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ON GENERALIZED *w*-CLOSED SETS IN ASSOCIATED WEAK SPACES

Won Keun Min* **and Young Key Kim**

Department of Mathematics
Kangwon National University
Chuncheon 200-701, Korea

Department of Mathematics
MyongJi University
Yongin 449-728, Korea

Abstract

The purpose of this note is to introduce the notion of generalized w_τ-closed $(w_\tau\text{-open})$ sets in an associated w_τ-space and to study its properties. In particular, we find the conditions of continuous functions to preserve generalized w_τ-closed sets or generalized w_τ-open sets.

1. Introduction

Siwiec [13] introduced the notions of weak neighborhoods and weak base in a topological space. We introduced the weak neighborhood systems defined by using the notion of weak neighborhoods in [8]. The weak

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*Corresponding author

neighborhood system induces a weak neighborhood space which is independent of neighborhood spaces [2] and general topological spaces [1]. The notions of weak structure, *w*-space, *W*-continuity and W^*-continuity were investigated in [9]. In fact, the set of all *g*-closed subsets [3] in a topological space is a kind of weak structure. Moreover, in [10], we introduced the notion of an associated weak space (simply, associated w_τ-space) containing a given topology τ. The one purpose of our research is to generalize w_τ-open sets in an associated weak space w_τ in the similar way introduced by Levine [3] in topological spaces. So we introduce the notion of generalize w_τ-open sets (generalize w_τ-closed sets) in an associated weak space w_τ and study its properties. In particular, we have the following theorems: (A) *If f is continuous and* W^**-closed, then for every* gw_τ*-open subset B in Y,* $f^{-1}(B)$ *is* gw_τ*-open*. (B) *If f is* W^**-continuous and closed, for every* gw_τ*-closed set A in X,* $f(A)$ *is* gw_τ*-closed*.

2. Preliminaries

Definition 2.1 [9]**.** Let *X* be a nonempty set. A subfamily w_X of the power set $P(X)$ is called a *weak structure* on *X* if it satisfies the following:

(1) $\emptyset \in w_X$ and $X \in w_X$.

(2) For $U_1, U_2 \in w_X$, $U_1 \cap U_2 \in w_X$.

Then the pair (X, w_X) is called a *w-space* on *X*. Then $V \in w_X$ is called a *w-open set* and the complement of a *w*-open set is a *w-closed set*.

The collection of all *w*-open sets (resp., *w*-closed sets) in a *w*-space *X* will be denoted by $WO(X)$ (resp., $WC(X)$). We set $W(x) = \{U \in WO(X) : x \in U\}$.

Let *S* be a subset of a topological space *X*. The closure (resp., interior) of *S* will be denoted by *clS* (resp., *intS*). A subset *S* of *X* is called a *preopen set* [6] (resp., α-open set [12], *semi-open* [4]) if $S \subset \text{int}(cl(S))$ (resp., $S \subset$

$\text{int}(cl(\text{int}(S))),\ S \subset cl(\text{int}(S)))$. The complement of a preopen set (resp., α-open set, *semi-open*) is called a *preclosed set* (resp., α-closed set, *semi-closed*). The family of all preopen sets (resp., α-open sets, semi-open sets) in X will be denoted by $PO(X)$ (resp., $\alpha(X)$, $SO(X)$). We know the family $\alpha(X)$ is a topology finer than the given topology on X.

Moreover, a subset S of X is said to be *g-closed* [3] if $cl(A) \subset U$ whenever $A \subset U$ and U is open in X.

Then the family $GO(X) = \{U \subseteq X : U$ is g-open$\}$, $O(X) = \{U \subseteq X : U$ is open$\}$ and $CL(X) = \{F \subseteq X : F$ is closed$\}$ are all weak structures on X. But $PO(X)$, $GPO(X)$ and $SO(X)$ are not weak structures on X. A subfamily m_X of the power set $P(X)$ of a nonempty set X is called a *minimal structure* on X [5] if $\varnothing \in w_X$ and $X \in w_X$. Thus clearly every weak structure is a minimal structure.

Let (X, w_X) be a w-space. For a subset A of X, the *w-closure* of A and the *w-interior* of A are defined as follows:

(1) $wC(A) = \bigcap\{F : A \subseteq F,\ X - F \in w_X\}$.

(2) $wI(A) = \bigcup\{U : U \subseteq A, U \in w_X\}$.

Theorem 2.2 [9]. *Let* (X, w_X) *be a w-space and* $A \subseteq X$. *Then the following things hold*:

(1) *If* $A \subset B$, *then* $wI(A) \subset wI(B)$; $wC(A) \subset wC(B)$.

(2) $wI(wI(A)) = wI(A)$; $wC(wC(A)) = wC(A)$.

(3) $wC(X - A) = X - wI(A)$; $wI(X - A) = X - wC(A)$.

(4) *If* A *is w-closed* (*resp*., *w-open*), *then* $wC(A) = A$ (*resp*., $wI(A) = A$).

3. Main Results

First, we recall the notion of an associated w-space with τ introduced in

[10]. Let X be a nonempty set and let (X, τ) be a topological space. A subfamily w of the power set $P(X)$ is called an *associated weak structure* (simply, w_τ) on X if $\tau \subseteq w$ and w is a weak structure. Then the pair (X, w_τ) is called an *associated w-space* with τ.

Definition 3.1. Let (X, w_τ) be an associated w-space with a topology τ and $A \subseteq X$. Then A is called a *generalized* w_τ*-closed set* (simply, gw_τ*-closed set*) if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is w-open.

Remark 3.2. (1) If $w_\tau = \tau$, then the generalized w_τ-closed set is exactly a generalized closed set in sense of Levine in [3].

(2) If w_τ is the family of all g-open sets in sense of Levine, then the generalized w_τ-closed set is exactly a g^*-closed set [14].

(3) Obviously, every w-closed set is generalized w_τ-closed, but in general, the converse is not true as the next example.

Example 3.3. Let $X = \{a, b, c, d\}$, a topology $\tau = \{\emptyset, \{b\}, X\}$ and a w-structure $w_X = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, d\}, X\}$ in X. For $A = \{a, c\}$, obviously A is gw_τ-closed but not w-closed.

We recall that: A is called a *generalized w-closed set* (simply, *gw-closed set*) [11] if $wC(A) \subseteq U$, whenever $A \subseteq U$ and U is w-open.

Remark 3.4. Let (X, w_τ) be an associated w-space with a topology τ and $A \subseteq X$. Since $wC(A) \subseteq cl(A)$, every gw_τ-closed set is gw-closed. But, the converse may not be true as the next example.

Example 3.5. In Example 3.3, let $A = \{d\}$. Then $wC(A) = A$, so A is gw-closed. Consider a w-open set $U = \{a, d\}$ such that $A \subseteq U$. Since $cl(A) = \{a, c, d\}$, A is not gw_τ-closed.

Theorem 3.6. *Let* (X, w_τ) *be an associated w-space with a topology* τ. *Then the union of two* gw_τ*-closed sets is a* gw_τ*-closed set*.

Proof. Let A and B be any two gw_τ-closed sets. Let G be any w-open set such that $A \bigcup B \subseteq G$. Then $A \subseteq G$ and $B \subseteq G$. Since A and B are gw_τ-closed sets, $cl(A) \subseteq G$ and $cl(B) \subseteq G$. So, $cl(A \bigcup B) = cl(A) \bigcup cl(B) \subseteq G$. Hence $A \bigcup B$ is gw_τ-closed. □

In general, the intersection of two gw_τ-closed sets is not gw_τ-closed:

Example 3.7. Let $X = \{a, b, c, d\}$, a topology $\tau = \{\varnothing, \{b\}, X\}$ and a w-structure $w_X = \{\varnothing, \{a, c\}, \{a\}, \{b\}, \{c\}, \{a, d\}, X\}$ in X. Now, consider $A = \{a, b, c\}$, and $B = \{a, c, d\}$. Then A and B are gw_τ-closed. But $A \bigcap B = \{a, c\}$ is not gw_τ-closed because $\{a, c\}$ is w-open and $cl(\{a, c\}) = \{a, c, d\}$.

Theorem 3.8. *Let (X, w_τ) be an associated w-space with a topology τ. Then if A is a gw_τ-closed set, then $cl(A) - A$ contains no non-empty w-closed set.*

Proof. Suppose that there is a w-closed set F such that $F \subseteq cl(A) - A$. Then $A \subseteq X - F$, and since $X - F$ is w-open and A is gw_τ-closed, $cl(A) \subseteq X - F$ and $F \subseteq X - cl(A)$. It implies that $F \subseteq cl(A) \bigcap (X - cl(A)) = \varnothing$. Hence, $F = \varnothing$. □

Corollary 3.9. *Let (X, w_τ) be an associated w-space with a topology τ. Then if A is a gw_τ-closed set, then $cl(A) - A$ contains no non-empty closed set.*

Proof. Since X is an associated w-space, every closed set is w-closed. The corollary is obtained by the above theorem. □

In Theorem 3.8, the converse is not true as shown in the next example.

Example 3.10. Let $X = \{a, b, c, d\}$, a topology $\tau = \{\varnothing, \{b, c\}, X\}$ and a w-structure $w_X = \{\varnothing, \{a\}, \{b, c\}, \{a, d\}, X\}$ in X. Consider $A = \{a\}$. Note $cl(A) = \{a, d\}$ and $cl(A) - A = \{a, d\} - \{a\} = \{d\}$. Since $\{d\}$ is not w-closed, $cl(A) - A$ contains no non-empty w-closed set, but A is not w-closed.

Theorem 3.11. *Let* (X, w_τ) *be an associated w-space with a topology* τ. *Then if A is a* gw_τ *-closed set and* $A \subseteq B \subseteq cl(A)$, *then B is* gw_τ *-closed*.

Proof. Let U be any w-open set such that $B \subseteq U$. Then $A \subseteq U$ and $cl(B) \subseteq cl(A)$. Since A is a gw_τ -closed set, $cl(B) \subseteq cl(A) \subseteq U$. It implies that B is gw_τ -closed set. □

Theorem 3.12. *Let* (X, w_τ) *be an associated w-space with a topology* τ. *Then if A is a* gw_τ *-closed set and* $A \subseteq B \subseteq wC(A)$, *then B is* gw_τ *-closed*.

Proof. From $wC(A) \subseteq cl(A)$ and Theorem 3.11, B is gw_τ -closed set. □

Corollary 3.13. *Let* (X, w_τ) *be an associated w-space with a topology* τ. *Then if A is a* gw_τ *-closed set and* $A \subseteq B \subseteq wC(A)$, *then B is gw-closed*.

Proof. It follows from the fact that every gw_τ -closed set is *gw*-closed. □

Definition 3.14. Let (X, w_τ) be an associated w-space with a topology τ and $A \subseteq X$. Then A is called a *generalized* w_τ *-open set* (simply, gw_τ *-open set*) if $X - A$ is gw_τ -closed.

Theorem 3.15. *Let* (X, w_τ) *be an associated w-space with a topology* τ *and* $A \subseteq X$. *Then A is* gw_τ *-open if and only if* $F \subseteq \text{int}(A)$ *whenever* $F \subseteq A$ *and F is w-closed*.

Proof. It follows from Definition 3.1. □

Theorem 3.16. *Let* (X, w_τ) *be an associated w-space with a topology* τ. *Then the intersection of two* gw_τ *-open sets is a* gw_τ *-open set*.

Proof. It is obvious from Theorem 3.6. □

In general, the union of two gw_τ -open sets is not gw_τ -open (see Example 3.7).

Theorem 3.17. *Let* (X, w_τ) *be an associated w-space with a topology* τ

and $A \subseteq X$. *Then if A is* gw_τ *-open, then* $U = X$, *whenever* $\text{int}(A) \cup (X - A) \subseteq U$ *and U is w-open.*

Proof. Let U be any w-open set and $\text{int}(A) \cup (X - A) \subseteq U$. Then $X - U \subseteq (X - \text{int}(A)) \cap A = cl(X - A) \cap A = cl(X - A) - (X - A)$. Since $X - A$ is gw_τ-closed, by Theorem 3.8, the w-closed set $X - U$ must be empty. Hence, $U = X$. □

Corollary 3.18. *Let* (X, w_τ) *be an associated w-space with a topology* τ *and* $A \subseteq X$. *Then if A is* gw_τ *-open, then* $U = X$, *whenever* $\text{int}(A) \cup (X - A) \subseteq U$ *and U is open.*

Proof. Since every open set is w-open, it follows from the above theorem. □

Theorem 3.19. *Let* (X, w_τ) *be an associated w-space with a topology* τ. *Then if A is a* gw_τ *-open set and* $wI(A) \subseteq B \subseteq A$, *then B is* gw_τ *-open.*

Proof. It is similar to the proof of Theorem 3.11. □

Theorem 3.20. *Let* (X, w_τ) *be an associated w-space with a topology* τ. *Then if A is a* gw_τ *-open set and* $\text{int}(A) \subseteq B \subseteq A$, *then B is* gw_τ *-open.*

Proof. Since $\text{int}(A) \subseteq wI(A)$, it is obtained from Theorem 3.19. □

Theorem 3.21. *Let* (X, w_τ) *be an associated w-space with a topology* τ. *Then if A is a* gw_τ *-closed set, then* $cl(A) - A$ *is* gw_τ *-open.*

Proof. Suppose that A is a gw_τ-closed set. Then by Theorem 3.8, the empty set is the only one w-closed subset of $cl(A) - A$. So, for the only w-closed subset $\varnothing$ of $cl(A) - A$, $\varnothing \subseteq cl(A) - A$ and $\varnothing \subseteq \text{int}(cl(A) - A)$. From Theorem 3.15, $cl(A) - A$ is gw_τ-open. □

Theorem 3.22. *Let* (X, w_τ) *be an associated w-space with a topology* τ. *Then if A is a* gw_τ *-open set, then* $\text{int}(A) \cup (X - A)$ *is* gw_τ *-closed.*

Proof. Suppose that A is a gw_τ-open set. Then by Theorem 3.17, the whole set X is the only one w-open set containing $\text{int}(A) \cup (X - A)$. So, $\text{int}(A) \cup (X - A) \subseteq X$ and $cl(\text{int}(A) \cup (X - A)) \subseteq X$. Hence, by definition of gw_τ-closedness, $\text{int}(A) \cup (X - A)$ is gw_τ-closed. □

Let (X, w_τ) be an associated w-space with a topology τ. For a subset A of X, gw_τ-closure of A and gw_τ-interior of A are defined as the following:

(1) $gw_\tau C(A) = \bigcap\{F : A \subseteq F,\ F \text{ is } gw_\tau\text{-closed}\}$.

(2) $gw_\tau I(A) = \bigcup\{U : U \subseteq A,\ U \text{ is } gw_\tau\text{-open}\}$.

Theorem 3.23. *Let (X, w_τ) be an associated w-space with a topology τ and $A \subseteq X$.*

(1) *If A is gw_τ-open, then $gw_\tau I(A) = A$.*

(2) *If A is gw_τ-closed, then $gw_\tau C(A) = A$.*

Proof. Obvious. □

But the converses in the above theorem are not always true as shown in the next example.

Example 3.24. In Example 3.7, let $F = \{a, c\}$. Since $\{a, b, c\}$ and $\{a, c, d\}$ are gw_τ-closed sets, $gw_\tau C(F) = \{a, c\}$. But from the fact that $\{a, c\}$ is w-open and $wC(\{a, c\}) = \{a, c, d\}$, F is not gw_τ-closed. Similarly, we can show that the converse of (2) in Theorem 3.23 is not true, in general.

Theorem 3.25. *Let (X, w_τ) be an associated w-space with a topology τ and $A, B \subseteq X$.*

(1) *If $A \subseteq B$, then $gw_\tau I(A) \subseteq gw_\tau I(B)$ and $gw_\tau C(A) \subseteq gw_\tau C(B)$.*

(2) $gw_\tau C(X - A) = X - gw_\tau I(A)$; $gw_\tau I(X - A) = X - gw_\tau C(A)$.

(3) *$x \in gw_\tau I(A)$ if and only if there exists a gw_τ-open set U containing x such that $U \subseteq A$.*

(4) *$x \in gw_\tau C(A)$ if and only if $A \cap V \neq \varnothing$ for all gw_τ-open set V containing x.*

Proof. Obvious. □

Theorem 3.26. *Let (X, w_τ) be an associated w-space with a topology τ and $A, B \subset X$.*

(1) $\varnothing = gw_\tau C(\varnothing)$.

(2) $A \subseteq gw_\tau C(A)$.

(3) $gw_\tau C(A \cup B) = gw_\tau C(A) \cup gw_\tau C(B)$.

(4) $gw_\tau C(gw_\tau C(A)) = gw_\tau C(A)$.

Proof. (1) and (2) are obvious.

(3) It is obvious that $gw_\tau C(A \cup B) \supseteq gw_\tau C(A) \cup gw_\tau C(B)$. We only show that $gw_\tau C(A \cup B) \subseteq gw_\tau C(A) \cup gw_\tau C(B)$. Suppose that $x \notin gw_\tau C(A) \cup gw_\tau C(B)$. Then there exist gw_τ-closed sets F_1 and F_2 such that $x \notin F_1$ and $A \subseteq F_1$; $x \notin F_2$ and $B \subseteq F_2$. So $x \notin F_1 \cup F_2$ and $A \cup B \subseteq F_1 \cup F_2$. From Theorem 3.6, $F_1 \cup F_2$ is gw_τ-closed, and $x \notin gw_\tau C(A \cup B)$. So $gw_\tau C(A \cup B) \subseteq gw_\tau C(A) \cup gw_\tau C(B)$.

(4) It is sufficient to show that $gw_\tau C(gw_\tau C(A)) \subseteq gw_\tau C(A)$. For any gw_τ-closed set F satisfying $A \subseteq F$, since $gw_\tau C(A) \subseteq gw_\tau C(F) = F$,

$$gw_\tau C(gw_\tau C(A)) = \bigcap\{K : gw_\tau C(A) \subseteq K,\ K \text{ is } gw_\tau\text{-closed}\}$$

$$\subseteq \bigcap\{F : A \subseteq F,\ F \text{ is } gw_\tau\text{-closed}\} = gw_\tau C(A).$$ □

Theorem 3.27. *Let (X, w_τ) be an associated w-space with a topology τ and $A, B \subset X$.*

(1) $X = gw_\tau I(X)$.

(2) $gw_\tau I(A) \subseteq A$.

(3) $gw_\tau I(A \cap B) = gw_\tau I(A) \cap gw_\tau I(B)$.

(4) $gw_\tau I(gw_\tau I(A)) = gw_\tau I(A)$.

Proof. These are easily obtained by Theorem 3.25 and Theorem 3.26. □

Finally, we have a topology induced by gw_τ -open sets as the following:

Theorem 3.28. *Let* (X, w_τ) *be an associated w-space with a topology* τ. *Then the family* $w_\tau^* = \{U \subseteq X : U = gw_\tau I(U)\}$ *is a topology containing the weak structure w, that is,* $\tau \subseteq w \subseteq w_\tau^*$.

Proof. It is easily obtained by Theorem 3.27. □

We recall that: Let (X, w_X) be a *w*-space and $A \subseteq X$. Then *A* is called a *generalized w-open set* (simply, *gw-open set*) [11] if $X - A$ is *gw*-closed.

Then A is generalized *w*-open if and only if $F \subseteq wI(A)$ whenever $F \subseteq A$ and *F* is *w*-closed.

Let $f : (X, w_\tau) \to (Y, \mu)$ be a function on an associated *w*-space with τ and a topological space (Y, μ). Then *f* is said to be

(1) *WO-continuous* [10] if for $x \in X$ and $V \in O(f(x))$, there is $U \in W(x)$ such that $f(U) \subseteq V$;

(2) W^* *-continuous* [9] if for every $A \in W(f(x))$, $f^{-1}(A)$ is in $W(x)$.

Theorem 3.29 [10]. *Let* $f : (X, w_\tau) \to (Y, \mu)$ *be a function on an associated w-space with* τ *and a topological space* (Y, μ). *Then the following statements are equivalent*:

(1) *f is WO-continuous*.

(2) $f(wC(A)) \subseteq cl(f(A))$ *for* $A \subseteq X$.

(3) $wC(f^{-1}(V)) \subseteq f^{-1}(cl(V))$ *for* $V \subseteq Y$.

(4) $f^{-1}(\mathrm{int}(V)) \subseteq wI(f^{-1}(V))$ *for* $V \subseteq Y$.

Let X and Y be w-spaces. A function $f:(X, w_X) \to (Y, w_Y)$ is said to be

(1) W^**-closed* [11] if for every w-closed set F in X, $f(F)$ is a w-closed set in Y.

(2) *quasi-*W^**-closed* [11] if for $A \subseteq X$, $wC(f(A)) \subseteq f(wC(A))$.

In fact, there is no any relation between the notions of W^*-closed function is quasi-W^*-closed function.

Theorem 3.30. *Let* $f:(X, w_X) \to (Y, w_Y)$ *be a function on w-spaces X and Y. Then the following statements hold*:

(1) *If f is WO-continuous and* W^**-closed, then for every* gw*-open subset B in Y,* $f^{-1}(B)$ *is gw-open.*

(2) *If f is continuous and* W^**-closed, then for every* gw_τ*-open subset B in Y,* $f^{-1}(B)$ *is* gw_τ*-open.*

Proof. (1) Let B be any gw_τ-open subset B in Y and F be a w-closed set in X such that $F \subseteq f^{-1}(B)$. Now, we show that $F \subseteq wI(f^{-1}(B))$. Since f is W^*-closed, $f(F)$ is w-closed. Moreover, since B is gw_τ-open, $f(F) \subseteq \text{int}(B)$. From Theorem 3.29, it follows that $F \subseteq f^{-1}(\text{int}(B)) \subseteq wI(f^{-1}(B))$. Hence, $f^{-1}(B)$ is gw-open.

(2) It is similar to the proof of (1). □

Corollary 3.31. *Let* $f:(X, w_X) \to (Y, w_Y)$ *be a function on w-spaces X and Y. Then the following statements hold*:

(1) *If f is WO-continuous and* W^**-closed, then for every open subset B in Y,* $f^{-1}(B)$ *is gw-open.*

(2) *If f is continuous and* W^**-closed, then for every open subset B in Y,* $f^{-1}(B)$ *is* gw_τ*-open.*

Proof. From the fact that every open set is gw_τ-open set and the above theorem, the things are obtained. □

Theorem 3.32. *Let* $f : (X, w_X) \to (Y, w_Y)$ *be a function on w-spaces X and Y. Then the following statements hold*:

(1) *If f is* W^**-continuous and closed, for every* gw_τ*-closed set A in X,* $f(A)$ *is* gw_τ*-closed.*

(2) *If f is* W^**-continuous and quasi-*W^**-closed, for every* gw_τ*-closed set A in X,* $f(A)$ *is gw-closed.*

Proof. (1) Let *A* be any gw_τ-closed subset *A* in *X*, and *U* be a *w*-open set in *Y* such that $f(A) \subseteq U$. Now, we show that $cl(f(A)) \subseteq U$. Since *f* is W^*-continuous and *A* is *gw*-closed, $f^{-1}(U)$ is *w*-open and $cl(A) \subseteq f^{-1}(U)$. Since *f* is closed, $cl(f(A)) \subseteq f(cl(A)) \subseteq ff^{-1}(U) \subseteq U$, and hence, $f(A)$ is gw_τ-closed.

(2) Let *A* be any gw_τ-closed subset *A* in *X*, and *U* be a *w*-open set in *Y* such that $f(A) \subseteq U$. Now, we show that $cl(f(A)) \subseteq U$. Since *f* is W^*-continuous and *A* is *gw*-closed, $f^{-1}(U)$ is *w*-open and $cl(A) \subseteq f^{-1}(U)$. Since *f* is quasi-$W^*$-closed, $wC(f(A)) \subseteq f(cl(A)) \subseteq ff^{-1}(U) \subseteq U$, and hence, $f(A)$ is *gw*-closed. □

We get directly the following corollary:

Corollary 3.33. *Let* $f : (X, w_X) \to (Y, w_Y)$ *be a function on w-spaces X and Y. Then the following statements hold*:

(1) *If f is* W^**-continuous and closed, for every closed set A in X,* $f(A)$ *is* gw_τ*-closed.*

(2) *If f is* W^**-continuous and quasi-*W^**-closed, for every closed set A in X,* $f(A)$ *is gw-closed.*

4. Conclusion

In this paper, we introduced the notion of generalized *w*-closed (*w*-open) sets in an associated weak space and studied some basic properties. In Theorem 3.30, particularly, we established that if f is continuous and W^*-closed (resp., *WO*-continuous and W^*-closed), then for every gw_τ-open subset B in Y, $f^{-1}(B)$ is gw_τ-open (resp., *gw*-open). In the next research, we will intensively investigate the notions of functions from an associated weak space to an associated weak space satisfying for every gw_τ-open subset (or open set) in the codomain, its preimage is gw_τ-open.

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