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THE EWMA MOVING AVERAGE CONTROL CHART FOR EXPONENTIAL DISTRIBUTION USING MULTIPLE DEPENDENT STATE SAMPLING

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Abstract

In this manuscript, we designed moving average control chart for exponential distribution using EWMA statistic using multiple dependent state (MDS) sampling. The necessary measures for in-control and out of control process are given for the proposed control chart. The proposed control chart is efficient than the existing control chart in detecting the shift in the process. The application of the proposed control chart is discussed with the help of simulation data.

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1. Introduction

The manufacturing of the product according to given target (mean) is one of the important issues in quality control problems. This target achievement may impossible if the cause of variations is not removed from the manufacturing process. As when cause of variation occurs, the process will change from the target value. In this situation, there is need to remove the cause of variations quickly. The control charts are one of the important tools that detect the process changes and inform the operators about the shift of the process. The process works at the target minimizes the non-conforming. The main goal of control chart is identify the cause of variation and chance variations. So, control chart is one of the important tools that also maximize the profit of the company.

It is well known that the Shewhart control charts are suitable to detect the large shift in the process. Therefore, some alternative control charts that are much effective to detect the smaller shift in the manufacturing process are moving average (MA) chart, CUSUM and EWMA charts. These control charts use two types of information that is current and past information to make the decision about the process. The use of such information makes this type of control chart as a powerful tool in minimizing the false alarm rate. These control charts have the ability to detect a smaller shift which cannot be detected in the Shewhart control chart. A wide study of MA, CUSUM and EWMA charts are available in the literature for specific situations. For a more details about MA control charts and applications in variety of fields can be seen in Montgomery [21], Roberts [24], Shamma and Shamma [26], Zhang [39], Khoo and Wang [15], Shamma et al. [27], Chen and Yang [10], Sparks [29], Dyer et al. [12], Khoo [14], Wong et al. [31], Yu and Wu [36], Zhang et al. [42], Zhang and Chen [40], Zhang et al. [41], Khoo and Yap [16], Zandi et al. [37] and Cheng and Huang [11].

The MA charts existing in the literature are designed by assuming that the quality under study follows the normal distribution with know or unknown standard deviation. But, practically, the data of variable of interest may have skewed shape. For skewed data, the use of normal distribution may cause to increase the false alarm rate and non-conforming products. Furthermore, the use of normal distribution is only justified when the data of variable of interest is gathered in subgroups. If the data is not in subgroup, it may skew data. In this situation, the use of exponential distribution is suggested (Santiago and Smith [25]). Santiago and Smith [25] presented the designing of t-chart for the exponential distribution using the transformation suggested by Johnson and Kotz [13]. Aslam et al. [3] designed t-chart under the repetitive sampling. Later on, Aslam et al. [4] proposed t-chart using EWMA statistic. The applications of t-chart and EWMA chart can be studied in Lucas and Saccucci [18], Lowry et al. [17], Mohammed [19], Mohammed and Laney [20], Zhang et al. [38], Yang et al. [35], Al-Refaie [2], Abbasi [1], Yang et al. [34] and Yang [33].

So for, the control charts are proposed using the single and double sampling. Recently, Aslam et al. [7] proposed t-chart using repetitive sampling and Aslam et al. [3] designed chart using multiple dependent states (MDS) sampling. So, they introduced other sampling schemes in the area of quality control than the single and double sampling schemes. Both suggested schemes are more efficient than the single sampling plan. For more detail, reader may refer to Aslam et al. [3] and Aslam et al. [4]. MDS sampling was introduced by Wortham and Baker [32]. This sampling scheme is much popular in the area of acceptance sampling plan. Balamurali and Jun [9] proposed the variable sampling plan using MDS sampling and showed that it performs better than the single sampling plan. Aslam et al. [5, 8] proposed the MDS plan for process capability index and process loss function, respectively. Recently, Aslam et al. [3] proposed t-chart using MDS sampling. More details about it can be seen in Vaerst [30] and Soundararajan and Vijayaraghavan [28].

By exploring the literature, we note that not work on MA t-chart using MDS sampling. In this paper, we will address this issue and design the t-chart by assuming that the time between events follows the exponential distribution. We will provide the necessary measures of the proposed control chart for practical use. We will provide a simulation study to show the efficiency of chart using MDS sampling over the t-chart using single sampling. The rest of the paper is set as follows: the designing of proposed chart is given in next section; comparative study is given in Section 3, the simulation study is given in Section 4 and concluding remarks are given in the last section.

2. Designing of Proposed Chart

Suppose that time between events T follows the exponential distribution with following probability density function (pdf):

$$f(t) = \frac{1}{\theta} e^{-\frac{t}{\theta}}, \quad t > 0, \tag{1}$$

where θ is the scale parameter of the exponential distribution. Johnson and Kotz [13] suggested for T, the transformation $T^* = T^{1/\beta}$ follows the Weibull distribution with shape parameter β and the scale parameter $\theta^{1/\beta}$. Further, Nelson [23] suggested that when $\beta = 3.6$, the distribution becomes approximate normal distribution.

The proposed control chart is stated as follows:

Step 1. Select a sample of size n at time (or subgroup) i and measure the quality characteristics of n items, denoted by T_{ij}, $j = 1, ..., n$. Calculate the sample mean of the transformed variable $\overline{T}_i^*$:

$$\overline{T}_i^* = \frac{1}{n} \sum_{j=1}^{n} T_{ij}^{1/3.6}.$$

Step 2. The moving average (MA) statistic of span w at time i is computed from the current and the past subgroups averages is defined as

$$MA_i = \frac{\overline{T}_i^* + \overline{T}_{i-1}^* + \cdots + \overline{T}_{i-w+1}^*}{w}.$$

Then, calculate the following EWMA statistic, denoted by M_i, having a smoothing constant λ at time i:

$$M_i = \lambda MA_i + (1 - \lambda) M_{i-1}.$$

Step 3. Declare the process as in-control if $LCL_2 \leq M_i \leq UCL_2$. Declare the process to be out-of-control if $M_i \geq UCL_1$ or $M_i \leq LCL_1$. Otherwise, go to Step 4.

Step 4. Declare the process is in-control if i proceeding subgroups declared the process as in-control. Otherwise, declare the process to be out-of-control.

Khoo and Wang [15] designed moving average control chart for normal distribution. The proposed control chart is the extension of Khoo and Wang [15] control chart for non-normal distribution. Khan et al. [22] designed moving average control chart for the exponential distribution. So, the proposed chart is improved version of Khan et al. [22] control chart for exponential distribution. The proposed chart reduces to Khan et al. [22] chart when $i = 0$. The mean and variance of M_i are given as follows:

$$E[M_i] = \theta_0^* \Gamma\left(1 + \frac{1}{3.6}\right) \tag{2}$$

and

$$\mathrm{Var}[M_i] = \frac{\left(\frac{\lambda}{2-\lambda}\right)\theta_0^{*2}\left[\Gamma\left(1+\frac{2}{3.6}\right) - \Gamma^2\left(1+\frac{1}{3.6}\right)\right]}{nw}, \tag{3}$$

where $\theta_0^* = \theta_0^{1/3.6}$.

The proposed chart is based on four control limits. These limits are given as follows:

$$LCL_1 = \theta_0^*\left\{\Gamma\left(1+\frac{1}{3.6}\right) - k_1\sqrt{\frac{\left(\frac{\lambda}{2-\lambda}\right)\left[\Gamma\left(1+\frac{2}{3.6}\right) - \Gamma^2\left(1+\frac{1}{3.6}\right)\right]}{nw}}\right\}, \tag{4}$$

$$LCL_2 = \theta_0^*\left\{\Gamma\left(1+\frac{1}{3.6}\right) - k_2\sqrt{\frac{\left(\frac{\lambda}{2-\lambda}\right)\left[\Gamma\left(1+\frac{2}{3.6}\right) - \Gamma^2\left(1+\frac{1}{3.6}\right)\right]}{nw}}\right\}, \tag{5}$$

$$UCL_2 = \theta_0^* \left\{ \Gamma\left(1+\frac{1}{3.6}\right) + k_2\sqrt{\frac{\left(\frac{\lambda}{2-\lambda}\right)\left[\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma^2\left(1+\frac{1}{3.6}\right)\right]}{nw}} \right\}, \quad (6)$$

$$UCL_1 = \theta_0^* \left\{ \Gamma\left(1+\frac{1}{3.6}\right) + k_1\sqrt{\frac{\left(\frac{\lambda}{2-\lambda}\right)\left[\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma^2\left(1+\frac{1}{3.6}\right)\right]}{nw}} \right\}. \quad (7)$$

The four control limits can be rewritten as follows:

$$LCL_1 = \theta_0^* LL_1, \quad (8)$$

$$LCL_2 = \theta_0^* LL_2, \quad (9)$$

$$UCL_2 = \theta_0^* UL_2, \quad (10)$$

$$UCL_1 = \theta_0^* UL_1, \quad (11)$$

where

$$LL_1 = \Gamma\left(1+\frac{1}{3.6}\right) - k_1\sqrt{\frac{\left(\frac{\lambda}{2-\lambda}\right)\left[\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma^2\left(1+\frac{1}{3.6}\right)\right]}{nw}}, \quad (12)$$

$$LL_2 = \Gamma\left(1+\frac{1}{3.6}\right) - k_2\sqrt{\frac{\left(\frac{\lambda}{2-\lambda}\right)\left[\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma^2\left(1+\frac{1}{3.6}\right)\right]}{nw}}, \quad (13)$$

$$UL_2 = \Gamma\left(1+\frac{1}{3.6}\right) + k_2\sqrt{\frac{\left(\frac{\lambda}{2-\lambda}\right)\left[\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma^2\left(1+\frac{1}{3.6}\right)\right]}{nw}}, \quad (14)$$

$$UL_1 = \Gamma\left(1+\frac{1}{3.6}\right) + k_1\sqrt{\frac{\left(\frac{\lambda}{2-\lambda}\right)\left[\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma^2\left(1+\frac{1}{3.6}\right)\right]}{nw}}. \quad (15)$$

By following Aslam et al. [3], the necessary measures when the process is in-control are given as follows. We suppose that θ_0 is the mean of in-control process. The probability of in-control at θ_0 is derived as follows:

$$\begin{aligned} P_{in} &= P(LCL_2 \le M_i \le UCL_2) \\ &\quad + \{P(LCL_1 < M_i < LCL_2) + P(UCL_2 < M_i < UCL_1)\} \\ &\quad \times \{P(LCL_2 \le M_i \le UCL_2)\}^i. \end{aligned} \tag{16}$$

The equation (16) can be rewritten as

$$P_{in} = a + (b + c) * a^i, \tag{17}$$

where $a = P(LCL_2 \le M_i \le UCL_2)$, $b = P(LCL_1 < M_i < LCL_2)$ and $c = P(UCL_2 < M_i < UCL_1)$.

For the in-control process, we have

$$\begin{aligned} a_0 &= P(LCL_2 \le M_i \le UCL_2 \mid \theta = \theta_0) \\ &= P\left(\frac{LCL_2 - E[DM_i]}{\sqrt{\mathrm{Var}[DM_i]}} \le Z_{DM} \le \frac{UCL_2 - E[DM_i]}{\sqrt{\mathrm{Var}[DM_i]}}\right). \end{aligned} \tag{18}$$

After simplification, a_0 can be given as follows:

$$a_0 = 2\Phi(k_2) - 1. \tag{19}$$

Similarly,

$$b_0 = P(LCL_1 \le M_i \le LCL_2 \mid \theta = \theta_0) = 2\{\Phi(k_1) - \Phi(k_2)\}, \tag{20}$$

$$c_0 = P(UCL_2 \le M_i \le UCL_1 \mid \theta = \theta_0). \tag{21}$$

So, the probability of in-control for in-control process is given as Aslam et al. [3]

$$P_{in}^0 = a_0 + (b_0 + c_0) * a_0^i \tag{22}$$

or

$$P_{in}^{0} = \{2\Phi(k_2) - 1\} + [2\{\Phi(k_1) - \Phi(k_2)\}] * \{2\Phi(k_2) - 1\}^{i}. \tag{23}$$

The performance of any control chart is judged on the basis of average run length (ARL). The ARL follows the geometric distribution as one is interested to see when the first subgroup is out of control. For the in-control process, ARL, say ARL_0 is given as

$$ARL_0 = \frac{1}{1 - P_{in}^{0}}. \tag{24}$$

It is now supposed that the process mean has shifted from θ_0 to θ_1. The mean and variance of M_i for the shifted process are given as follows:

$$E(M_i) = \theta_0^{*} c^{1/3.6} \cdot \Gamma\left(1 + \frac{1}{3.6}\right),$$

$$\mathrm{Var}(M_i) = \mathrm{Var}(M_i) = \frac{\left(\frac{\lambda}{2-\lambda}\right) c^{2/3.6} \theta_0^{*2} \left[\Gamma\left(1 + \frac{2}{3.6}\right) - \Gamma^2\left(1 + \frac{1}{3.6}\right)\right]}{nw}.$$

The probability of in-control for the shifted process is given as follows:

$$P_{in}^{1} = a_1 + (b_1 + c_1) * a_1^{i}, \tag{25}$$

where

$$a_1 = \Phi\left(\frac{(1 - c^{\frac{1}{3.6}}) \cdot \Gamma\left(1 + \frac{1}{3.6}\right) + k_2 \sqrt{\frac{\left(\frac{\lambda}{2-\lambda}\right)\left[\Gamma\left(1 + \frac{2}{3.6}\right) - \Gamma^2\left(1 + \frac{1}{3.6}\right)\right]}{nw}}}{\sqrt{\frac{\left(\frac{\lambda}{2-\lambda}\right) c^{2/3.6}\left[\Gamma\left(1 + \frac{2}{3.6}\right) - \Gamma^2\left(1 + \frac{1}{3.6}\right)\right]}{nw}}}\right)$$

$$- \Phi\left(\frac{(1 - c^{\frac{1}{3.6}}) \cdot \Gamma\left(1 + \frac{1}{3.6}\right) - k_2 \sqrt{\frac{\left(\frac{\lambda}{2-\lambda}\right)\left[\Gamma\left(1 + \frac{2}{3.6}\right) - \Gamma^2\left(1 + \frac{1}{3.6}\right)\right]}{nw}}}{\sqrt{\frac{\left(\frac{\lambda}{2-\lambda}\right) c^{2/3.6}\left[\Gamma\left(1 + \frac{2}{3.6}\right) - \Gamma^2\left(1 + \frac{1}{3.6}\right)\right]}{nw}}}\right),$$

$$b_1 = \Phi\left(\frac{(1-c^{\frac{1}{3.6}})\cdot\Gamma\left(1+\frac{1}{3.6}\right) - k_2\sqrt{\frac{\left(\frac{\lambda}{2-\lambda}\right)\left[\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma^2\left(1+\frac{1}{3.6}\right)\right]}{nw}}}{\sqrt{\frac{\left(\frac{\lambda}{2-\lambda}\right)c^{2/3.6}\left[\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma^2\left(1+\frac{1}{3.6}\right)\right]}{nw}}} \right)$$

$$-\Phi\left(\frac{(1-c^{\frac{1}{3.6}})\cdot\Gamma\left(1+\frac{1}{3.6}\right) - k_1\sqrt{\frac{\left(\frac{\lambda}{2-\lambda}\right)\left[\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma^2\left(1+\frac{1}{3.6}\right)\right]}{nw}}}{\sqrt{\frac{\left(\frac{\lambda}{2-\lambda}\right)c^{2/3.6}\left[\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma^2\left(1+\frac{1}{3.6}\right)\right]}{nw}}} \right),$$

$$c_1 = \Phi\left(\frac{(1-c^{\frac{1}{3.6}})\cdot\Gamma\left(1+\frac{1}{3.6}\right) + k_1\sqrt{\frac{\left(\frac{\lambda}{2-\lambda}\right)\left[\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma^2\left(1+\frac{1}{3.6}\right)\right]}{nw}}}{\sqrt{\frac{\left(\frac{\lambda}{2-\lambda}\right)c^{2/3.6}\left[\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma^2\left(1+\frac{1}{3.6}\right)\right]}{nw}}} \right)$$

$$-\Phi\left(\frac{(1-c^{\frac{1}{3.6}})\cdot\Gamma\left(1+\frac{1}{3.6}\right) + k_2\sqrt{\frac{\left(\frac{\lambda}{2-\lambda}\right)\left[\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma^2\left(1+\frac{1}{3.6}\right)\right]}{nw}}}{\sqrt{\frac{\left(\frac{\lambda}{2-\lambda}\right)c^{2/3.6}\left[\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma^2\left(1+\frac{1}{3.6}\right)\right]}{nw}}} \right).$$

The ARL for the out of control process is given as follows:

$$ARL_1 = \frac{1}{1 - P_{in}^1}. \tag{26}$$

Let r_0 be the specified ARL. The values of ARL_1 are presented for various values of specified parameters in Tables 1-6. Table 1 is presented for $r_0 = 300$, $w = 3$, $n = 5$ and $i = 2$. Table 2 is given for $r_0 = 370$, $w = 3$, $n = 5$ and $i = 2$. Table 3 is constructed when $r_0 = 300$, $w = 4$, $n = 5$ and

$i = 2$. Table 4 is made for $r_0 = 370$, $w = 3$, $n = 5$ and $i = 2$. Table 5 is given when $r_0 = 300$, $w = 5$, $n = 5$ and $i = 2$ and Table 6 is given for $r_0 = 370$, $w = 3$, $n = 5$ and $i = 2$.

Table 1. The values of ARL when $r_0 = 300$; $w = 3$; $n = 5$ and $i = 2$

						Existing chart	Existing chart
	$k_1 = 3.1056$; $k_2 = 2.1981$					Khoo and Wang [15]	Khan et al. [22]
	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$	$\lambda = 1$, $i = 0$	$\lambda = 0.1$, $i = 0$
c	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1
1	300.00	300.00	300.00	300.00	300.00	300.00	300.00
1.01	259.42	275.47	281.18	284.11	285.89	290.63	263.67
1.02	187.03	229.54	247.28	257.00	263.14	278.66	197.91
1.03	122.60	178.71	206.96	223.89	235.15	264.69	137.63
1.04	77.82	133.74	167.40	189.59	205.27	249.34	93.76
1.05	49.48	98.25	132.61	157.44	176.08	233.21	64.26
1.07	21.31	52.75	81.15	105.23	125.50	200.62	31.88
1.09	10.34	29.28	49.81	69.57	87.87	169.97	17.23
1.1	7.55	22.25	39.37	56.73	73.49	155.90	13.07
1.2	1.42	3.18	6.13	10.13	15.11	65.29	2.15
1.3	1.04	1.42	2.21	3.40	5.04	30.72	1.15
1.5	1.00	1.02	1.13	1.34	1.68	10.06	1.00
2	1.00	1.00	1.00	1.00	1.02	2.43	1.00

Table 2. The values of ARL when $r_0 = 370$; $w = 3$; $n = 5$ and $i = 2$

c	$k_1 = 3.20536$; $k_2 = 2.213309$					
	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$	$\lambda = 1$
	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1
1	370.01	370.01	370.01	370.01	370.01	370.01
1.01	318.05	338.55	345.86	349.61	351.90	356.54
1.02	226.26	280.00	302.56	314.95	322.78	339.43
1.03	145.83	215.80	251.40	272.82	287.12	319.60
1.04	90.93	159.63	201.61	229.48	249.25	298.00
1.05	56.83	115.86	158.23	189.13	212.49	275.51
1.07	23.73	60.72	94.97	124.41	149.41	230.76
1.09	11.22	32.97	57.21	80.93	103.15	189.63
1.1	8.10	24.80	44.82	65.48	85.67	171.11
1.2	1.43	3.31	6.53	10.98	16.61	59.94
1.3	1.04	1.44	2.27	3.55	5.33	24.00
1.5	1.00	1.02	1.13	1.36	1.71	6.52
2	1.00	1.00	1.00	1.01	1.02	1.65

Table 3. The values of ARL when $r_0 = 300$; $w = 4$; $n = 5$ and $i = 2$

c	$k_1 = 3.2422$; $k_2 = 2.1243$					
	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$	$\lambda = 1$
	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1
1	300.00	300.00	300.00	300.00	300.00	300.00
1.01	249.79	270.38	277.85	281.71	284.07	288.87
1.02	165.05	214.74	236.66	248.97	256.85	273.85
1.03	98.11	156.98	189.37	209.65	223.51	255.96
1.04	57.01	109.94	145.59	170.55	188.89	236.29
1.05	33.73	75.85	109.52	135.71	156.37	215.85
1.07	13.25	36.66	60.93	83.43	103.62	175.77
1.09	6.23	18.93	34.63	51.25	67.79	140.05
1.1	4.57	14.03	26.53	40.48	54.99	124.41
1.2	1.17	2.10	3.81	6.28	9.53	38.33
1.3	1.01	1.17	1.58	2.25	3.22	14.45
1.5	1.00	1.00	1.04	1.14	1.31	3.99
2	1.00	1.00	1.00	1.00	1.00	1.30

Table 4. The values of ARL when $r_0 = 370$; $w = 4$; $n = 5$ and $i = 2$

c	$k_1 = 3.23307$; $k_1 = 2.197$					
	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$	$\lambda = 1$
	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1
1	370.00	370.00	370.00	370.00	370.00	370.00
1.01	305.98	332.14	341.66	346.58	349.59	355.74
1.02	199.81	261.74	289.33	304.90	314.89	336.53
1.03	117.54	189.73	229.93	255.29	272.69	313.73
1.04	67.70	131.88	175.58	206.45	229.25	288.77
1.05	39.71	90.38	131.28	163.35	188.81	262.95
1.07	15.29	43.16	72.28	99.43	123.94	212.70
1.09	7.03	21.99	40.67	60.55	80.43	168.38
1.1	5.09	16.18	31.01	47.63	65.00	149.12
1.2	1.19	2.25	4.19	7.04	10.83	44.78
1.3	1.01	1.20	1.66	2.41	3.51	16.51
1.5	1.00	1.00	1.05	1.16	1.35	4.37
2	1.00	1.00	1.00	1.00	1.00	1.34

Table 5. The values of ARL when $r_0 = 300$; $w = 5$; $n = 5$ and $i = 2$

c	$k_1 = 3.1918$; $k_2 = 2.1451$					
	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$	$\lambda = 1$
	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1
1	300.00	300.00	300.00	300.00	300.00	300.00
1.01	240.82	265.47	274.60	279.35	282.26	288.24
1.02	147.67	201.71	226.89	241.41	250.84	271.57
1.03	81.73	139.94	174.52	197.10	212.95	251.43
1.04	44.91	93.29	128.78	154.97	174.92	229.32
1.05	25.53	61.71	93.24	119.23	140.62	206.56
1.07	9.63	28.04	48.73	69.08	88.21	163.06
1.09	4.53	13.97	26.50	40.52	55.15	125.86
1.1	3.37	10.26	19.98	31.43	43.89	110.11
1.2	1.08	1.69	2.86	4.62	7.01	30.30
1.3	1.00	1.09	1.35	1.80	2.46	10.90
1.5	1.00	1.00	1.01	1.07	1.17	3.06
2	1.00	1.00	1.00	1.00	1.00	1.17

Table 6. The values of ARL when $r_0 = 370$; $w = 5$; $n = 5$ and $i = 2$

c	$k_1 = 3.121998$; $k_2 = 2.28478$					
	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$	$\lambda = 1$
	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1
1	370.00	370.00	370.00	370.00	370.00	370.00
1.01	295.82	326.57	338.01	343.97	347.64	355.17
1.02	181.11	247.30	278.42	296.45	308.21	334.17
1.03	100.79	171.52	213.75	241.49	261.05	308.89
1.04	55.76	114.70	157.79	189.70	214.09	281.25
1.05	31.78	76.19	114.50	146.04	172.05	252.93
1.07	11.85	34.81	60.19	84.95	108.15	199.17
1.09	5.42	17.27	32.82	50.03	67.85	153.55
1.1	3.95	12.60	24.72	38.84	54.08	134.31
1.2	1.11	1.85	3.30	5.48	8.45	37.03
1.3	1.00	1.11	1.43	1.98	2.80	13.16
1.5	1.00	1.00	1.02	1.09	1.22	3.50
2	1.00	1.00	1.00	1.00	1.00	1.21

We note following trends in-control chart parameters:

1. For other fixed values, as r_0 changes from 300 to 370, ARL_1 increases.

2. For other fixed values, as w changes, ARL_1 decreases.

The following algorithm is used for the construction of tables:

Step 1. Pre-assigned the values of r_0, n and i.

Step 2. Determine k_1 and k_2 for which $ARL_0 \geq r_0$.

Step 3. Determine ARL_1 using k_1 and k_2 obtained in step 2.

3. Comparative Study

The efficiency of the proposed control chart over Khoo and Wang [15] and Khan et al. [22] charts will be discussed in this section. The proposed chart reduces to Khoo and Wang [15] chart when $\lambda = 1$ and $i = 0$ and to

Khan et al. [22] chart when $i = 0$. We will use ARL_1 to assess the performance of three control charts. To compare the performance, we set the same values of three control charts. To save the space, we give the comparison when $ARL_0 = 300$, $w = 3$; $n = 5$ and $i = 2$ in Table 1.

3.1. Proposed Chart Vs Khoo and Wang [15]

The proposed control chart is extension of chart by Khoo and Wang [15]. The proposed chart reduces to Khoo and Wang [15] chart when $\lambda = 1$ and $i = 0$. The values of ARL_1 of Khoo and Wang [15] are reported in Table 1. From Table 1, it can be seen that the proposed chart provides smaller values of ARLs as compared to Khoo and Wang [15] chart. For example, when $c = 1.05$, the ARL from proposed chart is 49 when $\lambda = 0.1$ while it is 233 from the Khoo and Wang [15] chart.

3.2. Proposed Chart Vs Khan et al. [22]

The proposed control chart is extension of chart by Khan et al. [22]. The proposed chart reduces to Khan et al. [22] chart when $i = 0$. The values of ARL_1 of Khan et al. [22] are reported in Table 1. From Table 1, it can be seen that the proposed chart provides smaller values of ARLs as compared to Khan et al. [22] chart. For example, when $c = 1.05$, the ARL from proposed chart is 49 when $\lambda = 0.1$ while it is 64 from Khan et al. [22] chart. Therefore, the proposed chart is more sensitive to detect a small shift as compared to the existing charts.

4. Simulation Study

In this section, we give the application of the proposed control chart using the simulated data. The data is generated from the exponential distribution with $\theta_0 = 2$. First 20 observations are generated from the in-control process at $\theta_0 = 2$ and next 20 observations are generated from the shifted process with shift $c = 1.2$. Suppose that $\lambda = 0.5, w = 3$ and $r_0 = 370$. The values of M_i are reported in Table 7.

Table 7. Simulated data

Sr#	T_1	T_2	T_3	T_4	T_5	$\overline{T}_i^*$	MA	M_i
1	0.223835	0.529759	3.156334	3.110693	5.289962	1.16663	1.16663	1.146612
2	0.499212	0.054954	0.482619	2.106359	1.355417	0.881204	1.023917	1.085265
3	1.066275	3.587956	2.542989	0.371839	3.835665	1.19048	1.079438	1.082351
4	1.815991	0.755931	1.786726	1.031576	4.449751	1.160595	1.077427	1.079889
5	5.627436	4.035522	2.655814	0.098605	0.966587	1.183402	1.178159	1.129024
6	0.129316	0.107351	4.035618	0.173862	5.719163	0.963239	1.102412	1.115718
7	2.77963	0.659648	0.364896	1.547849	3.908321	1.112867	1.086503	1.10111
8	0.849083	2.832678	2.815852	2.374947	5.288918	1.296812	1.124306	1.112708
9	2.73101	0.286306	1.362637	0.746874	0.113292	0.917281	1.108987	1.110848
10	6.64609	0.074971	1.63058	2.270614	3.689635	1.203542	1.139212	1.12503
11	0.562552	1.69614	4.002467	1.215161	4.885575	1.217941	1.112921	1.118975
12	1.847217	1.263335	0.226917	5.109178	10.97291	1.286733	1.236072	1.177524
13	2.931397	0.951441	4.616622	1.678085	0.867087	1.195937	1.233537	1.20553
14	0.979084	4.806125	1.269712	1.287139	0.241686	1.071205	1.184625	1.195078
15	2.469307	0.09035	0.044918	3.10824	1.452921	0.940049	1.069064	1.132071
16	1.301797	1.929851	1.983521	1.100729	1.08633	1.107241	1.039498	1.085785
17	4.075068	0.282377	2.116137	1.323182	1.195466	1.108875	1.052055	1.06892
18	0.88442	2.029554	2.105873	0.670384	1.678379	1.092624	1.102913	1.085916
19	0.300838	2.144151	0.903918	3.961263	1.635381	1.107358	1.102952	1.094434
20	3.01746	7.364843	1.680283	1.11483	2.20493	1.306342	1.168775	1.131604
21	3.857215	0.436394	1.102329	0.016699	3.545217	1.003759	1.139153	1.135379
22	0.161768	2.975889	1.735933	2.124294	1.305106	1.086372	1.132158	1.133768
23	0.000451	1.353728	3.584609	4.254631	0.984669	1.024391	1.038174	1.085971
24	0.28597	1.334301	0.719812	1.369587	2.927476	1.028274	1.046346	1.066159
25	1.353624	1.026518	0.801932	3.441383	0.342923	1.037597	1.030088	1.048123
26	0.354417	1.556234	2.982025	2.117558	2.780597	1.159043	1.074972	1.061547
27	6.285165	1.114259	1.681656	1.547325	0.350157	1.145641	1.114094	1.08782
28	7.50E+00	9.28E-04	1.02E+00	3.44E-01	1.11E+01	1.118203	1.140962	1.114391
29	2.752679	1.517755	2.4661	3.584233	0.592265	1.204571	1.156138	1.135265
30	1.832967	0.846837	0.533101	0.673657	12.25072	1.175928	1.166234	1.150749
31	4.092768	1.326069	10.33858	1.05367	0.354708	1.247697	1.209399	1.180074

32	1.632122	7.277467	0.845662	0.260229	2.151913	1.152216	1.191947	1.18601
33	1.275021	2.047248	0.347065	0.574513	0.958614	0.976197	1.12537	1.15569
34	0.828376	1.421919	15.90661	0.596708	0.795842	1.202659	1.110357	1.133024
35	0.046886	5.704367	1.976724	0.598712	0.633727	1.001203	1.06002	1.096522
36	0.268786	0.96575	8.184218	0.106111	0.241539	0.937575	1.047146	1.071834
37	13.68504	0.255237	1.522573	5.264787	2.282881	1.344109	1.094296	1.083065
38	5.021468	1.253948	1.860619	3.578008	3.893866	1.340482	1.207389	1.145227
39	4.546627	1.754568	2.134424	3.028189	0.391652	1.211514	1.298702	1.221964
40	3.253493	0.334907	8.740172	2.714592	2.870874	1.322389	1.291462	1.256713

The four control limits for this data are given as follows:

$$UCL_1 = 1.2535,$$

$$UCL_2 = 1.2036,$$

$$LCL_2 = 0.9812,$$

$$LCL_1 = 0.9314.$$

The tabulated ARL is about 17 from Table 2. We plotted the values of M_i on control chart in Figure 1.

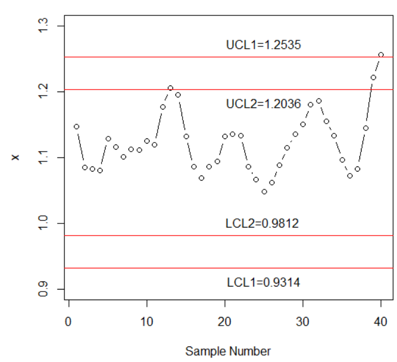

Figure 1. The proposed control chart for simulated data.

The proposed control chart effectively detects the out-of-control shift at 18th observations. The same values of M_i are plotted on the existing control chart in Figure 2.

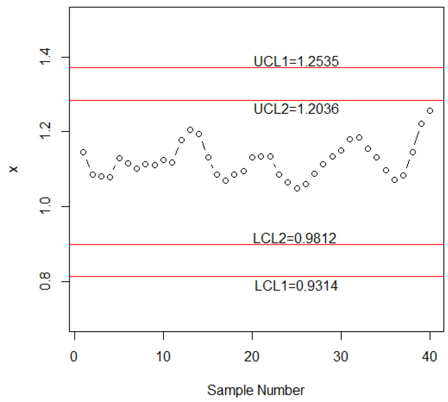

Figure 2. The existing control chart for simulated data.

We note that the existing control chart is unable to detect the shift in the process. The value of M_i lie within control limits. This study show the superiority of the proposed chart over the existing control chart.

5. Concluding Remarks

In this paper, MDS sampling scheme is introduced in the designing of MA control chart by assuming that the time between events follows the exponential distribution. The necessary measures are given for the proposed control chart. The tables for various specified parameters are given for the practical use. A simulation study is given to show the efficiency of proposed chart over the existing chart. The proposed chart has ability to detect a small shift in the process. A quick indication helps the supervisor to bring back the process in in-control state. A comparison of the proposed chart over the existing chart is provided. We conclude that the proposed chart is more sensitive to detect out of control for small shift. So, the use of proposed chart in the industry for monitoring of the process can minimize the non-conforming product. The proposed chart will be extended for some other distribution as future research.

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