equation of $p(K)$ is

$$p(K) = \sum_{i=0}^{\left\lfloor \frac{K-1}{2} \right\rfloor} \pi_i$$

$$= \sum_{i=0}^{\left\lfloor \frac{K-1}{2} \right\rfloor} \binom{K-i-1}{i} \left(\frac{1}{6} \times 1\right)^i \times \frac{1}{2} \times \left(\frac{1}{3}\right)^{K-2i-1}, \ K = 1, 2, 3, ... \quad (7)$$

Now, we can get the expected number of cell crossings when UE entered the current LA until it moves into a new LA:

$$E[K] = \sum_{K=1}^{\infty} Kp(K). \quad (8)$$

When the mean of the cell residence times is $\frac{1}{\theta}$, the mean of the LA residence time t_{LA} is obtained as

$$E[t_{LA}] = \frac{E[K]}{\theta} = \sum_{K=1}^{\infty} Kp(K) \times \frac{1}{\theta}. \quad (9)$$

Let t_K be the LA residence time of UE, given that it resides in an LA with K cell crossings. Then t_K is the sum of K cell residence times (each follows an exponential distribution with mean $\frac{1}{\theta}$), and so t_K follows a gamma distribution with shape parameter K and scale parameter $\frac{1}{\theta}$, gamma $\left(K, \frac{1}{\theta}\right)$ [8]. As a result, the mean and variance of t_K $(K \ge 1)$ can be obtained easily:

$$E[t_K] = \frac{K}{\theta}, \tag{10}$$

$$V[t_K] = \frac{K}{\theta^2}. \tag{11}$$

Using

$$E[t_K^2] = V[t_K] + (E[t_K])^2 = \frac{K(K+1)}{\theta^2}, \quad K = 1, 2, 3, ..., \tag{12}$$

we can get the variance of t_{LA}:

$$V[t_{LA}] = E[t_{LA}^2] - (E[t_{LA}])^2 = \sum_{k=1}^{\infty} p(K) E[t_K^2] - \left(\frac{E[K]}{\theta}\right)^2$$

$$= \frac{1}{\theta^2}\left\{\sum_{K=1}^{\infty} K(K+1)p(K) - (E[K])^2\right\}. \tag{13}$$

3. Numerical Results

To verify the analytical result, computer simulations are performed by RAPTOR [3]. Figure 2 shows a flow chart for our computer simulations:

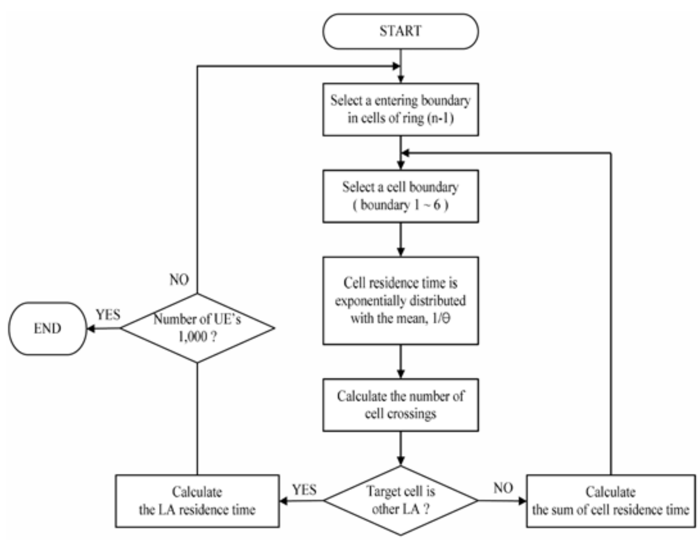

Figure 2. Simulation model.

In our computer simulations, the following are considered:

- UE enters a cell in ring $(n-1)$.

- UE stays in the cell during an exponentially distributed time.

- Whenever UE moves into one of 6 neighboring cells, its number of cell crossings and LA residence time are updated.

- Whenever UE moves into a cell, it is checked whether the cell is in new LA. If so, then its number of cell crossings and LA residence time are stored.

- The above process is repeated for 1,000 UEs.

- The above simulations are performed 10 times, and the final results for the LA residence time for 10,000 UEs are obtained.

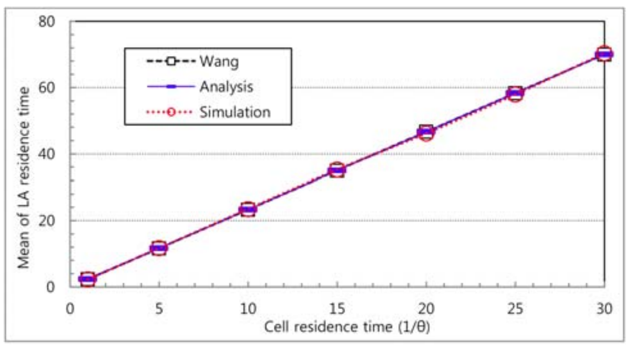

Figure 3.1. Mean of the LA residence times (radius = 2).

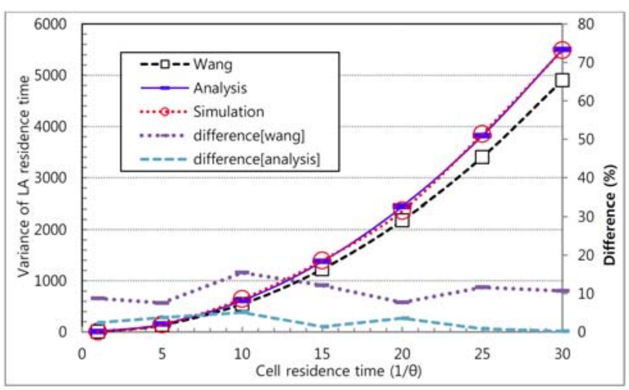

Figure 3.2. Variance of the LA residence time (radius = 2).

Figure 3

Figure 3 shows the mean and variance of LA residence times for radius = 2. In the figure, three cases of results by Wang et al.'s method, analytical method, and simulation method are shown. We can see that the three methods give the same mean, but Wang et al.'s method gives variance different from that of the analytical method and simulation method. Note that, in Figure 3.2, the difference ratio of our analytical method to the simulation method $\left(\frac{|\text{Analysis} - \text{Simulation}|}{\text{Simulation}}\right)$ is just 2.47%, but the difference ratio of Wang et al.'s method to the simulation method $\left(\frac{|\text{Wang} - \text{Simulation}|}{\text{Simulation}}\right)$ is 10.57%.

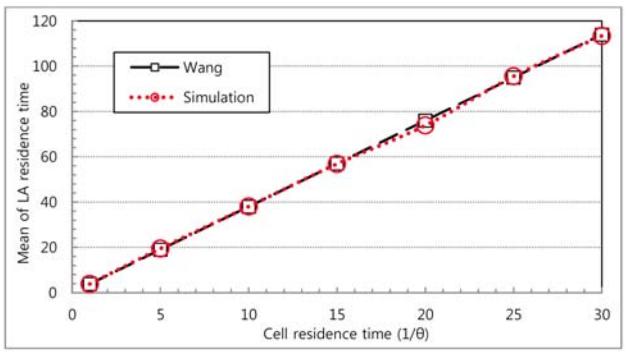

Figure 4.1. Mean of the LA residence times (radius = 3).

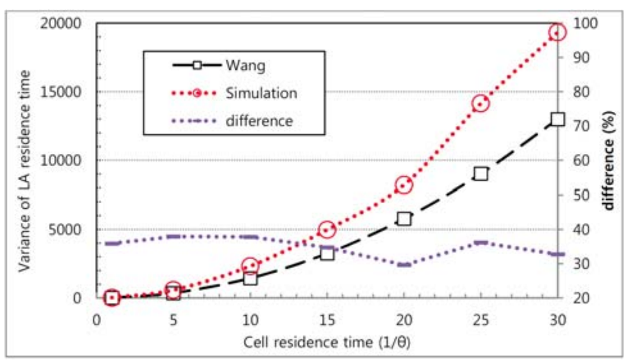

Figure 4.2. Variance of the LA residence time (radius = 3).

Figure 4

Figure 4 shows the mean and variance of LA residence times for radius = 3. We can see that Wang et al.'s method gives a variance different from that of the simulation method. In Figure 4.2, the difference ratio of Wang et al.'s method to the simulation method $\left(\frac{|\,\text{Wang} - \text{Simulation}\,|}{\text{Simulation}}\right)$ is 35.00%.

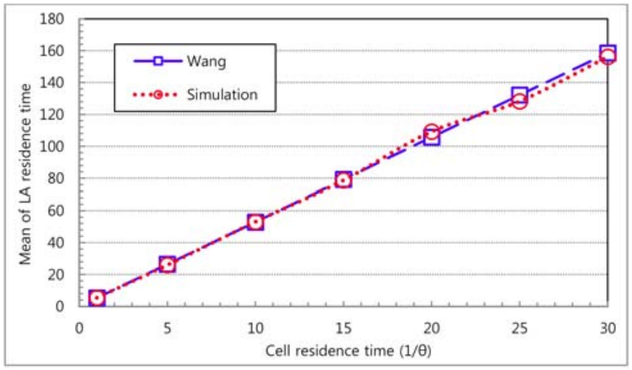

Figure 5.1. Mean of the LA residence times (radius = 4).

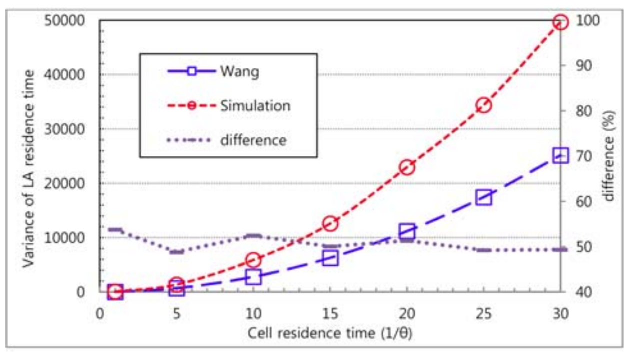

Figure 5.2. Variance of the LA residence time (radius = 4).

Figure 5

Figure 5 shows the mean and variance of LA residence times for radius = 4. We can also see that Wang et al.'s method gives a variance different from that of the simulation method. In Figure 5.2, the difference ratio of Wang et al.'s method to the simulation method $\left(\frac{|\,\text{Wang} - \text{Simulation}\,|}{\text{Simulation}}\right)$ is fully 50.71%.

From the above results, we can see that Wang et al.'s method gives a variance different from that of the analytical method and simulation method, and the difference ratio of Wang et al.'s method to the simulation method $\left(\frac{|\text{Wang} - \text{Simulation}|}{\text{Simulation}}\right)$ increases steeply as the number of cells in an LA increases. In conclusion, Wang et al. asserted that, if the cell residence time of UE follows an exponential distribution and a fluid-flow model is adopted, then LA residence time of UE also follows an exponential distribution is incorrect.

However, to test again if LA residence time follows an exponential distribution, a Kolmogorov-Smirnov (K-S) test is performed; that is, a test for distribution of a continuous random variable. Table 1 shows the results of the K-S test, performed for the case of radius = 2, for which the difference ratio is the smallest. According to the test results, the K-S test statistic for exponential distribution is less than significant (5%) in every case; so it is very difficult to say that LA residence time follows an exponential distribution [2, 9].

Table 1. Kolmogorov-Smirnov test statistic for exponential distribution (radius = 2)

Average cell residence time $(1/\theta)$	1	5	10	15	20	25	30
K-S test statistic	2.899	2.242	3.039	3.085	2.926	3.079	2.487
Asymptotic significance probability (2-tailed)	0.000*						

*Null hypothesis is rejected, since 0.000 is less than the significance level of 0.05 (H_0 : simulation results follow an exponential distribution)

In fact, according to the coefficient of variance δ for the simulation results, LA residence time shows some characteristics of gamma distribution or Weibull distribution rather than exponential distribution, especially as the number of cells in an LA increases (as radius increases).

4. Conclusions

In previous work by Wang et al., it was asserted that, if cell residence time of UE follows an exponential distribution and a fluid-flow model is adopted, then the location area (LA) residence time of UE also follows an exponential distribution. However, this result is apparently incorrect, because LA residence time of UE, which is composed of various combinations of cell residence times, cannot follow an exponential distribution.

To disprove that LA residence time follows an exponential distribution, we first obtained accurate analytical results for LA residence time for a small LA that is composed of 7 hexagonal cells, and then showed that the obtained LA residence time does not follow an exponential distribution. Furthermore, we performed computer simulations for various sized LAs, and then showed that, through a goodness-of-fit test, simulated LA residence time cannot follow an exponential distribution.

In conclusion, Wang et al. asserted that, if the cell residence time of UE follows an exponential distribution and a fluid-flow model is adopted, then LA residence time of UE also follows an exponential distribution, is incorrect. Therefore, it is necessary to obtain an accurate distribution of LA residence time in order to analyze the performance of location registration schemes in the network that is composed of VLRs. Further study will be performed for accurate distribution of LA residence times and the performance of location registration schemes considering network architecture.

Acknowledgement

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