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# HAUSDORFF PROPERTY OF SOME DERIVED GRAPHS 

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#### Abstract

A simple graph $G$ is said to be Hausdorff if for any two distinct vertices $u$ and $v$ of $G$, one of the following conditions holds: (1) At least one of $u$ and $v$ is isolated. (2) There exist two nonadjacent edges $e_{1}$ and $e_{2}$ of $G$ such that $e_{1}$ is incident with $u$ and $e_{2}$ is incident with $v$.

In this paper, we discuss the Hausdorff property of some graphs which are derived from the given graph.


## 1. Introduction

All graphs considered here are finite and simple. In this paper, we denote the set of vertices of $G$ by $V(G)$, the set of edges of $G$ by $E(G)$ and the minimum degree of $G$ by $\delta(G)$.

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The degree [6] of a vertex $v$ in a graph $G$, denoted by deg $v$, is the number of edges incident with $v$. A pendant vertex [7] in a graph $G$ is a vertex of degree one. The unique edge incident with a pendant vertex is the pendant edge [7] and the vertex adjacent to the pendant vertex is the support vertex. A vertex $v$ is isolated [6] if $\operatorname{deg} v=0$. By an empty graph [4], we mean a graph with no edges. A simple graph is said to be complete [1] if every pair of distinct vertices of $G$ are adjacent in $G$. A complete graph on $n$ vertices is denoted by $K_{n}$. The union [13] of two graphs $G_{1}$ and $G_{2}$ denoted by $G_{1} \cup G_{2}$ is the graph with vertex set $V\left(G_{1}\right) \cup V\left(G_{2}\right)$ and edge set $E\left(G_{1}\right) \cup E\left(G_{2}\right)$. The total graph [12] $T(G)$ of a graph $G$ is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent whenever they are either adjacent or incident in $G$. For a graph $G$, the splitting graph [8] $S(G)$ is obtained by adding to each vertex $v$ a new vertex $v^{\prime}$ such that $v^{\prime}$ is adjacent to every vertex that is adjacent to $v$ in $G$. The subdivision graph [5] $S D(G)$ of the graph $G$ is obtained from $G$ by inserting a new vertex of degree 2 on each edge of $G$.

A graph $G$ is said to be Hausdorff [11] if for any two distinct vertices $u$ and $v$ of $G$, one of the following conditions holds:
(1) At least one of $u$ and $v$ is isolated.
(2) There exist two nonadjacent edges $e_{1}$ and $e_{2}$ of $G$ such that $e_{1}$ is incident with $u$ and $e_{2}$ is incident with $v$.

From the definition of a Hausdorff graph, we have:
Remark 1. Let $G_{1}$ and $G_{2}$ be two isomorphic graphs, that is, $G_{1} \cong G_{2}$. If $G_{1}$ is Hausdorff, then $G_{2}$ is also Hausdorff.

Theorem 2 [11]. Let $G=(V(G) E(G))$ be a graph with $\delta(G) \geq 3$. Then $G$ is Hausdorff.

Theorem 3 [11]. Line graph of a Hausdorff graph is Hausdorff.

## 2. Splitting Graph, Subdivision Graph and Shadow Graph

Splitting graph $S(G)$ of a graph $G$ is one of the interesting graphs derived from the given graph $G$. The vertex set, $V(S(G))$, of $S(G)$ can be partitioned into $V(G) \cup H$, where $H$ consists of all newly added vertices corresponding to each vertex of $G$. Hereafter, we denote the vertex of $H$ corresponding to the vertex $v$ of $G$ by $v^{\prime}$.

$C_{4}$

$S\left(C_{4}\right)$

Figure 1. Cycle $C_{4}$ and its splitting graph $S\left(C_{4}\right)$.

Theorem 4. Let $G$ be a graph with no pendant vertices. Then the splitting graph, $S(G)$, of $G$ is Hausdorff. In particular, splitting graph of a Hausdorff graph is Hausdorff.

Proof. Let $u$ and $v$ be two distinct vertices of $S(G)$.
Case 1. $u$ and $v$ are in $V(G)$.
If $u$ and $v$ are adjacent vertices of $G$, then $u v^{\prime}$ and $v u^{\prime}$ are two nonadjacent edges of $S(G)$. Suppose $u$ is not adjacent to $v$. Since $G$ contains no pendant vertices, both $u$ and $v$ are adjacent to at least two vertices of $G$. Let $u_{1}$ be a vertex of $G$ adjacent to $u$. Let $v_{1}$ be a vertex of $G$ distinct from $u_{1}$ and is adjacent to $v$. Then $u u_{1}$ and $v v_{1}$ are two nonadjacent edges of $S(G)$.

Case 2. $u$ and $v$ are in $H$.
Let us suppose that $u$ and $v$ are the newly added vertices corresponding to the vertices $x$ and $y$ of $G$, respectively. That is, $u=x^{\prime}$ and $v=y^{\prime}$. If $x$ and $y$ are adjacent vertices of $G$, then $u y$ and $v x$ are two nonadjacent edges of $S(G)$. Suppose $x$ is not adjacent to $y$. Then, as in the proof of Case 1, we get two nonadjacent edges of $S(G)$ incident with $u$ and $v$, respectively.

Case 3. $u \in V(G)$ and $v \in H$.
Suppose $v=u^{\prime}$. Since $G$ contains no pendant vertices, $u$ is adjacent to at least two vertices, say $u_{1}$ and $u_{2}$ of $G$. Hence $u^{\prime}$ is adjacent to both vertices $u_{1}$ and $u_{2}$ in $G$. Therefore, $u u_{1}$ and $v u_{2}$ are two nonadjacent edges of $S(G)$.

Now, suppose $v \neq u^{\prime}$. Let $v$ be $x^{\prime}$ for some $x \in V(G)$. If $u$ and $x$ are adjacent vertices of $G$, then choose a vertex $w$ of $G$ distinct from $u$ and adjacent to $x$. Then $u x$ and $v w$ are two nonadjacent edges of $S(G)$. If $u$ is not adjacent to $x$, then there exist two distinct vertices $w_{1}$ and $w_{2}$ of $G$ such that $u$ is adjacent to $w_{1}$ and $x$ is adjacent to $w_{2}$. Then $u w_{1}$ and $v w_{2}$ are two nonadjacent edges of $S(G)$ incident with $u$ and $v$, respectively.

Hence the theorem.
Proposition 5. The splitting graph $S(G)$ of a graph $G$ with at least one pendant vertex cannot be Hausdorff.

Proof. Let $u$ be a pendant vertex of $G$ and let $v$ be its support vertex. Then $u^{\prime} v$ is a pendant edge of $S(G)$. Therefore, $S(G)$ is not Hausdorff.

Next, we consider the case of subdivision graphs. The vertex set $V(S D(G))$ of the subdivision graph $S D(G)$ is $V(G) \cup L$, where $L$ consists of all newly added vertices on each edge of $G$. We denote the vertices of $\operatorname{SD}(G)$ corresponding to the edges $e, f, \ldots$ of $G$ by $w_{e}, w_{f}, \ldots$, respectively. Cycle $C_{4}$ and its subdivision graph are shown in Figure 2:

$C_{4}$

$S D\left(C_{4}\right)$

Figure 2. Cycle $C_{4}$ and its subdivision graph $S D\left(C_{4}\right)$.
Theorem 6. If $G$ is a graph with no pendant vertices, then its subdivision graph $\operatorname{SD}(G)$ is Hausdorff.

Proof. Let $u$ and $v$ be two distinct vertices of $S D(G)$.
Case 1. $u$ and $v$ are vertices of $G$.
Since $G$ is a graph with no pendant vertices, we can choose a vertex $x$ of $G$ distinct from $v$ and adjacent to $u$. Similarly, choose a vertex $y$ of $G$ distinct from $u$ and adjacent to $v$ ( $x$ may be equal to $y$ ). Let $w_{e}$ be the new vertex added on the edge $e=u x$ and $w_{f}$ be the new vertex added on the edge $f=v y$. Then $u w_{e}$ and $v w_{f}$ are two nonadjacent edges of $S D(G)$.

Case 2. $u=w_{e}$ and $v=w_{f}$, for some edges $e$ and $f$ in $G$.
Since $u \neq v, e$ and $f$ are distinct. Therefore, there exist two distinct vertices $x$ and $y$ such that $e$ is incident with $x$ and $f$ is incident with $y$. Then $u x$ and $v y$ are two nonadjacent edges of $S D(G)$.

Case 3. $u \in V(G)$ and $v=w_{e}$, for some edge $e$ of $G$.
Since $G$ is a graph with no pendant vertices, there exists an edge $f$ of $G$ distinct from $e$ such that $f$ is incident with $u$ in $G$. Let $f=u x$ and $y$ be a vertex of $G$ distinct from $u$ and $x$ such that the edge $e$ is incident with $y$ in $G$. Then $u w_{f}$ and $v y$ are two nonadjacent edges of $S D(G)$.

Thus, for any two distinct vertices $u$ and $v$ of $\operatorname{SD}(G)$ there exist two nonadjacent edges $e_{1}$ and $e_{2}$ of $\operatorname{SD}(G)$ such that $e_{1}$ is incident with $u$ and $e_{2}$ is incident with $v$. Hence $\operatorname{SD}(G)$ is Hausdorff.

Note that, if $e=u v$ is a pendant edge of a graph $G$ with $u$ as its pendant vertex, then $u w_{e}$ is a pendant edge of its subdivision graph $\operatorname{SD}(G)$. Therefore, $S D(G)$ is not Hausdorff. So we have:

Proposition 7. If $G$ is a graph with a pendant vertex, then its subdivision graph $S D(G)$ cannot be Hausdorff.

Definition 8. The shadow graph [2] $D_{2}(G)$ of a connected graph $G$ is constructed as follows:

Take two copies $G^{\prime}$ and $G^{\prime \prime}$ of $G$. Denote the vertices of $G^{\prime}$ and $G^{\prime \prime}$ corresponding to the vertex $v$ of $G$ by $v^{\prime}$ and $v^{\prime \prime}$, respectively. Then the shadow graph $D_{2}(G)$ of $G$ is a graph whose vertex set is $V\left(G^{\prime}\right) \cup V\left(G^{\prime \prime}\right)$ and $e$ is an edge of $D_{2}(G)$ if $e$ is an edge of $G^{\prime}$ or an edge of $G^{\prime \prime}$ or it is an edge with one end a vertex $v^{\prime}$ of $G^{\prime}$ and the other end a neighbour of the vertex $v^{\prime \prime}$ of $G^{\prime \prime}$.


Figure 3. Cycle $C_{4}$ and its shadow graph $D_{2}\left(C_{4}\right)$.

Theorem 9. Let $G$ be any graph with $\delta(G) \geq 1$. Then its shadow graph $D_{2}(G)$ is Hausdorff.

Proof. Let $u$ and $v$ be two distinct vertices of $D_{2}(G)$. Suppose $u$ and $v$ belong to the copy $G^{\prime}$ of $G$. Then $u=x^{\prime}$ and $v=y^{\prime}$ for some vertices $x$ and $y$ of $G$. Suppose $x^{\prime}$ and $y^{\prime}$ are adjacent vertices of $G^{\prime}$. Then $x^{\prime} y^{\prime \prime}$ and $y^{\prime} x^{\prime \prime}$ are two nonadjacent edges of $D_{2}(G)$. Suppose $x^{\prime}$ and $y^{\prime}$ are two nonadjacent vertices of $G^{\prime}$. Since $\delta(G) \geq 1$, there exist vertices $p$ and $q$ of $G$ adjacent to the vertices $x$ and $y$, respectively. Then $x^{\prime} p^{\prime}$ and $y^{\prime} q^{\prime \prime}$ are two nonadjacent edges of $D_{2}(G)$. Suppose $u=x^{\prime}$ and $v=y^{\prime \prime}$ for some vertices $x, y$ of $G$. Since $\delta(G) \geq 1$, there exist vertices $p$ and $q$ which are adjacent to the vertices $x$ and $y$, respectively. Then $x^{\prime} p^{\prime}$ and $y^{\prime \prime} q^{\prime \prime}$ are two nonadjacent edges of $D_{2}(G)$. Hence the theorem.

## 3. Total Graph and Quasi-total Graph

Figure 4 shows that the total graph of a non-Hausdorff graph may be Hausdorff. Note that the graph $P_{2}$ is free from isolated vertices and $K_{2}$ is not a component of $P_{2}$. Theorem 10 shows that this result is true, in general. That is, if $\delta(G) \geq 1$ and $K_{2}$ is not a component of $G$, then its total graph $T(G)$ is Hausdorff.


Figure 4. Path $P_{2}$ and its total graph $T\left(P_{2}\right)$.

Theorem 10. Let $G$ be a graph with no isolated vertices. If $K_{2}$ is not a component of $G$, then its total graph $T(G)$ is Hausdorff.

Proof. Let $u$ and $v$ be two distinct vertices of $T(G)$.

Suppose $u, v \in V(G)$. Since $K_{2}$ is not a component of $G$, there exists a vertex $w$ of $G$ such that $w$ is adjacent to $u$ or $v$ or both. Without loss of generality, assume that $u$ is adjacent to $w$. Suppose $f$ is an edge incident with $v$. Then $u e$ and $v f$ are two nonadjacent edges of $T(G)$, where $e$ is the edge $u w$.

Now suppose $u, v \in E(G)$. Then there exist two distinct vertices $x$ and $y$ such that $u$ is incident with $x$ and $v$ is incident with $y$. Then $u x$ and $v y$ are two nonadjacent edges of $T(G)$.

Now suppose $u \in V(G)$ and $v \in E(G)$. Let $w$ be a vertex of $G$ adjacent to $u$ in $G$. Since $K_{2}$ is not a component of $G$, there exists an edge $f$ of $G$ incident with $v$ in $G$. Then $u w$ and $v f$ are two nonadjacent edges of $T(G)$. Therefore, $T(G)$ is Hausdorff.

## Example 11.


$C_{4}$

$T\left(C_{4}\right)$

Figure 5. Cycle $C_{4}$ and its total graph $T\left(C_{4}\right)$.

Theorem 12. Let $G$ be any graph. If $K_{2}$ is not a component of $G$, then its total graph $T(G)$ is Hausdorff. In particular, total graph of a Hausdorff graph is Hausdorff.

Proof. We have $V(G)=H \bigcup K$, where $H$ is the set all isolated vertices of $G$ and $K$ is the set all non-isolated vertices of $G$. So $T(G)=T(H) \cup T(K)$. By Theorem 10, $T(K)$ is Hausdorff. Since the total graph of an empty graph
is empty, $T(H)$ is also Hausdorff. Therefore, $T(G)$, being the union of Hausdorff graph is Hausdorff.

Another interesting graph that we can derive from the given graph is quasi-total graph.

Definition 13. Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$. The quasi-total graph [9] $P(G)$ of $G$ is a graph with vertex set $V(G) \cup E(G)$ and two vertices are adjacent if they correspond to two nonadjacent vertices of $G$ or to two adjacent edges of $G$ or to a vertex and an edge incident to it in $G$.


Figure 6. Cycle $C_{4}$ and its quasi-total graph $P\left(C_{4}\right)$.
Theorem 14. Let $G$ be a graph with $\delta(G) \geq 2$. Then its quasi-total graph $P(G)$ of $G$ is Hausdorff.

Proof. Let $u$ and $v$ be two distinct vertices of $P(G)$.
First of all, suppose both $u$ and $v$ are vertices of $G$. Since $\delta(G) \geq 2$, there exists a vertex $w$ of $G$ distinct from $v$ and adjacent to $u$ in $G$. Similarly, there exists a vertex $x$ of $G$ distinct from $u$ and adjacent to $v$ in $G$. Let $e=u w$ and $f=v x$. Then $u e$ and $v f$ are two nonadjacent edges of $P(G)$.

Now, suppose both $u$ and $v$ are edges of $G$. Then there exist two distinct vertices $x$ and $y$ such that $u$ is incident with $x$ and $v$ is incident with $y$. Then $u x$ and $v y$ are two nonadjacent edges of $P(G)$.

Finally, suppose $u$ is a vertex of $G$ and $v$ is an edge of $G$. Let $w$ be a vertex of $G$ distinct from $u$ such that the edge $v$ is incident with $w$. Since $\delta(G) \geq 2$, there exists a vertex $x$ distinct from $w$ and adjacent to $u$. Let $e=$ $u x$. Then $u e$ and $v w$ are two nonadjacent edges of $P(G)$. Hence the theorem.

Corollary 15. If $G$ is a Hausdorff graph with no isolated vertices, then its quasi-total graph $P(G)$ is Hausdorff.

The question then arises is that what happens to the quasi-total graph when we decrease the minimum degree of the graph $G$. Unfortunately, the result remains failed in certain cases. For example if $G \cong K_{2}$, then $P(G)$ $\cong P_{2}$, a path on two vertices which is not Hausdorff. Similarly, if $G$ is an empty graph on two vertices, then $P(G) \cong K_{2}$, which is also non-Hausdorff. This shows that $P(G)$ need not be Hausdorff if $G$ is a graph with $\delta(G)<2$.

But one can overcome this difficulty by giving some restrictions to the graph $G$.

Theorem 16. Let $G$ be a graph with no isolated vertices and $K_{2}$ is not a component of $G$. Then its quasi-total graph $P(G)$ is Hausdorff.

Proof. Let $u$ and $v$ be two distinct vertices of $P(G)$.
Suppose $u, v \in V(G)$. Since $G$ contains no isolated vertices and $K_{2}$ is not its component, there exist distinct edges $e$ and $f$ in $G$ incident with $u$ and $v$, respectively. Then $u e$ and $v f$ are two nonadjacent edges of $P(G)$.

Now suppose $u, v \in V(G)$. Then there exist two distinct vertices $x$ and $y$ such that $u$ is incident with $x$ and $v$ is incident with $y$. Then $u x$ and $v y$ are two nonadjacent edges of $P(G)$.

Now suppose $u \in V(G)$ and $v \in E(G)$. Since $K_{2}$ is not a component of $G$, there exists an edge $f$ of $G$ incident with $v$. Let $x$ be the end point of $f$ which is not incident with the edge $v$. If $e=u x$ is an edge of $G$, then $u e$ and
$v f$ are two nonadjacent edges of $P(G)$. Otherwise $u x$ and $v f$ are two nonadjacent edges of $P(G)$. Therefore, $P(G)$ is Hausdorff.

## 4. 1-quasi-total Graph and 2-quasi-total Graph

Definition 17. Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$. The 1-quasi-total graph [10], $Q_{1}(G)$ of $G$ is a graph with vertex set, $V\left(Q_{1}(G)\right)=V(G) \bigcup E(G)$ and two vertices $x, y$ of $Q_{1}(G)$ are adjacent if they satisfy one of the following conditions:
(1) $x, y$ are in $V(G)$ and $x y \in E(G)$.
(2) $x, y$ are in $E(G)$ and $x$ and $y$ are incident in $G$.


G

$\mathrm{Q}_{1}(G)$

Figure 7. Cycle $C_{4}$ and its 1-quasi-total graph $Q_{1}\left(C_{4}\right)$.

Remark 18. Since $Q_{1}(G)$ is the disjoint union of $G$ and $L(G)$, if $G$ is non-Hausdorff, then $Q_{1}(G)$ is non-Hausdorff.

Proposition 19. 1-quasi-total graph of a Hausdorff graph is Hausdorff.
Proof. Let $G$ be a Hausdorff graph. Then, by Theorem 3, its line graph $L(G)$ is also Hausdorff. Therefore, the 1-quasi-total graph $Q_{1}(G)$, being the union of Hausdorff graphs, is Hausdorff.

Definition 20. Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$. The 2-quasi-total graph [3], $Q_{2}(G)$ of $G$ is a graph with vertex set,
$V\left(Q_{2}(G)\right)=V(G) \cup E(G)$ and two vertices $x, y$ of $Q_{2}(G)$ are adjacent if they satisfy one of the following conditions:
(1) $x, y$ are in $V(G)$ and $x y \in E(G)$.
(2) $x$ is in $V(G), y$ is in $E(G)$ and $x$ and $y$ are incident in $G$.


G

$\mathrm{Q}_{2}(G)$

Figure 8. Cycle $C_{4}$ and its 2-quasi-total graph $Q_{2}\left(C_{4}\right)$.

Theorem 21. Let $G$ be a graph with $\delta(G) \geq 2$. Then 2-quasi-total graph, $Q_{2}(G)$, of $G$ is Hausdorff.

Proof. Let $u$ and $v$ be two distinct vertices of $Q_{2}(G)$.
First of all, suppose $u$ and $v$ are vertices of $G$. Since $\delta(G) \geq 2$, there exists a vertex $x$ distinct from $v$ such that $u$ is adjacent to $x$ in $G$. Similarly, there exists a vertex $y$ distinct from $u$ such that $v$ is adjacent to $y$ in $G$. Then $u e$ and $v f$ are two nonadjacent edges of $Q_{2}(G)$, where $e=u x$ and $f=v y$ are edges of $G$.

Now suppose $u$ and $v$ are edges of $G$. Let $x$ and $y$ be two distinct vertices of $G$ such that $u$ is incident with $x$ and $v$ is incident with $y$ in G . Therefore, $u x$ and $v y$ are two nonadjacent edges of $Q_{2}(G)$.

If $u$ is a vertex of $G$ and $v$ is an edge of $G$, then there exists a vertex $w$ of $G$ distinct from $u$ such that the edge $v$ is incident with $w$ in $G$. Since $\delta(G)$
$\geq 2$, there exists a vertex $x$ distinct from $w$ such that $u$ is adjacent to $x$ in $G$. Then $u x$ and $v w$ are two nonadjacent edges of $Q_{2}(G)$. Hence the theorem.

Let $u$ be a pendant vertex of $G$ with pendant edge $e=u v$. Then there does not exist nonadjacent edges $e_{1}$ and $e_{2}$ in $Q_{2}(G)$, incident with $u$ and $e$, respectively. Therefore, $Q_{2}(G)$ cannot be Hausdorff. So we have:

Proposition 22. If $G$ is a graph with at least one pendant vertex, then $Q_{2}(G)$ can never be Hausdorff. In particular, $Q_{2}\left(P_{n}\right)$ is non-Hausdorff for every $n$.

Theorem 23. The 2-quasi-total graph of a Hausdorff graph is Hausdorff.
Proof. We have $V(G)=H \bigcup K$, where $H$ is the set of all isolated vertices of $G$ and $K$ is the set of all non-isolated vertices of $G$. So $Q_{2}(G)=Q_{2}(H)$ $\cup Q_{2}(K)$. Since 2-quasi-total graph of an empty graph is empty, $Q_{2}(H)$ is Hausdorff. By Theorem 21, $Q_{2}(H)$ is Hausdorff. Therefore, $Q_{2}(G)$ is the union of two Hausdorff graphs. Hence it is Hausdorff.

## 5. Conclusions

In this paper, we have discussed conditions for splitting graph, subdivision graph, shadow graph, total graph, quasi-total graph, 1-quasi-total graph and 2-quasi-total graph of a given graph to be Hausdorff. It is proved that the splitting graph, subdivision graph, total graph, 1-quasi-total graph and 2-quasi-total graph of a Hausdorff graph are Hausdorff. It is also proved that if $G$ is a Hausdorff graph with no isolated vertices, then its quasi-total graph $P(G)$ is Hausdorff.

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