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EFFECT OF VOLUME FRACTION ALONG WITH CONCENTRATION PARAMETER IN THE DUSTY INCOMPRESSIBLE FLUID

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Abstract

The effect of finite volume fraction of suspended particulate matter on axially symmetrical jet mixing of incompressible dusty fluid has been considered. Assuming the velocity and temperature in the jet to differ slightly from the surrounding stream, a perturbation method has been employed to linearize the nonlinear differential equation. The linear equations have been solved by using Laplace transformation technique. Numerical computations have been made to discuss the

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magnitude of the longitudinal perturbated fluid velocity. It is observed that increase in concentration parameter of dust particle reduces the magnitude of fluid velocity significantly.

Nomenclature

(x, y, z)	: Space coordinates
(u, v, w)	: Velocity components of fluid phase
(u_p, v_p, w_p)	: Velocity components of particle phase
$(\bar{u}, \bar{v}, \bar{w})$	: Dimensionless velocity components of fluid phase
$(\bar{u}_p, \bar{v}_p, \bar{w}_p)$	: Dimensionless velocity components of particle phase
T	: Temperature of fluid phase
T_p	: Temperature of particle phase
C_{f_0}, C_{f_1}	: Skin friction coefficients at the lower and upper plates, respectively
C_p, C_s	: Specific heats of fluid and SPM, respectively
K	: Thermal conductivity
R_e	: Fluid phase Reynolds number
R_{e_p}	: Particle phase Reynolds number
E_c	: Eckert number
ρ	: Density of the fluid
ρ_p	: Density of the fluid particle
α	: Concentric parameter of dusty particle.
τ_m	: The momentum equilibration time
ϕ	: Volume fraction of the suspended particles

1. Introduction

Many researchers have solved the incompressible laminar jet mixing of a dusty fluid issuing from a circular jet with negligible volume fraction of SPM. However, the assumption is not justified when the fluid density is high or particle mass fraction is large. In the present paper, we studied a solution of longitudinal velocity of fluid phase boundary layer equations in axi-symmetrical jet mixing of an incompressible fluid along with the effect of volume fraction of SPM. Assuming the velocity and temperature in the jet to differ only slightly from that of the surrounding stream, a perturbation method has been employed to linearize the governing differential equations. The resulting linearized equations have been solved by using Laplace transformation technique.

2. Mathematical Formulation

The equation governing the study of two-phase boundary layer flow in axi-symmetric case can be written in cylindrical polar coordinates as

$$(1-\phi)\rho\left(u\frac{\partial u}{\partial z}+v\frac{\partial u}{\partial r}\right)=\frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{\rho_p}{\tau_m}(u_p-u). \tag{2.1}$$

To study the boundary layer flow, we introduce the dimensionless variables:

$$\bar{z}=\frac{z}{\lambda},\ \bar{r}=\frac{r}{(\tau_m\upsilon)^{1/2}},\ \bar{u}=\frac{u}{U},\ \bar{v}=v\left(\frac{\tau_m}{\upsilon}\right)^{1/2},\ \bar{u}_p=\frac{u_p}{U},$$

$$\bar{v}_p=v_p\left(\frac{\tau_m}{\upsilon}\right)^{1/2},\ \alpha=\frac{\rho_{p_0}}{\rho}=const.,$$

$$\bar{\rho}_p=\frac{\rho_p}{\rho_{p_0}},\ \bar{T}=\frac{T}{T_0},\ \bar{T}_p=\frac{T_p}{T_0},\ \lambda=\tau_m U,\ \tau_m=\frac{2}{3}\frac{C_p}{C_s}\frac{1}{p_r}\tau_T,\ p_r=\frac{\mu C_p}{K}.$$

Assuming the pressure at the exit equal to that of the surrounding stream, velocity and temperature in the jet slightly differ from that of the surrounding stream. The coefficient of viscosity μ and thermal conductivity K are assumed to be constant. Then it is possible to write $u = u_0 + u_1$, $v = v_1$, $T = T_0 + T_1$, where the subscripts '1' denote the perturbed values which are much smaller than the basic values with subscripts '0' of the surrounding stream, i.e., $u_0 \gg u_1$, $T_0 \gg T_1$.

Using the dimensionless variable and the perturbation method, (2.1) becomes:

$$(1-\phi)u_0 \frac{\partial u_1}{\partial z} = \frac{1}{r}\frac{\partial u_1}{\partial r} + \frac{\partial^2 u_1}{\partial r^2} + \alpha\rho_{p_1}(u_{p_0} - u_0). \tag{2.2}$$

The boundary conditions for u_1, v_1, u_{p_1} and v_{p_1} are

$$u_1(0, r) = \begin{cases} u_{10}, & r \le 1, \\ 0, & r > 1, \end{cases} \tag{2.3}$$

$$\frac{\partial u_1}{\partial r}(z, 0) = 0, \; u_1(z, \infty) = 0, \tag{2.4}$$

$$u_{p_1}(0, r) = \begin{cases} u_{p_{10}}, & r \le 1, \\ 0, & r > 1. \end{cases} \tag{2.5}$$

3. Method of Solution

The governing linearized equation (2.2) has been solved by using Laplace transform technique and the relevant conditions (2.3) to (2.5).

Since

$$(1-\phi)u_0 \frac{\partial u_1}{\partial z} = \frac{1}{r}\frac{\partial u_1}{\partial r} + \frac{\partial^2 u_1}{\partial r^2} + \alpha\rho_{p_1}(u_{p_0} - u_0).$$

We have

$$L\left\{(1-\phi)u_0 \frac{\partial u_1}{\partial z}\right\} = L\left\{\frac{1}{r}\frac{\partial u_1}{\partial r} + \frac{\partial^2 u_1}{\partial r^2} + \alpha\rho^*_{p_1}(u_{p_0} - u_0)\right\},$$

i.e.,

$$L\left\{(1-\phi)u_0\frac{\partial u_1}{\partial z}\right\} = L\left\{\frac{1}{r}\frac{\partial u_1}{\partial r}+\frac{\partial^2 u_1}{\partial r^2}\right\} + L\{\alpha\rho_{p_1}(u_{p_0}-u_0)\}.$$

Further,

$$L\left\{\frac{1}{r}\frac{\partial u}{\partial r}\right\} = p^2 U(z,\, s)$$

and

$$L\left\{\frac{\partial^2 u}{\partial r^2}\right\} = s^2 U(z,\, s) - su(z,\, 0) - u'(z,\, 0)$$

$$= s^2 U(z,\, s),$$

$$L\{\alpha\rho_{p_1}\} = \int_0^\infty \alpha\rho_{p_1}(z,\, r)e^{-sr}dr$$

$$= \alpha\rho^*_{p_1}(z,\, s).$$

Taking all above into account, we obtain

$$(1-\phi)u_0\frac{\partial u_1^*}{\partial z} = p^2 u_1^* + s^2 u_1^*(z,\, s) + \alpha\rho^*_{p_1}(u_{p_0}-u_0)$$

or

$$\frac{\partial u_1^*}{\partial z} + \frac{(p^2+s^2)}{(1-\phi)u_0}u_1^* = \alpha\frac{(u_{p_0}-u_0)}{(1-\phi)u_0}\rho^*_{\rho_1}$$

or

$$\frac{\partial u_1^*}{\partial z} + Ak^2 u_1^* = \alpha E\rho^*_{p_1}. \tag{3.1}$$

Equation (3.1) is a linear differential equation, where $A = \dfrac{p^2+s^2}{(1-\phi)u_0}$.

Let $p^2 + s^2 = k^2$. Then

$$u_1^* = e^{-\int AK^2 dz}\left\{\int \alpha E \rho_{p_1}^* e^{\int AK^2 dz} dz + B\right\}$$

$$= e^{-AK^2 z}\left\{\alpha E \int \rho_{p_1}^* e^{AK^2 z} dz + B\right\}$$

$$= e^{-AK^2 z}\left\{\alpha E \rho_{p_1}^* \frac{e^{AK^2 z}}{AK^2}\right\} + Be^{-Ak^2 z}$$

or

$$u_1^*(z,\ s) = \frac{\alpha E \rho_{p_1}^*(z,\ s)}{AK^2} + Be^{-Ak^2 z},$$

where $B = \dfrac{(up_0 - u_0)}{(1 - \phi)u_0}$.

Now that for $z = 0$,

$$u_1^*(0,\ s) = \frac{\alpha E}{AK^2}\rho_{p_1}^*(0,\ s) + B,$$

$$B = u_1^*(0,\ s) - \frac{\alpha E}{AK^2}\rho_{p_1}^*(0,\ s).$$

But

$$\rho_{p_1}^*(0,\ s) = \int_0^\infty \rho_{p_1}(0,\ r)e^{-sr} dr$$

$$= \rho_{p_{10}} \int_0^1 e^{-sr} dr$$

$$= \rho_{p_{10}} \left| \frac{e^{-sr}}{-s} \right|_0^1$$

$$= \rho_{p_{10}} \frac{(1 - e^{-s})}{s}$$

and

$$u_1^*(0,\ s) = \int_0^\infty u_1(0,\ r)e^{-sr}dr$$

$$= u_{10}\int_0^1 e^{-sr}dr = u_{10}\,\frac{(1-e^{-s})}{s}.$$

Therefore, we have $B = \left(u_{10} - \frac{\alpha E}{AK^2}\right)\frac{(1-e^{-s})}{s}$.

The solution of the above differential equation using Laplace transformation technique is as follows:

$$u_1^* = \left(u_{10} - \frac{\alpha E\rho^*_{p_1}}{Ak^2}\right)\left(\frac{1-e^{-s}}{s}\right)e^{-AK^2 z} + \frac{\alpha E\rho_{p_{10}}}{Ak^2}. \tag{3.2}$$

Taking Laplace inverse transformation of (3.2), we give

$$u_1(z,\ r) = L^{-1}\left\{\left(u_{10} - \frac{\alpha E\rho^*_{p_1}}{Ak^2}\right)\frac{(1-e^{-s})}{s}e^{-AK^2 z} + \frac{\alpha E\rho_{p_{10}}}{Ak^2}\right\}. \tag{3.3}$$

4. Discussion on Results and Conclusion

Numerical computations have been made by taking $P_r = 0.72$, $u_{10} = u_{p_{10}} = T_{10} = T_{p_{10}} = \rho_{p_{10}} = 0.1$, $\varphi = 0.01$. The velocity and temperature at the exit are taken nearly equal to unity. Figures 1, 2 and 3 show the profiles of longitudinal perturbation fluid velocity u_1 for $\alpha = 0.1, 0.2$ and 0.3 and for different values of Z. It is observed that the effect of increase in concentration parameter α of dust particles is to reduce the magnitude of u_1. Hence, we conclude that the consideration of finite volume fraction shows that the magnitude of fluid velocity reduces significantly towards the downstream direction.

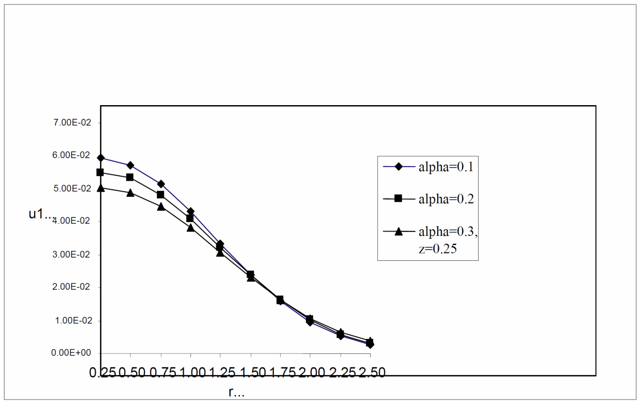

Figure 1. Profiles of longitudinal perturbation fluid velocity.

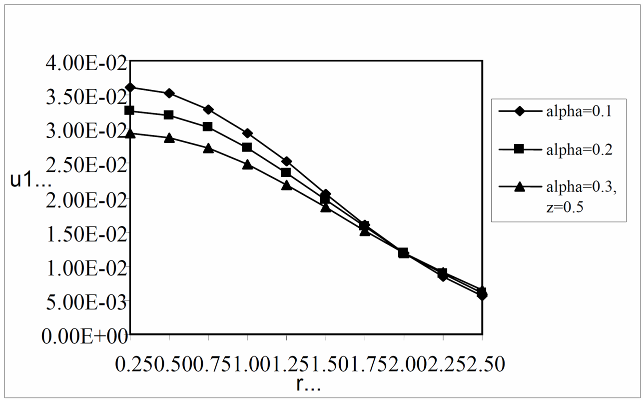

Figure 2. Profiles of longitudinal perturbation fluid velocity.

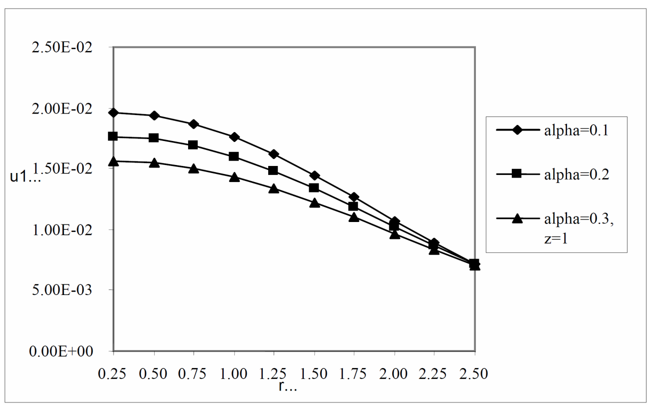

Figure 3. Profiles of longitudinal perturbation fluid velocity.

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