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ON SIZE MULTIPARTITE RAMSEY NUMBERS FOR SMALL $K_{s \times t}$ VERSUS PATHS

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Abstract

For given two graphs G_1 and G_2, and integer $s \geq 2$, the size Ramsey multipartite number $m_s(G_1, G_2) = t$ is the smallest integer such that every factorization of graph $K_{s \times t} := F_1 \oplus F_2$ satisfies the following condition: either F_1 contains G_1 as a subgraph or F_2 contains G_2 as a subgraph. In this paper, we determine that $m_2(K_{2 \times n}, P_{4n})$ for $n \geq 2$, and $m_3(K_{3 \times 2}, P_n)$ for $4 \leq n \leq 7$.

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1. Introduction

All graphs $G = (V, E)$ considered in this paper are finite graphs without loops and multiple edges. The order of the graph $G = (V, E)$ is denoted by $|V(G)|$ and the number of edges in the graph is denoted by $|E(G)|$. The graphs $K_{s,t}$ and $K_{s\times t}$ represent complete bipartite graph with partite sets of size *s* and *t*, and the complete multipartite graph consisting of *s* partite sets having exactly *t* vertices in each partite set, respectively.

The notion of size multipartite Ramsey numbers was introduced by Burger and van Vuuren [1] and Syafrizal et al. [5] by considering the two factorization of a $K_{s\times t}$ by fixing the size *s* of the uniform multipartite sets. More precisely, for given two graphs G_1 and G_2, and integer $s \geq 2$, the *size Ramsey multipartite number* $m_s(G_1, G_2) = t$ is the smallest integer such that every factorization of graph $K_{s\times t} := F_1 \oplus F_2$ satisfies the following condition: either F_1 contains G_1 as a subgraph or F_2 contains G_2 as a subgraph. Ramsey numbers of small paths versus certain classes of graphs have been studied by Sy et al. in [6, 7, 9, 10] and [11]. Motivated by these findings, we have attempted in this paper to find size multipartite Ramsey numbers for small balanced complete multipartite graphs. Sy and Baskoro [8] have obtained the size multipartite Ramsey numbers for balanced complete multipartite graphs as follows.

Theorem 1. *For integers* $j, n \geq 3$ *and* $b \geq 2$,

$$m_j(P_n, K_{j\times b}) \geq \begin{cases} (n-1)b, & \text{if } 3 \leq n \leq j, \\ jb, & \text{if } j < n < jb, \\ (j-1)\left\lfloor \dfrac{n-2}{2} \right\rfloor + b, & \text{if } n \geq jb. \end{cases}$$

Surahmat and Sy [4] have determined the stars-paths size bipartite Ramsey numbers $m_2(P_4, K_{1,n}) = n + 1$ for $n \geq 3$. Furthermore, Sy [12] determined the exact values of the size multipartite Ramsey numbers for trees versus paths as follows.

Theorem 2. *For integers* $n,\ s \geq 2$,

$$m_2(P_n, T_s) = \begin{cases} \left\lceil \frac{n+1}{2} \right\rceil, & \text{for } n \geq 10,\ s = 4, \\ \left\lceil \frac{n}{2} \right\rceil, & \text{for } (n \geq 3,\ s \leq 3) \text{ or } \left(n \geq 9,\ 4 < s \leq \left\lceil \frac{n}{2} \right\rceil\right), \\ s, & \text{for } \left(\left\lceil \frac{n}{2} \right\rceil < s < n < 2s\right) \text{ or } (s = n \text{ is odd}), \\ s - 1, & \text{for } s = n \text{ is even}, \\ n + \left\lfloor \frac{n}{2} \right\rfloor - 1, & \text{for } s = 2n, \\ \left\lceil \frac{s}{2} \right\rceil + \frac{n}{2} - 1, & \text{for } n < s,\ n \text{ is even}, \\ \left\lceil \frac{s}{2} \right\rceil + \left\lceil \frac{n}{2} \right\rceil - 2, & \text{for } n < s,\ n \text{ is odd}. \end{cases}$$

Recently, Lusiani et al. [2] have obtained the size multipartite Ramsey numbers for a combination of stars and cycles $m_j(S_m, C_n)$, where $3 \leq n \leq j$ and any $m \geq 3$.

2. Main Results

The first main result of this paper is the determination of the size bipartite Ramsey numbers for combination small balanced complete multipartite graph and paths on $n \geq 2$ vertices.

Theorem 3. *For integer* $n \geq 2$, $m_2(P_{4n}, K_{2\times n}) = 2n + 1$.

Proof. We show first that $m_2(P_{4n}, K_{2\times n}) \geq 2n + 1$. Consider any factorization $K_{2\times 2n} := G_1 \oplus G_2$, where $G_1 := K_{1,n-1} \cup K_{n-1,1}$. Clearly that $G_1 \not\supseteq K_{2\times n}$ and also $G_2 \not\supseteq P_{4n}$. Therefore, $m_2(P_{4n}, K_{2\times n}) \geq 2n + 1$ for all $n \geq 2$.

Now, we show that $m_2(P_{4n}, K_{2\times n}) \le 2n+1$. Let $F_1 \oplus F_2$ be any factorization of $K_{2\times(2n+1)}$ such that $K_{2\times n}$ is not a subgraph of F_1. We will show that P_{4n} is a subgraph of F_2. Thus, by Lemma 1 in [3], F_2 possesses a Hamiltonian path, so $F_2 \supseteq P_{4n}$. Therefore, $m_2(P_{4n}, K_{2\times n}) \le 2n+1$ for all $n \ge 2$. □

The second main result of this paper is determination of the size tripartite Ramsey numbers for combination small balanced complete multipartite graph and paths P_n, where $n = 4, 5, 6,$ or 7.

Theorem 4. *For positive integer n,* $4 \le n \le 7$, $m_3(P_n, K_{3\times 2}) = 6$.

Proof. For $3 \le n \le 5$ by Theorem 2 in [8], we have $m_3(P_n, K_{3\times 2}) = 6$.

Next, by Theorem 1 in [8], we have the lower bound $m_3(P_n, K_{3\times 2}) \ge 6$ for $6 \le n \le 7$.

To show the upper bound $m_3(P_n, K_{3\times 2}) \le 6$, consider $F \cong K_{3\times 6}$. Let $F_1 \oplus F_2$ be any factorization of F so that F_1 contains no P_n. We will show that F_2 contains $K_{3\times 2}$. Let P be a longest path in F, and Q be a longest path in graph $F \backslash V(P)$ and R be a longest path in graph $F \backslash V(P \cup Q)$. Let a and b be the end vertices of P. Let c and d be the end vertices of Q. Let e and f be the end vertices of R. Since P, Q, and R are the longest paths, $xy \notin E(F_1)$ for each $x, y \in \{a, b, c, d, e, f\}$. Let A, B, C be the partite sets of F. We consider two possibilities.

Case 1. If $a, b \in A$, $c, d \in B$, and $e, f \in C$, then $uv \notin E(F_1)$ for every $u, v \in \{a, b, c, d, e, f\}$. Thus, the set $\{a, b, c, d, e, f\}$ will induce a $K_{3\times 2}$ in F_2. Therefore $m_3(K_{3\times 2}, P_n) \le 6$ for $6 \le n \le 7$.

Case 2. If $a, b \notin A$, $c, d \notin B$, or $e, f \notin C$. Without loss of generality, we may assume $a \in A$ and $b \in B$, $c \in B$ and $d \in C$, and $e \in C$ and $f \in C$. Clearly that $ac, ae, ce \notin F_1$. Since $|A| = |B| = |C| = 6$, there are three vertices, namely $u \in A$, $v \in B$, and $w \in C$ such that $uv, uw, vw \notin F_1$. Therefore, the set $\{a, u, c, v, e, w\}$ will induce a $K_{3\times 2}$ in F_2. Thus, $m_3(P_n, K_{3\times 2}) \leq 6$ for $6 \leq n \leq 7$. □

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