Advances and Applications in Fluid Mechanics

http://www.pphmj.com
http://dx.doi.org/10.17654/FM020010127
Volume 20, Number 1, 2017, Pages 127-139 **ISSN: 0973-4686**

SUCTION POTENTIAL AND WATER ABSORPTION FROM PERIODIC CHANNELS IN A HOMOGENEOUS SOIL WITH DIFFERENT ROOT UPTAKES

Imam Solekhudin

Department of Mathematics
Faculty of Mathematics and Natural Sciences
Universitas Gadjah Mada
Indonesia

Abstract

A problem involving steady water infiltration into a homogeneous soil with water absorption by plant roots is governed by Richards equation. In this paper, four different types of root-water uptake are considered. To study this equation more conveniently, the governing equation is transformed into a modified Helmholtz equation. The modified Helmholtz equation is then solved numerically using a dual reciprocity boundary element method (DRBEM) with a predictor-corrector scheme. Using the numerical solutions, water absorption by plant roots can be obtained.

1. Introduction

Water infiltration in homogeneous soils has been studied by numerous researchers. Some of such researchers are Batu [3], Lobo et al. [7], Clements and Lobo [5] and Solekhudin and Ang [8-10]. Batu studied infiltration problems from single and periodic flat channels [3]. Lobo et al. examined

Received: August 28, 2016; Accepted: January 2, 2017

2010 Mathematics Subject Classification: 76M15.

Keywords and phrases: water infiltration, root-water uptake, DRBEM, predictor-corrector.

infiltration from irrigation channels with impermeable inclusions [7]. Time dependent infiltration from a semicircular channel has been discussed by Clements and Lobo [5]. Infiltration from different geometries of channels has been examined by Solekhudin and Ang [9]. Solekhudin and Ang [8, 10] also studied problems involving steady infiltration as well as time dependent infiltration from periodic trapezoidal channels with water absorption by plant roots. However, in these studies, water absorption by different types of root uptake was not taken into account.

In this paper, we investigate water absorption by plant roots with different root uptake. This study is a continuation of our previous study in [8]. For completeness of this paper and the convenience of readers, some basic equations and methods described in our previous study are reproduced.

2. Problem Formulation

Using a Cartesian coordinate system $OXYZ$ with OZ vertically positive downwards, we consider a homogeneous soil, Pima clay loam (PCL), in the region $Z \geq 0.$ For every unit length in the OY direction, the channel has a sunken surface area of $2L$ square units. Between two adjacent channels, a row of crops, with roots of depth Z_m and width $2X_m$, are planted. The distance between two consecutive rows of crops is $2(L + D)$. This description is illustrated in Figure 1:

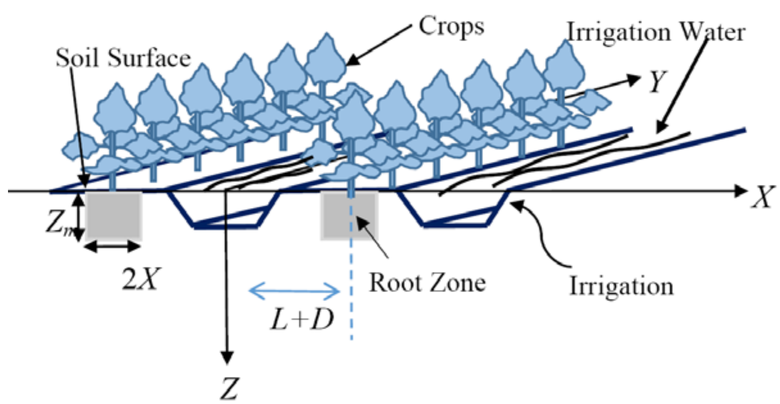


Figure 1. Periodic trapezoidal channels with root zones.

The channels are filled with water. It is assumed that the geometries of the channels and the root distribution do not vary in the OY direction and are symmetrical about the planes $X = \pm k(L + D)$ for $k = 0, 1, 2, \ldots$. Supply of water is only from the channels in uniform fluxes, v_0. Given this situation, we wish to determine water absorption or water uptake by different types of root distribution and root uptake from the homogeneous soil stated above. The types of root uptake are adopted from Vrugt et al. [12].

Due to the symmetry of the problem, there will be no flux across the plane $X = \pm k(L + D)$. Hence, it is sufficient to consider a semi-infinite region bounded by $0 \leq X \leq L + D$ and $Z \geq 0$. This region is represented by R bounded by C. From the assumption above, flux across the surface of channel is v_0, and flux across the surface of soil outside the channel is 0. Fluxes across $X = 0$ and $X = L + D$ are 0. The derivatives $\partial\Theta/\partial X \to 0$ and $\partial\Theta/\partial Z \to 0$ as $X^2 + Z^2 \to \infty$ [3].

3. Basic Equations

In this section, we briefly derive a modified Helmholtz equation from the governing equation of steady water infiltration. The modified Helmholtz equation is then solved numerically using a dual reciprocity method (DRM) with a predictor-corrector scheme. For the detail of the derivation and the method used, readers may refer to paper [8]. The governing equation of steady infiltration with root-water uptake is the Richards equation

$$\frac{\partial}{\partial X}\left(K\frac{\partial \psi}{\partial X}\right) + \frac{\partial}{\partial Z}\left(K\frac{\partial \psi}{\partial Z}\right) - \frac{\partial K}{\partial Z} = S(X, Z, \psi), \tag{1}$$

where K is the hydraulic conductivity, ψ is the suction potential and S is the root-water uptake function, which also can be written as

$$S(X, Z, \psi) = \gamma(\psi)S_m(X, Z), \tag{2}$$

here γ is the water stress response function and S_m is the normalized root-water uptake distribution [12]. Using the dimensionless matric flux potential (MFP) Θ,

$$\Theta = \int_{-\infty}^{\psi} K(t)dt,$$

exponential relationship between K and ψ,

$$K = K_s e^{\alpha\psi},$$

and dimensionless variables

$$x = \frac{\alpha}{2}X;\ z = \frac{\alpha}{2}Z;\ \Phi = \frac{\pi\Theta}{v_0 L};\ u = \frac{2\pi}{v_0\alpha L}U;\ v = \frac{2\pi}{v_0\alpha L}V;\ f = \frac{2\pi}{v_0\alpha L}F,$$

suction potential ψ can be written in terms of α, Θ and K_s as

$$\psi = \frac{1}{\alpha}\ln\left(\frac{\alpha\Theta}{K_s}\right), \tag{3}$$

and equation (1) can be written as

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial z^2} - 2\frac{\partial\Phi}{\partial z} = \gamma^*(\Phi)s^*(x, z), \tag{4}$$

where $\gamma^*(\Phi)s^*(x, z)$ is the dimensionless root-water uptake function. The flux normal in terms of Θ is

$$F = -\frac{\partial\Theta}{\partial X}n_1 + \left(\alpha\Theta - \frac{\partial\Theta}{\partial Z}\right)n_2.$$

Using the transformation

$$\Phi = \phi e^z,$$

equation (4) can be written as

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2} - \phi = \gamma^*(\phi)s^*(x, z)e^{-z}. \tag{5}$$

Equation (5) is a modified Helmholtz equation. To solve the infiltration problems stated above, we solve equation (5) subject to boundary conditions:

$$\frac{\partial \phi}{\partial n} = \frac{2\pi}{\alpha L} e^{-z} - n_2 \phi \text{ on the surface of the channel,}$$

$$\frac{\partial \phi}{\partial n} = -\phi \text{ on the surface of soil outside the channel,}$$

$$\frac{\partial \phi}{\partial n} = 0, \; x = 0 \text{ and } z \geq 0,$$

$$\frac{\partial \phi}{\partial n} = 0, \; x = b \text{ and } z \geq 0,$$

$$\frac{\partial \phi}{\partial n} = -\phi, \; 0 \leq x \leq b \text{ and } z = \infty,$$

using a DRM and a predictor-corrector scheme, numerical values of ϕ can be obtained. From the values of ϕ, numerical values of ψ and S can be computed.

4. Results and Discussion

In this section, some numerical results of suction potential and water absorption or root-water uptake are presented for steady infiltration from periodic trapezoidal channels with four different types of root-water uptake. In this study, we set $L = D = 50\,\text{cm}$, and the width and the depth of the channels are $4L/\pi$ and $3L/2\pi$, respectively. The potential transpiration rate, T_{pot}, is 4cm/day, which was used by Li et al. in their studies [6]. The homogeneous soil considered in the present study is Pima clay loam (PCL). The values of experimental parameters α and K_s of the soil are $0.014\,\text{cm}^{-1}$ and 9.9cm/day. These values are as reported by Amoozegar-Fard et al. [1] and Bresler [4]. Again, the normal flux over the surface of the channels is assumed to be constant, $F = -v_0$. Following Basha [2], the value of v_0 in this study is $0.75K_s$.

We assume that root zones have the same width and depth of 100cm. We consider four different types of root uptake. The types of the roots are named Root A, Root B, Root C and Root D. Root A has high density at the surface of the soil as $Z^* = 0$. As the roots go deeper into the soil, their density decreases to zero in *OZ* direction. In the direction opposite to *OX*, this root also has similar pattern as $X^* = 0$. Root B has similar fashion as Root A in the direction opposite to *OX* direction. However, in *OZ* direction, the root density increases until it reaches a limit at the depth of 20cm below the surface of soil. From this level to the end point of the root zone, the root density decreases to zero. Following the description for Root A and Root B, the density pattern of Root C and Root D can be determined using the fitting parameters in Table 1. These fitting parameters are adopted from Vrugt et al. [12].

Table 1. Fitting parameters of four different types of root uptake

Root type	Fitting parameters					
	Z_m	X_m	Z^*	X^*	P_Z	P_X
Root A	100cm	50cm	0cm	0cm	1.0	1.0
Root B	100cm	50cm	20cm	0cm	1.0	1.0
Root C	100cm	50cm	0cm	25cm	1.0	4.0
Root D	100cm	50cm	20cm	25cm	5.0	2.0

The root-water stress response function γ used here is identical to that reported by Utset et al. [11], which can be seen graphically in Figure 2. The value of h_3 for $T_{pot} = 0.4\,\text{cm/day}$ is interpolated from $h_{3,a}$ and $h_{3,b}$, and we have $h_3 = -470$.

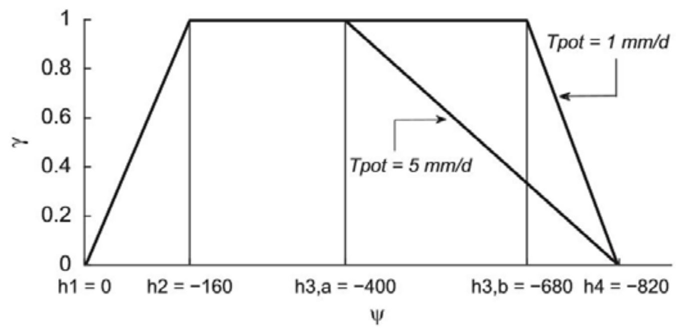
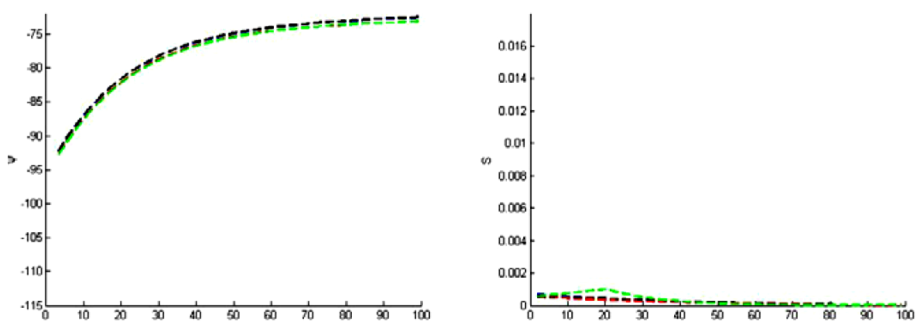


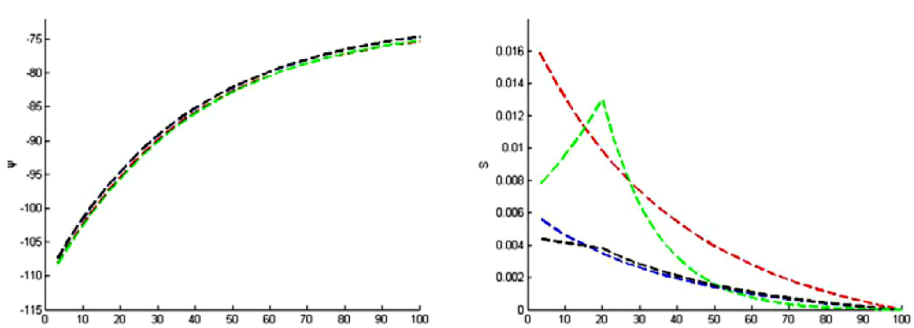
Figure 2. Graph of root-water stress response function reported by Utset et al. [11].

The DRBEM with the predictor-corrector scheme is employed to obtain numerical solutions to equation (5). Using the numerical solutions, numerical values of suction potential, ψ, may be computed employing equation (3). Substituting ψ to equation (2) yields values of root uptake function, S. To implement the DRBEM, the domain must be bounded by a simple closed curve. The domain is set to be between $z = 0$ and $z = 4$, sufficient depth for boundary conditions to be applied without significant impact to values of Φ in the domain. The number of line segments on the boundary is 202, and interior points chosen as collocation points are 619 points. These numbers are chosen in such a way that an optimum computational time and the convergence of the values of Φ are achieved after several computational experiments. Some of the results are presented graphically in Figures 3 and 4.

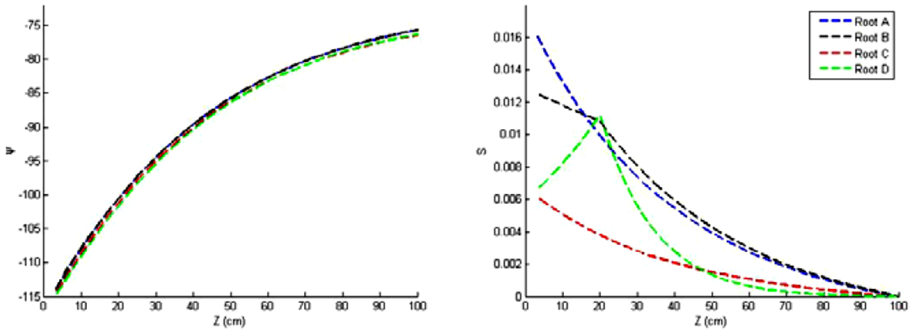
Figure 3 shows values of suction potential and corresponding values of root uptake at selected values of X along the root zone for four different types of root uptake. Specifically, Figure 3(a) shows ψ and S at $X = 55\,\text{cm}$. Graph of ψ and S at $X = 75\,\text{cm}$ and $X = 95\,\text{cm}$ are shown in Figures 3(b) and 3(c), respectively. It can be seen that values of ψ in the soil with Root A or Root B are higher than those with Root C or Root D. This means that higher decrease in ψ occurs when crops with Root C or Root D are planted.



(a) $X = 55\text{cm}$

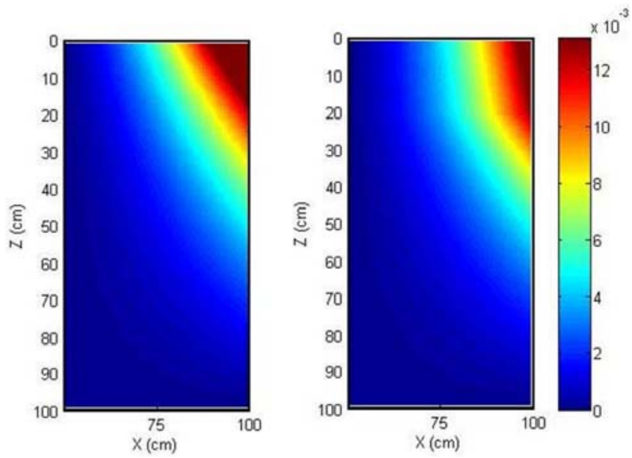


(b) $X = 75\text{cm}$

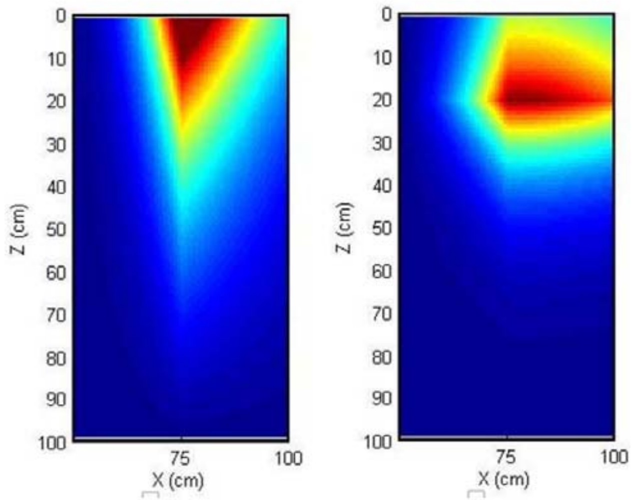


(c) $X = 95$cm

Figure 3. Suction potential and corresponding root-water uptake function at selected values of X along the root zone.



(a) Root A (b) Root B



(c) Root C (d) Root D

Figure 4. Distribution of root-water uptake over the root zone.

At a fixed value of Z, highest value of ψ is at $X = 55\,\mathrm{cm}$. This result indicates that at a fixed value of Z, higher values of ψ are at area nearer the channels. It can also be seen that ψ increases as Z increases, due to the assumption that there is no flux across the surface of soil outside the channels. Since the three values of X are located outside the channels, the surface of soil (Z near to zero) is the driest part of the soil. Driest soil results in lowest ψ.

Values of root-water uptake function S vary in X and root-water uptake types. At $X = 55\,\mathrm{cm}$, area near the end of the root zone, values of root-water uptake S are close to zero. This result is as predicted, as the density of the roots at this area is almost zero. At $X = 75\,\mathrm{cm}$ at the surface of soil, Root C and Root D result in higher uptake than Root C and Root D. It can also be seen that at the surface of the soil, water uptake by plants with Root C is the highest, as $X^* = 25\mathrm{cm}$ and $Z^* = 0\mathrm{cm}$. This means that the highest uptake occurs on the surface of the channels at the distance 25cm from the crops.

For Root D, it can be seen that maximum uptake occurs at $Z = 20\text{cm}.$ This indicates that the maximum uptake occurs at location 25cm from the crops and 20cm below the surface of soil.

At $X = 95\text{cm},$ location near the crops, values of S from Root A and Root B are higher than Root C and Root D, at the surface of soil. These results are expected, as $X^* = 0\text{cm},$ which means that maximum uptakes are at location nearest the crops. As before, since values of Z^* for Root A and Root B are 0cm and 20cm, respectively, maximum uptakes of Root A and Root B are at the surface of the soil and at location 20cm below the surface of soil, respectively.

Figure 4 shows surface plot of S over the root zone. It can be seen clearly that maximum uptakes of Root A and Root B are at $X = 100\text{cm},$ as $X^* = 0\text{cm}.$ The uptake getting lower as X goes further from 100cm. A similar pattern also occurs as Z goes deeper. For Root C and Root D, maximum uptakes occur at $X = 75\text{cm},$ as $X^* = 25\text{cm}.$ The uptake getting lower whenever X getting further from $X = 75\text{cm}.$ While Root C has optimum uptake on the surface of soil, Root D has optimum uptake at a level 20cm below the surface of soil. These results are as predicted, as the values of Z^* for Root C and Root D are 0cm and 20cm, respectively.

From the results, we can examine the difference in the total amount of water absorbed between the root types quantitatively from the root zone. This root zone with length of 100cm along OY direction is considered. The total amount of water absorbed from the zone is computed using formula

$$100 \times \int_0^{100} \int_{50}^{100} S(X, Z, \psi) dXdZ.$$

Since $\gamma(\psi)$ may not be determined analytically, the formula is computed numerically. To do so, each zone is divided into 100×100 rectangular regions with breadth Δx and length Δz. For the fixed rectangular region, the value of S is a constant. Using this numerical method, numerical values of water absorbed by the plant roots with different uptake are summarized in Table 2:

Table 2. Water absorbed from the soil by different types of root uptakes

	Root type			
	Root A	Root B	Root C	Root D
Amount of water absorbed (cm^3/day)	1192.34970	1182.72245	1168.69895	1206.92375

From Table 2, it can be seen that Root D absorbs more water than the other types of root uptake. We can also see that the lowest uptake occurs when crops with Root C are planted.

5. Concluding Remarks

A problem involving steady infiltration from periodic trapezoidal channels with root-water uptake of different types of root uptake was considered. The problem was solved numerically using a dual reciprocity method (DRM) with a predictor-corrector scheme. The method was employed to obtain numerical values of suction potential and water absorption by plant roots.

The results obtained indicate that maximum uptake occurs at the densest part of the root zone. Highest water absorption in this study occurs when crops with Root D are planted.

References

[1] A. Amoozegar-Fard, A. W. Warrick and D. O. Lomen, Design nomographs for trickle irrigation system, J. Irrig. Drain. Eng. 110 (1984), 107-120.

[2] H. A. Basha, Multidimensional linearized nonsteady infiltration toward a shallow water table, Water Resour. Res. 36 (2000), 2567-2573.

[3] V. Batu, Steady infiltration from single and periodic strip sources, Soil Science Society of America Journal 42 (1978), 544-549.

[4] E. Bresler, Analysis of trickle irrigation with application to design problems, Irrig. Sci. 1 (1978), 3-17.

[5] D. L. Clements and M. Lobo, A BEM for time dependent infiltration from an irrigation channel, Engineering Analysis with Boundary Elements 34 (2010), 1100-1104.

[6] K. Y. Li, R. D. Jong and J. B. Boisvert, An exponential root-water-uptake model with water stress compensation, J. Hydrol. 252 (2001), 189-204.

[7] M. Lobo, D. L. Clements and N. Widana, Infiltration from irrigation channels in a soil with impermeable inclusions, ANZIAM J. 46 (2005), C1055-C1068.

[8] I. Solekhudin and K. C. Ang, A DRBEM with a predictor-corrector scheme for steady infiltration from periodic channels with root-water uptake, Engineering Analysis with Boundary Elements 36 (2012), 1199-1204.

[9] I. Solekhudin and K. C. Ang, A dual reciprocity boundary element method for steady infiltration problems, ANZIAM J. 54 (2013), 171-180.

[10] I. Solekhudin and K. C. Ang, A Laplace transform DRBEM with a predictor-corrector scheme for time dependent infiltration from periodic channels with root-water uptake, Engineering Analysis with Boundary Elements 50 (2015), 141-147.

[11] A. Utset, M. E. Ruiz, J. Garcia and R. A. Feddes, A SWACROP-based potato root water uptake function as determined under tropical conditions, Potato Res. 43 (2000), 19-29.

[12] J. A. Vrugt, J. W. Hopmans and J. Šimunek, Calibration of a two dimensional root water uptake model, Soil Sci. Soc. Am. J. 65 (2001), 1027-1037.