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CERTAIN NEW UNIFIED INTEGRALS ASSOCIATED WITH THE GENERALIZED *k*-BESSEL FUNCTION

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Abstract

The main aim of this paper is to establish generalized integral formulas involving the generalized *k*-Bessel function $w_{k,\nu,b,c}^{\gamma,\lambda}(z)$ to obtain the results in terms of Fox-Wright function. Certain special cases of the main results presented here with some known identities are also pointed out.

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1. Introduction

The k-Pochhammer symbol and k-gamma function was introduced by Diaz and Pariguan [11] as follows:

$$(\gamma)_{n,k} := \begin{cases} \dfrac{\Gamma_k(\gamma + nk)}{\Gamma_k(\gamma)}, & (k \in \mathbb{R};\ \gamma \in \mathbb{C}\backslash\{0\}), \\ \gamma(\gamma + k)\cdots(\gamma + (n-1)k), & (n \in \mathbb{N};\ \gamma \in \mathbb{C}). \end{cases} \tag{1.1}$$

They gave the relation with the classical Euler's gamma function (see [10, 18]) as:

$$\Gamma_k(\gamma) = k^{\frac{\gamma}{k}-1}\Gamma\left(\frac{\gamma}{k}\right). \tag{1.2}$$

Clearly, for $k = 1$, (1.1) reduces to the classical Pochammer symbol and Euler's gamma function, respectively (see [16]).

Recently, Romero et al. [10] (see, also [18]) introduced the k-Bessel function of the first kind for $\alpha, \lambda, \gamma, v \in \mathbb{C}$ and $\Re(\lambda) > 0, \Re(v) > 0$ as follows:

$$J_{k,v}^{(\gamma),(\lambda)}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{n,k}}{\Gamma_k(\lambda n + v + 1)} \frac{(-1)^n \left(\frac{z}{2}\right)^n}{(n!)^2}. \tag{1.3}$$

The Fox-Wright function ${}_p\Psi_q(z)$ with p numerator and q denominator parameters, such that $a_1, \ldots, a_p \in \mathbb{C}$ and $b_1, \ldots, b_q \in \mathbb{C}\backslash\mathbb{Z}_0^-$ is defined by (see, for detail, [12, 13, 14]):

$${}_p\Psi_q(z) = {}_p\Psi_q\left[\begin{matrix} (a_i, \alpha_i)_{1,p} \\ (b_j, \beta_j)_{1,q} \end{matrix} \middle| z \right] = \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p} \Gamma(a_i + \alpha_i n)}{\prod_{j=1}^{q} \Gamma(b_j + \beta_j n)} \frac{z^n}{n!}, \tag{1.4}$$

under the condition

$$\sum_{j=1}^{q} \beta_j - \sum_{i=1}^{p} \alpha_i > -1. \tag{1.5}$$

In particular, when $a_i = b_j = 1$ $(i = 1, \ldots, p;\ j = 1, \ldots, q)$, immediate reduces to generalized hypergeometric function ${}_pF_q(p, q \in \mathbb{N}_0)$ (see, for details [15]):

$$ {}_p\Psi_q(z) = {}_p\Psi_q\left[\begin{matrix}(\alpha_1, 1), \ldots, (\alpha_p, 1); \\ (\beta_1, 1), \ldots, (\beta_q, 1);\end{matrix}\ z\right] $$

$$ = \frac{\prod_{j=1}^{p} \Gamma(\alpha_j)}{\prod_{j=1}^{q} \Gamma(\beta_j)}\ {}_pF_q\left[\begin{matrix}\alpha_1, \ldots, \alpha_p; \\ \beta_1, \ldots, \beta_q;\end{matrix}\ z\right]. \tag{1.6} $$

In terms of the k-Pochhammer symbol $(\gamma)_{n,k}$ defined by (1.1), we introduce more generalized form of k-Bessel function $w_{k,v,b,c}^{\gamma,\lambda}(z)$ as follows: For $\alpha, \lambda, \gamma, v, c, b \in \mathbb{C}$ and $\Re(\lambda) > 0$, $\Re(v) > 0$, we have

$$ w_{k,v,b,c}^{\gamma,\lambda}(z) = \sum_{n=0}^{\infty} \frac{(-1)^n c^n (\gamma)_{n,k}}{\Gamma_k\left(v + \lambda n + \frac{b+1}{2}\right)} \frac{\left(\frac{z}{2}\right)^{v+2n}}{(n!)^2}. \tag{1.7} $$

In this paper, we derive a class of integral involving the generalized form of k-Bessel function $w_{k,\nu,b,c}^{\gamma,\lambda}(z)$. Moreover, we give certain special cases as the corollaries. For the present investigation, we need the following result of Lavoie and Trottier [17]:

$$ \int_0^1 x^{\alpha-1}(1-x)^{2\beta-1}\left(1-\frac{x}{3}\right)^{2\alpha-1}\left(1-\frac{x}{4}\right)^{\beta-1} dx = \left(\frac{2}{3}\right)^{2\alpha} \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, $$

$$ (\Re(\alpha) > 0,\ \Re(\beta) > 0). \tag{1.8} $$

For various other investigations involving certain special functions, which were motivated essentially by the work of Rakha et al. [8], the interested reader may be referred to several recent papers on the subject (see, for example, [3, 4, 5, 6, 7] and the references cited in each of these papers).

2. Main Results

Two generalized integral formulas involving generalized form of k-Bessel function (1.7), are established here, which expressed in terms of Fox-Wright function (1.6) by inserting with the suitable argument in the integrand of (1.8).

Theorem 2.1. *Let* $\rho,\ j,\ v,\ c,\ b \in \mathbb{C}$ *with* $\Re(\nu) > -1$ *and* $\Re(\rho + j) > 0$, $\Re(\rho + v) > 0$ *and* $x > 0$

$$\int_0^1 x^{\rho+j-1}(1-x)^{2\rho-1}\left(1-\frac{x}{3}\right)^{2(\rho+j)-1}\left(1-\frac{x}{4}\right)^{\rho-1}$$

$$\times\, w_{k,v,b,c}^{\gamma,\lambda}\left(y\left(1-\frac{x}{4}\right)(1-x)^2\right)dx$$

$$= \frac{\left(\frac{y}{2}\right)^{v}\Gamma(\rho+j)\left(\frac{2}{3}\right)^{2(\rho+j)}}{\Gamma\left(\frac{\gamma}{k}\right)k^{\frac{v}{k}+\frac{b+1}{2k}-1}}\,{}_2\Psi_3$$

$$\times\left[\begin{matrix} \left(\frac{\gamma}{k},1\right),\ (\rho+v,\ 2); \\ (1,1),\ \left(\frac{v}{k}+\frac{b+1}{2k},\ \frac{\lambda}{k}\right),\ (2\rho+v+j,\ 2); \end{matrix}\ -\frac{y^2 c}{4k^{\frac{\lambda}{k}-1}}\right]. \tag{2.1}$$

Proof. Let $\mathcal{S}$ be the left-hand side of (2.1). Now applying the formula (1.7) to the integrand of (2.1) and by interchanging the order of integration and summation, which is verified by uniform convergence of the given series under the given condition in Theorem 2.1, we have

$$\mathcal{S} = \sum_{n=0}^{\infty}\frac{(-1)^n c^n (\gamma)_{n,k}(y/2)^{v+2n}}{(n!)^2\Gamma_k\left(\lambda n + v + \frac{b+1}{2}\right)}$$

$$\times\int_0^1 x^{\rho+j-1}(1-x)^{2(\rho+v+2n)-1}\left(1-\frac{x}{3}\right)^{2(\rho+j)-1}\left(1-\frac{x}{4}\right)^{\rho+v+2n-1}dx.$$

By considering the condition given in 2.1, since

$\Re(v) > -1,\ \Re(\rho + v + 2n) > \Re(\rho + v) > 0,\ \Re(\rho + j) > 0,\ (k \in \mathbb{N}_0 := \mathbb{N} \cup \{0\})$

and applying (1.8),

$$\mathcal{S} = \sum_{n=0}^{\infty} \frac{(-1)^n c^n (\gamma)_{n,k} (y/2)^{v+2n}}{(n!)^2 \Gamma_k\left(\lambda n + v + \frac{b+1}{2}\right)} \left(\frac{2}{3}\right)^{2(\rho+j)} \frac{\Gamma(\rho+j)\Gamma(\rho+v+2n)}{\Gamma(2\rho+j+v+2n)}.$$

Using the relation (1.1) and (1.2), we get

$$\mathcal{S} = \sum_{n=0}^{\infty} \frac{(-1)^n c^n \Gamma_k(\gamma+nk)(y/2)^{v+2n}}{(n!)^2 \Gamma_k(\gamma)\Gamma_k\left(\lambda n + v + \frac{b+1}{2}\right)} \left(\frac{2}{3}\right)^{2(\rho+j)} \frac{\Gamma(\rho+j)\Gamma(\rho+2n+v)}{\Gamma(2\rho+j+v+2n)}$$

$$= \frac{\left(\frac{y}{2}\right)^v \Gamma(\rho+j)\left(\frac{2}{3}\right)^{2(\rho+j)}}{\Gamma\left(\frac{\gamma}{k}\right) k^{\frac{v}{k}+\frac{b+1}{2k}-1}}$$

$$\times \sum_{n=0}^{\infty} \frac{\Gamma\left(\frac{\gamma}{k}+n\right)\Gamma(\rho+v+2n)(-1)^n c^n k^{n-\frac{\lambda n}{k}}\left(\frac{y}{2}\right)^{2n}}{\Gamma(1+n)\Gamma\left(\frac{v}{k}+\frac{b+1}{2k}+\frac{\lambda}{k}n\right)\Gamma(2\rho+j+v+2n)n!}$$

which upon using the definition (1.6), we get the desired result. □

Theorem 2.2. *For* $\rho, j, v \in \mathbb{C}$ *with* $\Re(v) > -1$ *and* $\Re(\rho + j) > 0$, $\Re(\rho + v) > 0$ *and* $x > 0$

$$\int_0^1 x^{\rho-1}(1-x)^{2(\rho+j)-1}\left(1-\frac{x}{3}\right)^{2\rho-1}\left(1-\frac{x}{4}\right)^{(\rho+j)-1} w_{k,v,b,c}^{\gamma,\lambda}\left(yx\left(1-\frac{x}{3}\right)^2\right)dx$$

$$= \frac{\left(\frac{y}{2}\right)^v \Gamma(\rho+j)\left(\frac{2}{3}\right)^{2(\rho+v)}}{\Gamma\left(\frac{\gamma}{k}\right) k^{\frac{v}{k}+\frac{b+1}{2k}-1}}\, {}_2\Psi_3$$

$$\times \left[\begin{array}{c} \left(\frac{\gamma}{k}, 1\right), (\rho + v, 2); \\ (1, 1), \left(\frac{v}{k} + \frac{b+1}{2k}, \frac{\lambda}{k}\right), (2\rho + v + j, 2); \end{array} - \frac{4y^2 c}{81 k^{\frac{\lambda}{k}-1}} \right]. \tag{2.2}$$

Proof. Let $\mathcal{L}$ be the left-hand side of (2.1). Now applying the formula (1.7) to the integrand of (2.2) and by interchanging the order of integral and summation, which is verified by uniform convergence of the given series under the given condition in Theorem 2.1, we have

$$\mathcal{L} = \sum_{n=0}^{\infty} \frac{(-1)^n c^n (\gamma)_{n,k} (yx/2)^{v+2n}}{(n!)^2 \Gamma_k\left(\lambda n + v + \frac{b+1}{2}\right)}$$

$$\times \int_0^1 x^{\rho+v+2n-1} (1-x)^{2(\rho+j)-1} \left(1 - \frac{x}{3}\right)^{2(2n+\rho+v)-1} \left(1 - \frac{x}{4}\right)^{\rho+j-1} dx.$$

By considering the condition given in Theorem 2.2, since

$$\Re(v) > -1, \Re(\rho + v + 2n) > \Re(\rho + v) > 0, \Re(\rho + j) > 0, (k \in \mathbb{N}_0 := \mathbb{N} \cup \{0\})$$

and applying (1.8),

$$\mathcal{L} = \sum_{n=0}^{\infty} \frac{(-1)^n c^n (\gamma)_{n,k} (y/2)^{v+2n}}{(n!)^2 \Gamma_k\left(\lambda n + v + \frac{b+1}{2}\right)} \left(\frac{2}{3}\right)^{2(2n+\rho+v)} \frac{\Gamma(\rho + j)\Gamma(\rho + 2n + v)}{\Gamma(2\rho + j + v + 2n)}.$$

Now using the relation (1.1) and (1.2), we obtain the following expression:

$$\mathcal{L} = \sum_{n=0}^{\infty} \frac{(-1)^n c^n \Gamma_k(\gamma + nk) (y/2)^{v+2n}}{(n!)^2 \Gamma_k(\gamma) \Gamma_k\left(\lambda n + v + \frac{b+1}{2}\right)} \left(\frac{2}{3}\right)^{2(\rho+v+2n)} \frac{\Gamma(\rho + j)\Gamma(\rho + v + 2n)}{\Gamma(2\rho + j + v + 2n)}$$

$$= \frac{\left(\frac{y}{2}\right)^{v} \Gamma(\rho + j) \left(\frac{2}{3}\right)^{2(\rho+v)}}{\Gamma\left(\frac{\gamma}{k}\right) k^{\frac{v}{k} + \frac{b+1}{2k} - 1}}$$

$$\times \sum_{n=0}^{\infty} \frac{\Gamma\left(\frac{\gamma}{k}+n\right)\Gamma(\rho+v+2n)(-1)^n c^n k^{n-\frac{\lambda n}{k}} (y/2)^{2n}\left(\frac{2}{3}\right)^{4n}}{\Gamma(1+n)\Gamma\left(\frac{v}{k}+\frac{b+1}{2k}+\frac{\lambda}{k}n\right)\Gamma(2\rho+j+v+2n)n!}$$

which upon using the definition (1.6), we get the desired result. □

3. Special Cases

In this section, we present certain special cases of generalized form of k-Bessel function (1.7). Further we obtain three corollaries as special cases of obtained theorem in Section 2.

Case 1. If we set $b = c = 1$ in (1.7), yields another definition of k-Bessel function

$$J_{k,\nu}^{\gamma,\lambda}(z) = \sum_{n=0}^{\infty} \frac{(-1)^n (\gamma)_{n,k}}{\Gamma_k(v+\lambda n+1)} \frac{(z/2)^{v+2n}}{(n!)^2}. \tag{3.1}$$

Case 2. If we set $b = -c = 1$ and $b = 1$ in (1.7), yields k-modified Bessel function

$$I_{k,v}^{\gamma,\lambda}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{n,k}}{\Gamma_k(v+\lambda n+1)} \frac{(z/2)^{v+2n}}{(n!)^2}. \tag{3.2}$$

Case 3. If we set $b - 1 = c = 1$ in (1.7), yields new definition of k-spherical Bessel function

$$K_{k,v}^{\gamma,\lambda}(z) = \sum_{n=0}^{\infty} \frac{(-1)^n (\gamma)_{n,k}}{\Gamma_k\left(v+\lambda n+\frac{3}{2}\right)} \frac{(z/2)^{v+2n}}{(n!)^2}. \tag{3.3}$$

Corollary 3.1. *Let the conditions of Theorem* 2.1 *be satisfied*. *Then the following integral formula holds true*:

$$\int_0^1 x^{\rho+j-1}(1-x)^{2\rho-1}\left(1-\frac{x}{3}\right)^{2(\rho+j)-1}\left(1-\frac{x}{4}\right)^{\rho-1}$$

$$\times J_{k,\nu}^{\gamma,\lambda}\left(y\left(1-\frac{x}{4}\right)(1-x)^2\right)dx$$

$$=\frac{\left(\frac{y}{2}\right)^{\nu}\Gamma(\rho+j)\left(\frac{2}{3}\right)^{2(\rho+j)}}{\Gamma\left(\frac{\gamma}{k}\right)k^{\frac{\nu}{k}+\frac{1}{k}-1}}\,{}_2\Psi_3$$

$$\times\left[\begin{matrix}\left(\frac{\gamma}{k},1\right),(\rho+\nu,2);\\ (1,1),\left(\frac{\nu}{k}+\frac{1}{k},\frac{\lambda}{k}\right),(2\rho+\nu+j,2);\end{matrix}\ -\frac{y^2}{4k^{\frac{\lambda}{k}-1}}\right]. \tag{3.4}$$

Corollary 3.2. *Let the conditions of Theorem* 2.1 *be satisfied*. *Then the following integral formula holds true*:

$$\int_0^1 x^{\rho+j-1}(1-x)^{2\rho-1}\left(1-\frac{x}{3}\right)^{2(\rho+j)-1}\left(1-\frac{x}{4}\right)^{\rho-1}$$

$$\times I_{k,\nu}^{\gamma,\lambda}\left(y\left(1-\frac{x}{4}\right)(1-x)^2\right)dx$$

$$=\frac{\left(\frac{y}{2}\right)^{\nu}\Gamma(\rho+j)\left(\frac{2}{3}\right)^{2(\rho+j)}}{\Gamma\left(\frac{\gamma}{k}\right)k^{\frac{\nu}{k}+\frac{1}{k}-1}}\,{}_2\Psi_3$$

$$\times\left[\begin{matrix}\left(\frac{\gamma}{k},1\right),(\rho+\nu,2);\\ (1,1),\left(\frac{\nu}{k}+\frac{1}{k},\frac{\lambda}{k}\right),(2\rho+\nu+j,2);\end{matrix}\ \frac{y^2}{4k^{\frac{\lambda}{k}-1}}\right]. \tag{3.5}$$

Corollary 3.3. *Let the conditions of Theorem* 2.1 *be satisfied*. *Then the following integral formula holds true*:

$$\int_0^1 x^{\rho+j-1}(1-x)^{2\rho-1}\left(1-\frac{x}{3}\right)^{2(\rho+j)-1}\left(1-\frac{x}{4}\right)^{\rho-1}$$

$$\times K_{k,v}^{\gamma,\lambda}\left(y\left(1-\frac{x}{4}\right)(1-x)^2\right)dx$$

$$= \frac{\left(\frac{y}{2}\right)^v \Gamma(\rho+j)\left(\frac{2}{3}\right)^{2(\rho+j)}}{\Gamma\left(\frac{\gamma}{k}\right)k^{\frac{v}{k}+\frac{3}{2k}-1}}\,{}_2\Psi_3$$

$$\times\left[\begin{matrix}\left(\frac{\gamma}{k},1\right),(\rho+v,\ 2); \\ (1,1),\left(\frac{v}{k}+\frac{3}{2k},\frac{\lambda}{k}\right),(2\rho+v+j,\ 2);\end{matrix}\ -\frac{y^2}{4k^{\frac{\lambda}{k}-1}}\right]. \tag{3.6}$$

Corollary 3.4. *Let the conditions of Theorem* 2.2 *be satisfied. Then the following integral formula holds true*:

$$\int_0^1 x^{\rho-1}(1-x)^{2(\rho+j)-1}\left(1-\frac{x}{3}\right)^{2\rho-1}\left(1-\frac{x}{4}\right)^{(\rho+j)-1} J_{k,v}^{\gamma,\lambda}\left(yx\left(1-\frac{x}{3}\right)^2\right)dx$$

$$= \frac{\left(\frac{y}{2}\right)^v \Gamma(\rho+j)\left(\frac{2}{3}\right)^{2(\rho+v)}}{\Gamma\left(\frac{\gamma}{k}\right)k^{\frac{v}{k}+\frac{1}{2k}-1}}\,{}_2\Psi_3$$

$$\times\left[\begin{matrix}\left(\frac{\gamma}{k},1\right),(\rho+v,\ 2); \\ (1,1),\left(\frac{v}{k}+\frac{1}{2k},\frac{\lambda}{k}\right),(2\rho+v+j,\ 2);\end{matrix}\ -\frac{4y^2}{81k^{\frac{\lambda}{k}-1}}\right]. \tag{3.7}$$

Corollary 3.5. *Let the conditions of Theorem* 2.2 *be satisfied. Then the following integral formula holds true*:

$$\int_0^1 x^{\rho-1}(1-x)^{2(\rho+j)-1}\left(1-\frac{x}{3}\right)^{2\rho-1}\left(1-\frac{x}{4}\right)^{(\rho+j)-1} I_{k,v}^{\gamma,\lambda}\left(yx\left(1-\frac{x}{3}\right)^2\right)dx$$

$$= \frac{\left(\frac{y}{2}\right)^v \Gamma(\rho+j)\left(\frac{2}{3}\right)^{2(\rho+v)}}{\Gamma\left(\frac{\gamma}{k}\right)k^{\frac{v}{k}+\frac{1}{2k}-1}}\,{}_2\Psi_3$$

$$\times \left[\begin{matrix} \left(\frac{\gamma}{k}, 1\right), (\rho + \nu, 2); \\ (1, 1), \left(\frac{\nu}{k} + \frac{1}{2k}, \frac{\lambda}{k}\right), (2\rho + \nu + j, 2); \end{matrix} \frac{4y^2}{81k^{\frac{\lambda}{k}-1}} \right]. \tag{3.8}$$

Corollary 3.6. *Let the conditions of Theorem* 2.2 *be satisfied. Then the following integral formula holds true*:

$$\int_0^1 x^{\rho-1}(1-x)^{2(\rho+j)-1}\left(1-\frac{x}{3}\right)^{2\rho-1}\left(1-\frac{x}{4}\right)^{(\rho+j)-1} K_{k,\nu}^{\gamma,\lambda}\left(yx\left(1-\frac{x}{3}\right)^2\right)dx$$

$$= \frac{\left(\frac{y}{2}\right)^{\nu}\Gamma(\rho+j)\left(\frac{2}{3}\right)^{2(\rho+\nu)}}{\Gamma\left(\frac{\gamma}{k}\right)k^{\frac{\nu}{k}+\frac{3}{2k}-1}} {}_2\Psi_3$$

$$\times \left[\begin{matrix} \left(\frac{\gamma}{k}, 1\right), (\rho + \nu, 2); \\ (1, 1), \left(\frac{\nu}{k} + \frac{3}{2k}, \frac{\lambda}{k}\right), (2\rho + \nu + j, 2); \end{matrix} -\frac{4y^2}{81k^{\frac{\lambda}{k}-1}} \right]. \tag{3.9}$$

Remark. If we set $k = 1$ and $\lambda = 1 = \gamma$ in (3.4) to (3.9), yields the known results for case 1 (see [2]) and new results for familiar functions as defined in [1, 9]. Most (if not all) of the results, which we have investigated in this paper for the special cases, can indeed be considered analogously in a rather simple and straightforward manner. The details involved may, therefore, be left as an exercise for the interested reader.

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