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DEVELOPMENT OF ALGORITHMS AND SERRATION MODEL TO ADD DETAILS TO THE LEAF MARGIN OF THE LEAF SHAPE MODELING BY USING OFFSET APPROACH

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Abstract

The aim of this research is to propose method to add serration known as details to the leaf margin. We assume teeth if present are always found all around the margin, from the top to the bottom. For this purpose, the offset of the original shape is used. The main parts of the leaf are represented with several *B*-spline curves which also represent the offset. The details are applied as an offset to the underlying curve. A new algorithm is proposed to represent the pattern of the details. A new algorithm how the pattern combines to the margin is also explained. The results of the drawings are divided into three categories: satisfactory, acceptable and unsatisfactory. Expert botanist is referred to assess the drawing result. The findings show that the geometry of the teeth is satisfactory, except for some minor distortion. As far the implication, this research allows novice botanists and amateurs to readily see a picture which they might find it hard to visualize before.

1. Introduction

The *multiresolution* representations have been developed based on wavelet for parametric curves [1]. It can be generalized to tensor-product surfaces, and to surfaces of arbitrary topological type [2]. Meanwhile, for the early general *multiresolution* editing or deformation techniques especially for parametric curves by using *B-splines* have been explored in [3] and [4]. Further for curve editing methods in *multiresolution*, the research has been carried out in [5] focusing on the constraint of constant enclosed area. In [6], is discussed the preservation of the length in *multiresolution* editing of the curve, while in [7] is presented the three dimensional representation.

To add the details to the leaf margin, the *multiresolution* approach which uses wavelets was tried in [3]. However, their approach was limited by the number of control points. Hence, it would not generate the desired number of control points. Alternatively, two *B-spline* curves are applied, where one of them is the *offset* curve. Many articles such as [8-11] have discussed the offset of *B-spline* curves. Some research like [12] relates surface to determine the offset. Most of the literatures explain that the *offset* curve can be defined by new control points, where each control point is the *offset* of a control point of the original curve. Almost similar work is found in [13] and [14] which use a geometric algorithm [15] to produce a normal *B-spline* curve. However, in order to generate the details onto *B-spline* curves requires complex mathematical understanding. Instead, the use of an *offset* point from equally spaced points on the *B-spline* curve is more practical.

2. To Model the Details to the Leaf Margin

In general, to add details to the leaf margin, the teeth are produced in a straight line (which is a part of the composite curve), and then wrapped onto the composite curve. The idea is to have the first sampling points that are equally spaced for the composite curves, while the second sampling points are for the margin which involve parts of the composite curves. Then they are combined together with a formula which is explained later to make a drawing of the leaf. While adding the teeth, the beginning and end points are specified at certain locations on the leaf. This is because, based on the observation of the flora, the teeth are not always found along the whole of the leaf margin. In most cases, the teeth were found along a certain part of the leaf margin. For this research, we focus on *dentate*.

Let us consider a simple example of a leaf description in [16] which refers to an *ovate* leaf with *dentate* teeth on the leaf margin. In this example, the basic *ovate* shape is unchanged while the *dentate* geometrically is the modification of a basic *ovate* shape. This justifies the fact that there is something in a form of $D(u) + m(u)$, where u is the equally spaced sampling point, $D(u)$ is the basic leaf shape of the composite curve, and $m(u)$ is the modification. When adding the teeth to the leaf margin, it is necessary for the teeth to be locally oriented with the leaf margin. Particularly, if the leaf is pointed in a certain direction, then it would be necessary for the teeth to also point in that direction. This results in an equation in a form of $D(u) + d(u)N(u)$, where u is the equally spaced sampling point, $D(u)$ is the basic leaf shape of the composite curve, $d(u)$ is the height of a tooth, and $N(u)$ normal to the shape which makes the tooth locally oriented. The $d(u)$

also can be said to be the *pattern* in a straight line which represents *dentate* geometry. It is then necessary to take that pattern and put it on the existing leaf margin. Therefore, that makes the teeth locally pointed in the right direction.

Firstly, we compute the arc length of the leaf margin, particularly, the *B-spline* curves. If $A = [x(t_A), y(t_A)]$ and $B = [x(t_B), y(t_B)]$, then the distance along the curve between A and B is s. If $s(t)$ is the length of the curve segment between the points on the curve corresponding to the value $t = t_0$, then the length of the *B-spline* curve is calculated by $s(t) = \int_{t_0}^{t} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt.$ As the leaf is composed of the tip, side and base, the arc length is therefore computed separately to discover the length for each item. The symbol L denotes the whole length of the *B-spline* curve. Once the length of the leaf margin has been determined, the pattern (*dentate*) is then set in a straight line. The amount of teeth on the straight line is then determined. The *dentate* is set to have a width (w) and height (d), where both w and d are real numbers. By using w and L, the amount of teeth (E) can be determined on the straight line approximately by using the expression $E = \frac{L}{w}.$ For simplicity, the E value was rounded. In order to draw the teeth, it is necessary to discover the amount of point needed for each tooth. It was decided to use equally spaced points p with the amount of $p = 10E + 1$ for each tooth for *dentate* as shown in Figure 2.1.

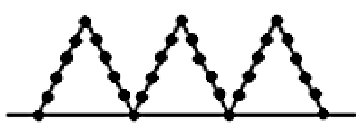

Figure 2.1. Dentate teeth.

Depending on E, the amount of equally spaced points needed on the straight line and on the *B-spline* curve (leaf margin) is determined. The *bisection* method is used to map the arc length value to a parameter value within the parameter interval. In particular, mappings from equally spaced arc length values are used as $0, \frac{1}{p-1}L, \frac{2}{p-1}L, \ldots, \frac{p-2}{p-1}L, \frac{p-1}{p-1}L.$ Next is an explanation of how *dentate* is generated. Let *u* be the equally spaced sampling point. The principle used to produce *dentate* is as

$$d(u) = \begin{cases} d = u & \text{if } u_{1, 3, \ldots, p-2} \le u \le u_{2, 4, \ldots, p-1}; \\ d = u_p - u & \text{if } u_{2, 4, \ldots, p-1} \le u \le u_{3, 5, \ldots, p} \end{cases}$$

and this is illustrated in Figure 2.2.

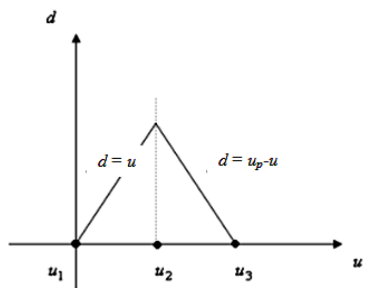

Figure 2.2. Dentate.

In order to produce *dentate*, it is necessary to take into account where the teeth are found on the leaf margin. There are two possibilities. Firstly, the teeth are found at the beginning and at the end of the leaf margin. Secondly, the teeth begin somewhere at a distance from the top and end at some distance from the bottom of the leaf. Observation from the flora shows that most of the cases agreed with the second possibility. In order to decide the position where the teeth should start and stop, it was decided that the teeth should start $1/8$ of the arc length from the top, and stop $1/6$ of the arc length from the bottom. To add the teeth to the leaf margin, the following algorithm applies:

(1) to produce the first sample points, the first curve of the whole leaf is sampled with 100 equally points, denoted as p_i; $i = 1, \ldots, 100,$

(2) to produce the second, the teeth are generated by using the *offset* approach, producing points t_j; $j = 1, 2, ..., k$; where k is an integer which is determined by the type of the teeth,

(3) the sample curve A_i is given by

$$A_i = [p_1, p_2, ..., \max(p_i) < t_j, t_1, t_2, ..., t_k, \min(p_i) < t_k, ..., p_{100}].$$

3. Drawing Evaluation and Results

70 samples of leaf description which have *dentate* are carefully chosen from botanical books [16-20]. To produce an outline drawing, a program in Matlab has been written to test the idea. An expert botanist was referred to assess the drawing results. The quality of the leaf drawing is divided into three categories which are *satisfactory*, *acceptable* and *unsatisfactory* [21, 22], focuses to the teeth size. Table 3.1 shows the analysis of mean for the dimension of teeth size for all cases. The mean value of the overall teeth size for *dentate* is 2.65, which shows that it falls under satisfactory category.

Table 3.1. Mean score for dentate

Dimension	Mean score
Overall size for dentate	2.65

4. Conclusion

The main goal of this research is to develop pattern for *dentate* and to propose an algorithm to add the details to the leaf margin. The overall results show 88.3% of satisfactory cases. It shows that our approach is practical and successful. However, the result is not perfect due to some reasons. The positions where the teeth should start and stop are not always correct because they might vary among the species. However, the difference between the positions of the teeth is not too obvious. Besides, there are some distortions to the teeth when bending the straight line. However, as long as the teeth are small compared to the overall size of the leaf, the distortion is small. The system does not draw a precise mathematical figure. The distortion is not important in the overall picture. Finally, this research is perhaps a step to a fully automated plant identification tool to convert an image of a plant to its textual description.

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