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A SUPPLYING CHAIN SCHEDULING WITH SUBCONTRACTING AND DELIVERY

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Abstract

We address an analytical scheduling model with subcontracting and delivery. Each job can be scheduled either on a single machine at a manufacturer or outsourced to a subcontractor. For a given set of jobs, the decisions we need to make include the selection of the subset of jobs to be outsourced and the schedule of all the jobs. The objective function in our scheduling model is to minimize the weighted sum of the number of tardy jobs and the total cost. We show our scheduling problem is binary NP-hard, and present a dynamic programming algorithm for it.

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1. Introduction and Problem Description

Subcontracting has received a lot of attention in manufacturing management research. When the demand level is beyond the in-house production capacity, the manufacturer may outsource some orders to available subcontractors so that all orders can be completed as early as possible. Many factors need to be taken into account in making subcontracting decisions, such as production cost, subcontracting cost, customer demand, delivery lead times, etc. There are a handful of existing papers that discuss subcontracting under machine scheduling models. For example, Bertrand and Sridharan [1] studied a make-to-order manufacturing environment where orders arrive over time randomly, and can either be processed in-house on a single machine or outsourced. The objective is to maximize the utilization of in-house capacity while minimizing tardiness in fulfilling orders. Qi [4] studied the production scheduling problem for a two-stage flow shop where there are options of subcontracting some operations to subcontractors. He considered a minimum makespan objective and analyzed various models for different situations of subcontracting.

We address an analytical scheduling model for a firm with an option of subcontracting in this paper. We assume that there is a single machine at the manufacturer's plant, and there is a subcontractor, who has a sufficient number of identical parallel machines, such that each of these machines will handle at most one job, possibly at a higher cost. Assuming that there is only one customer, who places n orders, i.e., there are n jobs to be processed. If job j is processed at the manufacturer's plant, then a processing time p_{0j}, a production cost w_{0j} and a delivery time s_{0j} are required. If job j is outsourced, then a processing time p_{1j}, a subcontracting cost w_{1j} and a delivery time s_{1j} are needed. Since there are plenty of vehicles at the processing places and each vehicle can transport one job at a time, each completed job can be transported directly from its processing place to the customer immediately. The delivery time of each job refers to the transportation time from its processing place to the customer. It is obvious that each job has two transportation times, which are determined by job's processing site. For convenience, the delivery time of job *j* is denoted by S_j, i.e., $S_j = s_{0j}$ if job *j* is scheduled on in-house machine and otherwise $S_j = s_{1j}$. Given a schedule, we denote C_j and d_j as the completion time and the due date of job *j*, respectively. All jobs are available at the time zero, and preemption is not allowed.

Let $U_j = 1$ if $C_j + S_j > d_j$ and $U_j = 0$ if $C_j + S_j \le d_j$. $\sum U_j$ denotes the number of tardy jobs. The objective function in our model is to minimize the weighted sum of the number of tardy jobs and the total cost. Using the notation introduced by Graham et al. [3], we denote the general form of our problem as $1 + \infty \| \lambda \sum U_j + (1 - \lambda)W$, where "1" indicates the number of the available in-house machines, "∞" indicates that the subcontractor has unlimited capacity, $\lambda \in (0, 1)$, and *W* denotes the whole sum of production cost and outsourcing cost. We can choose a weighting parameter $\lambda \in (0, 1)$, assign λ as the preference weight to $\sum U_j$, assign $(1 - \lambda)$ as the preference weight to *W*, and consider the weighted sum of these two measures. The key issue is then how to coordinate the in-house production and subcontracting in an efficient way.

The rest of the paper is organized as follows. In Section 2, we give the complexity analysis for the first problem $1 + \infty \| \lambda \sum U_j + (1 - \lambda)W$, and present a dynamic programming algorithm for it. Finally, we summarize our results in Section 3.

2. Problem $1 + \infty \| \lambda \sum U_j + (1 - \lambda)W$

Now, we show that the problem $1 + \infty \| \lambda \sum U_j + (1 - \lambda)W$ is NP-hard.

Theorem 2.1. *The problem* $1 + \infty \| \lambda \sum U_j + (1 - \lambda)W$ *is binary NP-hard*.

Proof. The proof can be done in polynomial reduction from the 2-partition problem [2], which is known to be NP-hard. The 2-partition problem is stated as follows:

2-partition. Given $n + 1$ integers $b_1, b_2, ..., b_n, B$ such that $\sum_j^n b_j = 2B,$ does there exist a subset $Q \subseteq N = \{1, 2, ..., n\}$ such that $\sum_{j \in Q} b_j = B$?

We construct the instance of the problem $1 + \infty \| \lambda \sum U_j + (1 - \lambda)W$ as follows:

- Number of jobs: $n + 1$.
- $\lambda = \dfrac{2B + 1}{2B + 2}.$
- $p_{0j} = b_j,\ p_{1j} = \dfrac{1}{2} p_{0j},$ for $j = 1, 2, ..., n,\ p_{0, n+1} = 2B,\ p_{1, n+1} = B.$
- $w_{0j} = b_j,\ w_{1j} = 2b_j,$ for $j = 1, ..., n,\ w_{0, n+1} = 4B,\ w_{1, n+1} = 0.$
- $s_{0j} = s_{1j} = 0;$ i.e., $S_j = 0;$ for $j = 1, 2, ..., n + 1.$
- $d_j = B,$ for $j = 1, 2, ..., n, n + 1.$
- Threshold value: $\dfrac{3B}{2B + 2}.$

It can be observed that the above construction can be done in polynomial time.

First, we assume that the partition problem has a solution $Q \subseteq N = \{1, 2, ..., n\}$ such that $\sum_{j \in Q} b_j = B.$ We construct a schedule by the following way: assign each job in $\{J_j : j \in Q\}$ to be scheduled on the in-house machine and outsourced all the other jobs. We use $C_{\max} =$

$\max_{j=1,2,\ldots,n,n+1}\{C_j + S_j\}$ to denote the common makespan. It is not hard to show that $C_{\max} = B$, $U_j = 0$, $j = 1, \ldots, n+1$, and $W = 3B$. Thus,

$$\lambda \sum U_j + (1-\lambda)W = (1-\lambda)3B = \frac{3B}{2B+2}.$$

Now, we suppose that there is a schedule π which objective function value is at most $\frac{3B}{2B+2}$, we will show that there exists a solution to the partition problem.

In fact, the job $n+1$ is outsourced. Otherwise,

$$\lambda \sum U_j + (1-\lambda)W \geq (1-\lambda) w_{0,n+1} = \frac{4B}{2B+2} > \frac{3B}{2B+2},$$

a contradiction.

Let Q to be the set of jobs scheduled on the in-house machine, we will show that $\sum_{j \in Q} b_j = B$.

If $\sum_{j \in Q} b_j < B$, then the completion time of the last job scheduled on the in-house machine is strictly less than B. According to the construction of the scheduling instance, it is easy to show that the job $n+1$ is the last one completed on the outsourcing machine, i.e.,

$$\max_{j \in N \setminus Q} p_{1j} \leq \max_{j \in N} p_{1j} \leq \sum_{j=1}^{n} \frac{1}{2} p_{0j} = B = p_{1,n+1}.$$

Thus, using the fact that $S_j = 0$ for $j = 1, 2, \ldots, n, n+1$, we have $C_{\max} = B$. As $d_j = B$, we obtain $U_j = 0$, $j = 1, \ldots, n, n+1$.

Since

$$W = \sum_{j \in Q} w_{0j} + \sum_{j \in N \setminus Q} w_{1j} + w_{1,n+1} = \sum_{j \in Q} b_j + \sum_{j \in N \setminus Q} 2b_j,$$

and $\sum_{j \in N \setminus Q} b_j > B,$ we have

$$\lambda \sum U_j + (1-\lambda) W = (1-\lambda)\left(\sum_{j \in Q} b_j + \sum_{j \in N \setminus Q} 2b_j \right)$$

$$= \frac{1}{2B+2}\left(\sum_{j=1}^{n} b_j + \sum_{j \in N \setminus Q} b_j \right)$$

$$= \frac{1}{2B+2}\left(2B + \sum_{j \in N \setminus Q} b_j \right) > \frac{1}{2B+2}(2B + B)$$

$$= \frac{3B}{2B+2},$$

a contradiction. If $\sum_{j \in Q} b_j > B,$ then we will obtain $\sum_{j=1}^{n+1} U_j > 1.$ By $\sum_{j \in Q} b_j \leq \sum_{j \in N} b_j = 2B,$ it follows that

$$\lambda \sum U_j + (1-\lambda) W$$

$$> \lambda + (1-\lambda) W = \lambda + (1-\lambda)\left(\sum_{j \in Q} b_j + \sum_{j \in N \setminus Q} 2b_j \right)$$

$$= \lambda + (1-\lambda)\left(2\sum_{j=1}^{n} b_j - \sum_{j \in Q} b_j \right) = \lambda + (1-\lambda)\left(4B - \sum_{j \in Q} b_j \right)$$

$$\geq \frac{2B+1}{2B+2} + \frac{1}{2B+2}(4B - 2B) = \frac{4B+1}{2B+2} > \frac{3B}{2B+2},$$

a contradiction. This implies the existence of a solution to the partition problem. □

Next, we design a dynamic programming algorithm to solve problem $1+\infty \| \lambda \sum U_j + (1-\lambda)W$, denoted as DP. Using the optimality of the problem $1\|\sum U_j$, we get the following evident lemma:

Lemma 2.1. *For problem* $1+\infty \| \lambda \sum U_j + (1-\lambda)W$, *there exists an optimal solution in which all on-time jobs are scheduled in the EDD order* (*i.e.*, *jobs are sequenced in the nondecreasing order of due times*), *and tardy jobs in-house are following the on-time jobs*.

Proof. It can be proved in interchange arguments. □

Based on the EDD property, we assume that jobs are indexed as $d_1 \le d_2 \le \cdots \le d_n$. We define (j, u, l, m) as a state describing a sub-schedule for jobs $1, 2, ..., j$, where (1) u is the load of the in-house machine, i.e., the total processing times of those jobs scheduled on the in-house machine, (2) l is the number of on-time jobs in-house and (3) m is the number of on-time jobs outsourced. Let $f(j, u, l, m)$ be the optimal value of the objective function for a sub-schedule described by (j, u, l, m).

If j is scheduled in-house, then we set

$$f^0(j, u, l, m)$$

$$= \begin{cases} f(j-1, u-p_{0j}, l-1, m) + (1-\lambda)w_{0j}, & \text{if } u + s_{0j} \le d_j; \\ f(j-1, u-p_{0j}; l, m) + \lambda + (1-\lambda)w_{0j}, & \text{otherwise.} \end{cases}$$

If j is outsourced, let

$$f^1(j, u, l, m) = \begin{cases} f(j-1, u, l, m-1) + (1-\lambda)w_{1j}, & \text{if } p_{1j} + s_{1j} \le d_j; \\ f(j-1, u, l, m) + \lambda + (1-\lambda)w_{1j}, & \text{otherwise.} \end{cases}$$

Furthermore

$$f(j, u, l, m) = \min\{f^0(j, u, l, m),\ f^1(j, u, l, m)\}.$$

Next, we give the initial conditions as follows:

$$f(1, u, l, m) = \begin{cases} \lambda + (1-\lambda) w_{01}, & u = p_{01},\ l = 0,\ m = 0; \\ (1-\lambda) w_{01}, & u = p_{01},\ l = 1,\ m = 0; \\ \lambda + (1-\lambda) w_{11}, & u = 0,\ l = 0,\ m = 0; \\ (1-\lambda) w_{01}, & u = 0,\ l = 0,\ m = 1; \\ +\infty, & \text{otherwise.} \end{cases}$$

Then the optimal solution is given by $\min_{u,l,m}\{f(n, u, l, m)\}$, where $u = 0, 1, 2, \ldots, P$; $l = 0, 1, 2, \ldots, n$; $m = 0, 1, 2, \ldots, n$, and P stands for the sum of processing times for all jobs. The running time of the dynamic programming algorithm DP is $O(n^2 P)$.

3. Conclusion

We have proposed an analytical scheduling model for the coordination of in-house production and outsourcing, whose objective function is to minimize the weighted sum of the number of tardy jobs and the total cost. In the further, we will investigate more complex models with multiple available subcontractors and batch processing.

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