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INVESTIGATION OF FLOW PRESSURE DISTRIBUTION OVER A STEPPED SPILLWAY

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Abstract

The use of stepped spillways has become increasingly widespread due to low manufacturing costs and high efficiency. Hence, relevant research has significantly developed. So far, most of these studies are carried out on stepped spillways with an even steps arrangement. Yet, any change in step arrangement can be considered as an alternative in

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some cases. In this study, flow was simulated for four types of stepped spillways with different step arrangements using Fluent. Numerical analysis was done using three types of turbulence models including k-ε (standard), k-ε (RNG) and k-ω (standard). These models are validated by comparing the results of the water level profile against laboratory data. Error values are computed. Based on root mean square error, the turbulence model k-ε (RNG) presented more approximate results as compared to the laboratory results, regardless of the arrangement of the steps. After selecting the optimal turbulence model, pressure distribution on the spillway was computed by means of various steps arrangements. Based on the results, it was observed that the trend of pressure distribution was the same for all of the spillways; that is, it is reduced until reaching the edge of the steps, then increased as a result of water falling and hitting against the step floor, and reduced again along the step. Pressure values are larger for spillways with a smaller number of larger steps.

1. Introduction

The view of using step-shape geometry in building spillways was developed long ago. Yet, with respect to factors such as the considerable effect of the steps on the flow energy depreciation, the detection of new technologies in using roller concrete, and the correlation between this method and the construction of the abovementioned spillway, stepped spillways have become widely used in many projects. The significant energy depreciation caused by steps reduces the excavation depth of the downstream stilling basin, the length of the stilling basin, and its side walls. Thus, a lot of financial sources is saved. Carrying out research on these types of spillways has drawn various researchers' interest [4]. When flow passes over stepped spillways, three types of distinct flow regimes can be defined:

(a) In a spilling flow regime, total flow height is divided into a set of perpendicular waterfalls. Water flow can have complete or partial hydraulic jump (depending on the horizontal length of the steps) in hitting successive steps. This regime occurs in low discharges and large steps heights. Energy depreciation is induced due to the contact between weather flow jet and the jet mixture on each step. It leads to the formation of total and/or partial hydraulic jump on each step.

(b) In sliding flow regime, steps act against flow with the same effect as the roughness of the bed. In this regime, a false bed forms. It connects the end edge of successive steps to each other. The majority of energy depreciation is caused by the emergence of circular flows below the false bed.

(c) Intermediate flow regime stands somewhere between the spilling flow and sliding flow regimes. Here, flow has no specific shape and form on the spillway [2].

Two-phasic flow spilling over stepped spillways is complex. Thus, many researchers have examined the function of these spillways using laboratory and numerical simulations. Cassidy [3] was the first researcher to analyze flow over Ogee spillways using Laplace's equation and finite difference methods. Using potential flow theory, he analyzed the flow over the spillway and the free area of water. He observed acceptable correspondence with laboratory results. Sorensen [13] and Chanson [4] conducted several studies on determining the type of flow over a stepped spillway. They also presented equations for setting the type of dominant flow and the extent of energy depreciation over spillways. Rajaratnam [10] presented an approximate method for predicting the characteristics of the sliding flow over stepped spillways. He extended flow equations by expressing the turbulent shear tension between sliding flow and the fluid locked between the step and flow. He used terms fluid friction coefficient and dynamic pressure. According to his studies, when critical depth > 0.8 equals the step height, dominant flow over a stepped spillway is of the sliding type. Olsen and Kjellesvig [9] have modeled two- and three-dimensional flows over Ogee spillways using numerical methods. They solved Navier-Stokes equations using the k-ε turbulence model. Doing so, they obtained Debi roughness coefficient for an Ogee spillway. They compared their results to empirical results. They introduced the error obtained 1 and 0.5 for the two- and three-dimensional models, respectively. They also computed a water level profile based on the horizontal flow profile using the reiteration method and correspondent networks. Then they compared their results to the results of physical models. They observed correspondence. Chen et al. [5] analyzed the dominant flow over a stepped spillway using the finite-volume method. They exploited a k-ε model. They measured free level, velocity, and pressure over stepped spillways based on the numerical simulation of turbulence. They compared their results to empirical results. Finally, they introduced the numerical simulation of turbulence as the useful method for the complex flow over stepped spillways. Tabbara et al. [14] modeled the flow over stepped spillways using the finite element method (ADINA) and the standard k-ε turbulence model. They measured the value of energy depreciation over each spillway. Cheng et al. [6] modeled the flow over stepped spillways using the mixing method and the k-ε turbulence model (RNG). Based on their research, it was demonstrated that the mixing method has many advantages in the numerical simulation of negative pressure and vortex flows over the steps of spillways. Dermawan and Legono [8] examined the residual energy and the relative drop of energy over the stepped spillway (slope = 45°, high = 100cm and width = 30cm). Then they divided the spillway area (from spillway's crest to spillway's toe) into 2, 4, 8, 16 and 32 steps in five stages. They obtained five types of Debi changes (from 1.73L/s to 6.15L/s) for each stage. Results showed that relative energy loss for each stage increased when Y_C/h decreased. Abbasi and Kamanbedast [1] numerically solved and simulated three groups of various types of stepped spillways using FLOW-3D. They studied the loss and compared it to the laboratory conditions examined in the past. They also presented and discussed equations for the relative loss and critical depth of flow over the stepped spillways. Daneshfaraz et al. [7] studied the finite element and the finite volume methods in simulating Ogee and stepped spillways. Both methods present good correspondence with experimental data. They observed that the errors of the finite volume method are less than those of the finite element method. Roushangar et al. [12] applied various methods for modeling energy loss in slipping and rotating flows along the stepped spillway through artificial neural networks and genetic programming (GEP) techniques using an empirical data set. Despite the importance of studying flow characteristics over stepped spillways, a general review of literature in this area shows a lack of sufficient and comprehensive studies on the subject. The aim of this study is to numerically investigate the water level profile and pressure distribution over four different types of stepped spillways with various arrangements of steps. In addition, the performance of three common turbulence models including k-ε (Standard), k-ε (RNG) and k-ω (standard) in numerical simulations of water characteristics over stepped spillway was evaluated. Then an optimal turbulence model was introduced.

2. Materials and Methods

2.1. Governing equations

Governing equations over a viscous incompressible fluid in a turbulent state are expressed by Reynolds averaged Navier-Stokes equations (RANS) in depth. They include continuity and motion equations [11]:

$$\frac{\partial u_i}{\partial x_i} = 0, \tag{1}$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_i} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + g_i + \frac{\partial}{\partial x_i} S_{ij}. \tag{2}$$

In the above equations, u_i is velocity component, P is total pressure, ρ is gravity acceleration in x_i direction, and S_{ij} is tension tensor, which is expressed as the following equation for a turbulent flow:

$$S_{ij} = \left[\rho(v + v_t)\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] - \left[\frac{2}{3}\rho(k + v_t)\frac{\partial u_i}{\partial x_i}\delta_{ij}\right], \tag{3}$$

where v_t is vortex viscosity. It is a function of turbulence and flow characteristics. S_{ij} is used for having an applied definition of vortex viscosity:

$$\delta_{ij} = \begin{cases} 1, & i = j, \\ 0, & i \neq j. \end{cases} \tag{4}$$

Shear tension in turbulent flows includes two terms. As well as the shear tension induced by average flow component, another shear tension is induced by the frequency components of velocity. They are known as Reynolds tensions [11]. They are expressed by the following equation:

$$S_{ij} = -\rho\overline{u_i'u_j'} = \rho v_i\left(\frac{\partial u_i}{\partial x} + \frac{\partial u_j}{\partial x}\right) - \frac{2}{3}\rho k\delta_{ij}. \tag{5}$$

Turbulence kinetic energy on a mass unit is expressed by equation (6):

$$k = \frac{1}{2}(\overline{u_1'^2} + \overline{u_j'^2} + \overline{u_k'^2}). \tag{6}$$

To solve the field of turbulent flow based on continuity and Reynolds equations, Reynolds tensions need to be modeled in the equations by a special method. Now, four X_S (velocity in three vertical directions and pressure) will be obtained for a 3D flow with four equations (continuity and motion in three dimensions). To dominate the above equations system, turbulence models are applied. To do so, three types of turbulence models k-ε (standard), k-ε (RNG) and k-ω (standard) are used [11].

2.2. Fluent software

Fluent is a software that analyzes flow in two- and three-dimensional terms. It applies Navier-Stokes and continuity equations. In case the flow is turbulent dominant equations will be transformed into Reynolds equations. Then one-, two- and five- or six-equation models are used for determining vortex viscosity. The type of the model is set by the user.

Flow field solution and equations discretion are based on the finite volume method. Schemes like upwind, second rank upwind, exponential, and quick are applied for the discretion of the transfer terms of the equations. Algorithms including simple, simple C, and PISO can be used for coupling velocity and pressure fields. To solve the transfer equation using the

Boussinesq approximation, this software applies two-equation turbulence models. The *k*-ε model is the most well-known two-equation model. This is a quasi-empirical model where the *k* equation is obtained by integrating Reynolds equations dominant on the flow. The ε equation is obtained based on empirical and experimental evidence as well as mathematical equations. It must be noted that, in extracting these equations, the flow is assumed to be totally turbulent. The effect of molecular viscosity is taken to be trivial. As well as having all of the *k*-ε (standard) model conditions, the *k*-ε (RNG) turbulence model has an extra term in ε equation. It enhances computation power for flows with quick strains. The effect of vortex on turbulence is included in the RNG model. It increases the accuracy of computations in vortex flows. RNG theory presents an analytical formula for the Prandtl number. Yet, this is determined by the user in the standard model.

2.3. Numerical solution, meshing, and boundary conditions

Design tables 2/1-111 (USACE-WES) were used for designing the spillway profile. The spillway body consisted of a vertical plane on the upstream and curved planes (curve radius $= 0.2H_d$ and $0.5H_d$) at the spillway's crest. The spillway body profile is defined using $X_n = KH_d^{1-n}y$. Using $K = 2$, $n = 1.85$ and $H_d = 5.08\text{cm}$, we will have equation (7) for the crest profile of the spillway. It must be said that H_d is considered to be the head of the design. The direct part of the spillway body is inclined about 60 (or slope = 1.73:1):

$$Y = 0.1256x^{1.85}. \tag{7}$$

In general, the downstream part of the profile after the spillway's crest includes three pieces. The first follows equation (7). The second piece is inclined about 60°. The third is a circular arch. It makes the transformation between the inclined direct line and the horizontal line related to the stillness basin. Four types of arrangements are considered over the respective stepped spillway. Table 1 shows the details of these four modes.

Table 1. Description of steps in four types of stepped spillways

Type of stepped spillway	Upper part		Lower part	
	Number of steps	Height (mm)	Number of steps	Height (mm)
I	6	19	12	19
II	12	9.5	24	9.5
III	6	19	24	9.5
IV	12	9.5	12	19

All four types of the stepped spillway with various configurations have undergone water head $1.5H_d$. In Tabbara et al. [14] experimental model, each spillway was built by joining two pieces of plexiglass. The upper part includes the spillway's crest area $(h = 1.3$ of the spillway height). The lower part includes the spillway's toe area $(h = 2.3$ of the spillway height). The total height of the spillway's model from the spillway's toe to the spillway's crest is about 380mm. The spillway profile curve (both on the top and at the bottom) follows the Ogee spillway profile. In each part, the large step height is 1/20th of the spillway height (19mm). The small step height is about 1/40th of the spillway height (9.5mm). Table 1 presents the configuration of the steps in all of the four types of spillways. Figure 1 shows the details of meshing for the type I stepped spillway. In Figure 1, vertical and horizontal axes indicate the dimensions of the model in meters.

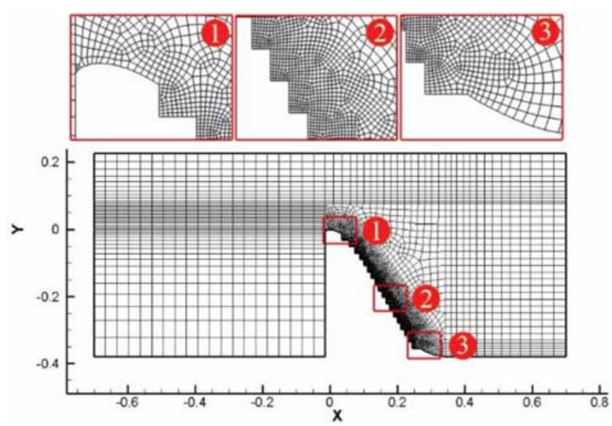


Figure 1. Meshing details for stepped spillway type I.

The models used square meshes. To increase velocity, it is structured in all areas, except on the spillway profile. It is unstructured on the spillway profile due to geometrical limitations. It must be noted that meshing is the same as the first mode for the second, third and fourth modes. In Figure 2, the boundary and basic conditions applied in numerical modeling in the software are illustrated.

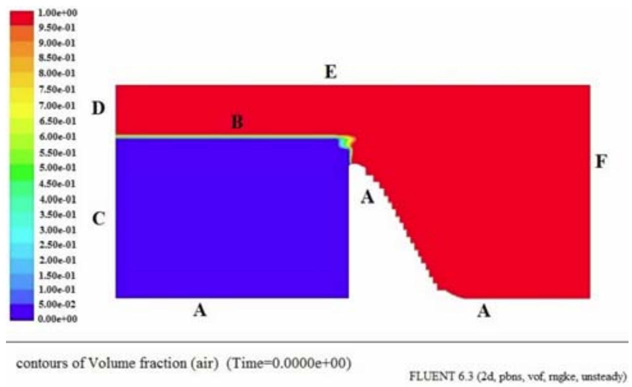


Figure 2. Boundary and basic conditions for stepped spillway type I.

Boundaries marked by letter *A* indicate the boundary condition of the fixed wall. It has formed the whole area of the floor and the spillway itself. The boundary condition of the wall is used in boundaries where the fluid is surrounded by a solid. The basic free water level is marked by letter *B*. Letter *C* is the entrance boundary of the consistent velocity of water $(v = 0.1065\,\text{m/s})$. Boundary *D* is the entrance boundary of the consistent velocity of air $(v = 0.00001\,\text{m/s})$. Boundaries *E* and *F* are, respectively, considered the entrance and exit boundaries of pressure. The entrance velocity of the water flow is computed by dividing flow Debi by the depth of the upstream flow obtained from experimental observations. These default definitions include for four states related to the stepped spillway.

The modeling trend in Fluent was such that, first, the basic boundary of water level is specified (based on Figure 2). Then equations with the time interval 0.005 continued until correlation were achieved. Correlation criterion for each variable was selected based on the residue of relative error (0.0001).

Fluent considers total flow field as the volume of separate controls. The integral of the dominant equations on the fluid flow is computed over control volume. Calculus equations are differentiated using various discrete schemes. To provide the geometry of the flow field, they are meshed using a Gambit preprocessor. The PRESTO scheme is also used for the discretion of pressure, quick scheme for the discretion of the displacement terms of momentum equations, the first rank upwind scheme for displacing turbulence equations, and also PISO algorithm for coupling velocity and pressure. Using sub-discount coefficients < 1 for pressure, momentum, and Reynolds tensions prevents from the divergence of the solution. It must be noted that the numerical model of this study is not totally convergent. Yet, analysis parameters get fixed after a while so that, after 2.5s, the analysis ends. During the solution operation, the time interval 0.001s was considered. Figure 3 shows the water level profile for the stepped spillway type I $(t = 2.5\text{s})$.

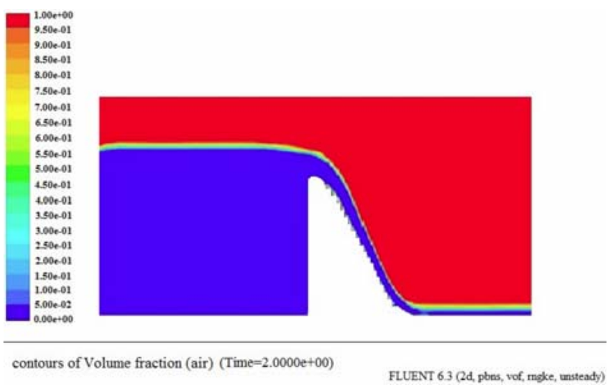


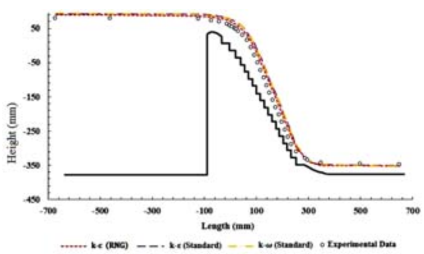
Figure 3. Water level profile for stepped spillway type I in 2.5s.

3. Results and Discussion

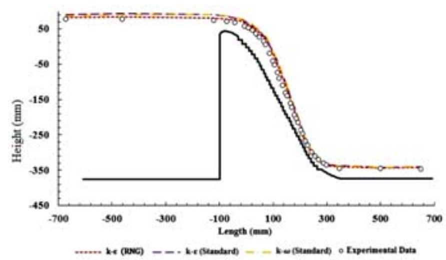
Results of the present analysis are examined in three parts, including the results of the water level profile, turbulence model errors and optimal turbulence model selection, and pressure distribution over the spillways in the various step arrangements in accordance with the optimal turbulence model. They are presented below.

3.1. Water level profile

In this subsection, in Figure 4, the water level profile for the stepped spillways types I, II, III and IV are presented. Results are presented based on three types of turbulence models: *k*-ε (standard), *k*-ε (RNG) and *k*-ω (standard). They are compared to experimental results of Tabbara et al. [14]. In these figures, the horizontal axis shows the longitudinal dimension (mm) and vertical axis shows water level height (mm).



(a)



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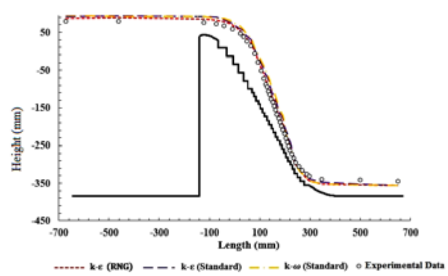

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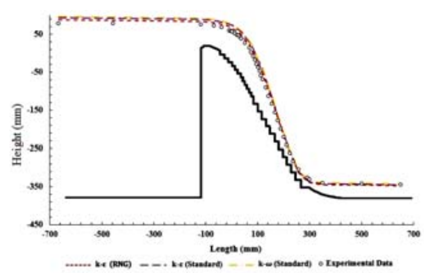


(d)

Figure 4. Results of the water level profile for a stepped spillway (a) type I, (b) type II, (c) type III, (d) type IV.

In this study, the water level profile is examined by the fluid volume method. In VOF method, a varying function known as α is used. This is the water volume component in a computational cell. If $\alpha = 1,$ then it shows that the cell is full of water. If $\alpha = 0,$ then it shows that the cell is full of air. For $0 < \alpha < 1,$ a percentage of the cell is water and another is air. Thus, considering the free level, in a certain volume component, it is possible to set the free level of the flow. In the present study, the free flow level in the volume component of water is defined as 0.5. By solving the continuity equation (8) for the water volume component α, it is possible to determine the volume component in the total field:

$$\frac{\partial \alpha}{\partial t} + u \frac{\partial \alpha}{\partial x} + v \frac{\partial \alpha}{\partial y} = 0. \tag{8}$$

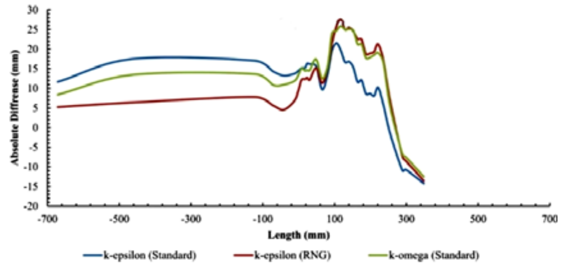
Based on Figures 5-7, the water level profile obtained was acceptable for all four types of the stepped spillway so that:

(1) Water level is closer to the spillway curve when passing over it. The curved line directly continues from the edge of the steps. It becomes curved at the bottom of the steps.

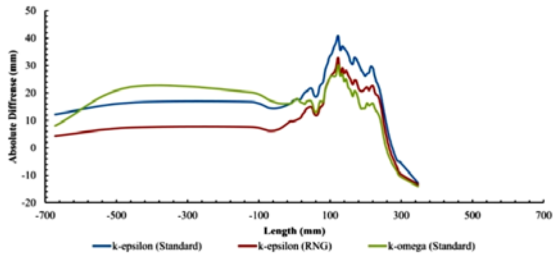
(2) In addition to the acceptable assessment of the water level profile obtained from numerical simulations during various turbulence models, its comparison with the experimental results shows agreement between numerical results and experimental results obtained with Tabbara et al. [14].

3.2. Turbulence models errors

The difference between the results of the respective turbulence models and experimental results is illustrated in Figure 5. Based on these figures where the horizontal axis indicates the longitudinal dimension (mm) and the vertical axis indicates the difference of the results from the experimental results of Tabbara et al. [14]. Further, as the vertical axis numbers (aside from their notation) approach to zero, better will be correspondence with the experimental results. Based on these diagrams, it is also possible to distinguish the area and the more efficient model.



(a)



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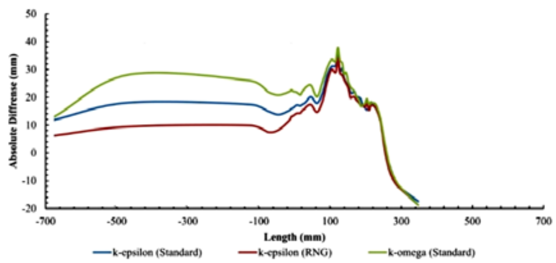

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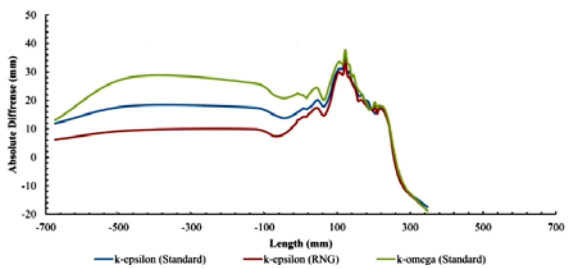


(d)

Figure 5. Differences between turbulence models results and laboratory results for a stepped spillway (a) Type I, (b) Type II, (c) Type III, (d) Type IV.

Root mean square error (RMSE) values are computed for respective turbulence models and experimental results of Tabbara et al. [14] by equation (9):

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N} (y_{mi} - y_{pi})^2}{N}}. \tag{9}$$

In this regard, N is the number of data, y_{mi} the numerical model values, and y_{pi} the experimental model values. The smaller the RMSE values are, the better the prediction of the numerical simulation. Table 2 shows the RMSE values for four types of stepped spillways.

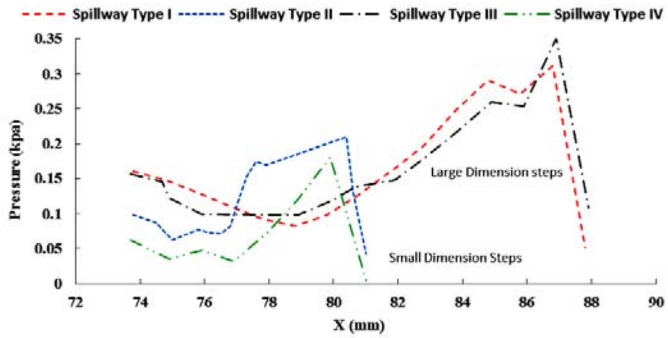
Table 2. RMSE for all four types of stepped spillways

Stepped spillway type	*k*-ε (standard)	*k*-ε (RNG)	*k*-ε (standard)
I	16.80	16.56	17.15
II	22.70	18.21	18.48
III	16.55	15.10	18.93
IV	14.26	11.03	17.68

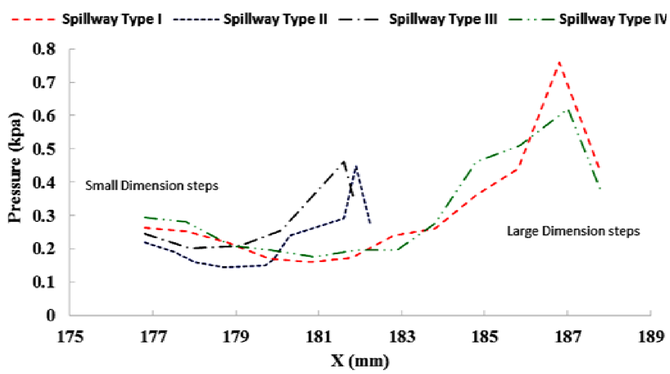
According to Table 2, the *k*-ε (RNG) turbulence model is selected as the desirable turbulence model in modeling the stepped spillways.

3.3. Pressure distribution over spillway

Negative pressure leads to destructive phenomenon cavitation. Hence, the examination of the pressure distribution over the spillways is considered among the main parameters in evaluating the hydraulic function of the construction. Figure 6 shows the diagram of the pressure distribution over the top and bottom parts of four models on *X* direction. As seen in all models, pressure decreases along with the direction so that the reduction reaches its peak value near the edge of the steps, then increases when water falls off the step to the next step floor, and again returns to its descending trend. By comparing the first and third types of the stepped spillways to the second and fourth types of steps, it can be found out that pressure in a stepped spillway with fewer steps and larger dimensions is larger than the one that has more steps with smaller dimensions.



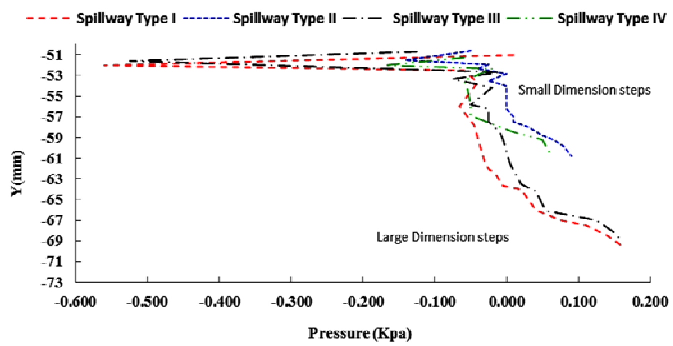
(a)



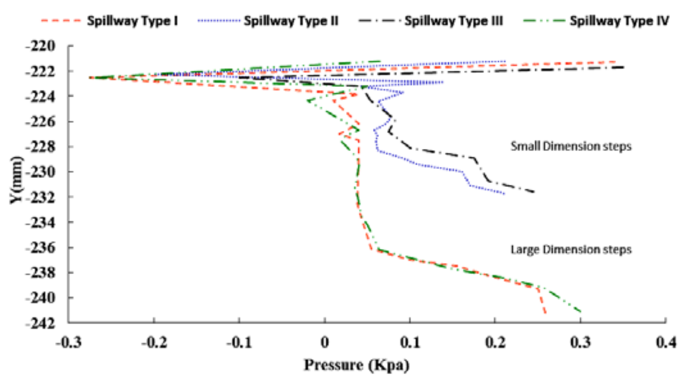
(b)

Figure 6. Pressure distribution diagram for four types of stepped spillways on *X* direction (a) upper section, (b) lower section.

Figure 7 shows the diagram of the pressure distribution over the top and bottom parts of four models on *Y* direction. As seen, pressure is minimum and negative at the edge of the step. It increases when the flow falls off. The spillways with various arrangements of the steps were compared. It was observed that the pressure values are the same as the values in *X* direction, that is, larger in a stepped spillway with fewer steps and larger dimensions.



(a)



(b)

Figure 7. Pressure distribution diagram for four types of stepped spillways on *Y* direction (a) upper section, (b) lower section.

4. Conclusion

In this study, four stepped spillways with various arrangements were simulated using Fluent. Turbulence models including k-ε (standard), k-ε (RNG) and k-ω (standard) were examined. First, the stepped spillways were analyzed based on three turbulence models. Results of the profile of the water levels were compared to the experimental results of Tabbara et al. [14]. This comparison was done by numerical simulations using various turbulence models. Numerical results showed acceptable correspondence with the experimental data. Investigating diagrams of numerical results against experimental results, the part of the area where the turbulent models have better performance can be identified. Generally, turbulence model k-ε (RNG) showed more acceptable results. It must be noted that other turbulence models lead to model divergence.

Based on the pressure distribution models obtained from k-ε (RNG) model simulation in two X and Y directions for two top and bottom parts of the spillway, it was seen that the trend of the pressure distribution is the same for four spillways. That is, pressure along the flow has a descending trend to the edge of the step. When water falls off, the flow has an ascending trend due to hitting against the step floor. Then it keeps on its descending trend. Studying the pressure distribution diagrams of the models in Y direction showed that minimum pressure occurs on the edge of the step. It is negative. Then it increases due to the flow fall. Generally, the pressure values for the spillway are larger with fewer numbers of steps with larger dimensions than for the reverse state.

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