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A STUDY OF STRATIFIED ROTATING EFFECTS ON HOMOGENEOUS TURBULENCE

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Abstract

A numerical investigation is performed in order to study the changes induced by the rotation and stratification effects on the behaviour of homogeneous incompressible turbulence. The dimensionless numbers relevant to this turbulence are the Froude, Rossby and B = Froude/Rossby. The computations cover B numbers ranging from 0.1 to infinity with constant Prandtl number, equal to unity. Spectrum quantity is explained to determine reasons for the changes on the turbulence as the B number is increased. The results obtained using the developed numerical model are in acceptable agreement with those performed by Cambon et al. [3].

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Introduction

Many works on the effects of rotation and stratification on the evolution of an incompressible homogeneous turbulence have been made (Hanazaki and Hunt [1], Iida and Nagano [2] and Cambon et al. [3]). Recent years have rather seen a real numerical success than an analytical one, in the prediction of homogeneous turbulence, evolving in the presence of a rotation or a stratification separately.

In spite of major existing difficulties, particularly in the relative case of a turbulence characterized by the relation between the number of Froude and the number of Rossby equal to the unity, the effects of the rotation and stratification are equal. It is important to indicate that the stratified rotating flows are often studied based on the theoretical concept of turbulence almost geophysics (Hanazaki and Hunt [1]). Such turbulence is characterized by the small numbers of *Ro* and *Fr* and a large time scale related to the effects of gravity and inertia. Actually, there are different cases. A purely stratified flow characterized by $B = 0$, when the number $B = 0.1$, the flow evolves in the presence of a dominant stratification $B = 10$ corresponds to a flow in a dominant rotation when the number B is very important, $B = \infty$, the flow evolves in the presence of a pure rotation. Finally, when the flow is considered non-dispersive, the flow corresponds to an equipartition between rotation and stratification.

Linear study in spectral space

We begin by defining the spectra $\hat{u}_i$, $\hat{p}$ and $\hat{\theta}$ from Fourier Transforms (FT) components u_i of the velocity fluctuation, the fluctuation of pressure p and temperature fluctuation $\hat{\theta}$, respectively:

$$\hat{u}_i(\underline{k}, t) = TF(u_i(\underline{x}, t)),$$

$$\hat{p}(\underline{k}, t)TF(p/\rho_0(\underline{x}, t)),$$

$$\hat{\theta} = TF(\alpha g\theta(\underline{x}, t)),$$

$\rho_0 =$ a reference density,

$\alpha =$ the dilatation coefficient of the fluid.

The equations describing these spectra are obtained by linearization which comes in the following forms (Cambon et al. [3]):

$$(\partial_t + u \cdot \nabla)u + 2\Omega n \times u + \nabla p - \nu \nabla^2 u = b, \tag{1}$$

$$(\partial_t + u \cdot \nabla)b - \chi \nabla^2 b = -N^2 n \cdot u, \tag{2}$$

$$\nabla \cdot u = 0. \tag{3}$$

In these equations, we add the parameters b, n and c, respectively, denote the buoyant force, kinematic viscosity and thermal diffusivity.

Thus, $N = \sqrt{agd\overline{T}/dx_3}$ defines the frequency of Brunt-Vaisala, U_1 is the horizontal component of the average speed and x_3 is the upward vertical axis.

These equations allow establishing equations of the spectrum evolution speed and temperature double correlations:

$$2e_{ij}(\underline{k})\delta(\underline{k} + \underline{k'}) = \overline{\hat{u}_i(\underline{k'})\hat{u}_j(\underline{k})} + \overline{\hat{u}_i(\underline{k})\hat{u}_j(\underline{k'})}, \tag{4}$$

$$2e_{\theta}(\underline{k})\delta(\underline{k} + \underline{k'}) = \overline{\theta(\underline{k'})\theta(\underline{k})}, \tag{5}$$

$$2e_{i\theta}(\underline{k})\delta(\underline{k} + \underline{k'}) = \overline{\hat{u}_i(\underline{k'})\hat{\theta}(\underline{k})} + \overline{\hat{u}_i(\underline{k})\hat{\theta}(\underline{k'})}. \tag{6}$$

The incompressibility condition (1) results are, for these spectra, the relations $k_i e_{ij} = 0$, $k_i e_{i\theta} = 0$ which allow to hold six basic functions, we make the choice of spectra e_{11}, e_{22}, e_{33} and $e_{3\theta}$.

These are their evolution equations from equations (4)-(6):

$$\frac{d}{dt} e_{11} = -\frac{1}{Re} k^2 [e_{11}] - \frac{k_1 k_3}{k^2} [e_{\theta 1}] + \frac{1}{Ros} \frac{k_3 k_2}{k^2} [e_{31}] - \frac{1}{Ros} \frac{k_3^2}{k^2} [e_{21}], \tag{7}$$

$$\frac{d}{dt}e_{22} = -\frac{1}{Re}k^2[e_{22}] - \frac{k_2k_3}{k^2}[e_{\theta 2}] - \frac{1}{Ros}\frac{k_3k_1}{k^2}[e_{32}] - \frac{1}{Ros}\frac{k_3^2}{k^2}[e_{12}], \quad (8)$$

$$\frac{d}{dt}e_{33} = \frac{k_1^2}{k_3^2}\frac{d}{dt}e_{11} + \frac{k_2^2}{k_3^2}\frac{d}{dt}e_{22} + \frac{k_1k_2}{k_3^2}\left[\frac{d}{dt}e_{12} + \frac{d}{dt}e_{21}\right], \quad (9)$$

$$\frac{d}{dt}e_{\theta 3} = \frac{k_1^2}{k_3}\frac{d}{dt}e_{\theta 1} + \frac{k_2}{k_3}\frac{d}{dt}e_{\theta 2}. \quad (10)$$

Materials and Methods

The equations are first written, equivalently, in Lagrangian formulation in spectral space. The time integration of these equations is then performed along the spectral trajectories. We use a simple time integration scheme, accurate to second order:

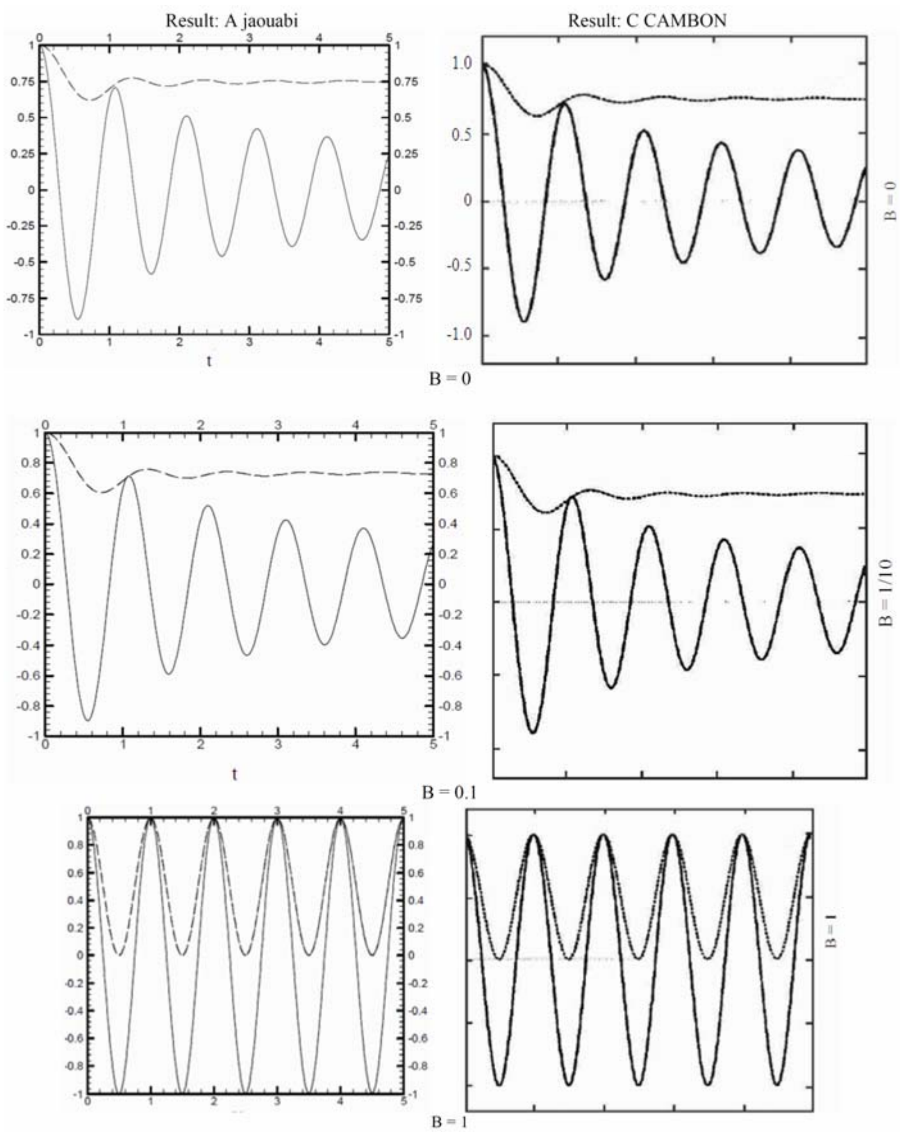
$$f(t + \Delta t) = f(t) + \Delta t f'(t) + (\Delta t)^2 \frac{f''(t)}{2}. \quad (11)$$

It should be noted that in this integration scheme, second order derivatives are obtained analytically by a simple derivation applied to different terms of equations. The spectral space is discretized in all three directions in spherical coordinates: logarithm of the modulus of the wave number k is distributed with a constant pitch between a maximum and minimum. The angular direction φ is distributed with a constant pitch on the interval $[0, 2\pi]$ and we introduce a variable $z = \cos\alpha$ distributed with a constant pitch on the interval $[-1, 1]$. Considering an initial isotropic turbulence, we take as three-dimensional spectrum of kinetic energy.

Results

We begin by presenting results for a Prandtl number $(Pr = 1)$ and different values of the number B. In Figure 1, we present the evolution of spectra $\langle u_1(0)u_1(t)\rangle$, $\langle u_2(0)u_2(t)\rangle$ and $\langle u_3(0)u_3(t)\rangle$ for different values of B. The numerical results we obtained are in agreement with those obtained by Cambon et al. [3].

Figure 2 shows the influence of the number *B* on the evolution of vertical heat flux. The results show the obvious effect of rotation on the heat flux, in fact when the rotation is enhanced heat flux tends to zero.



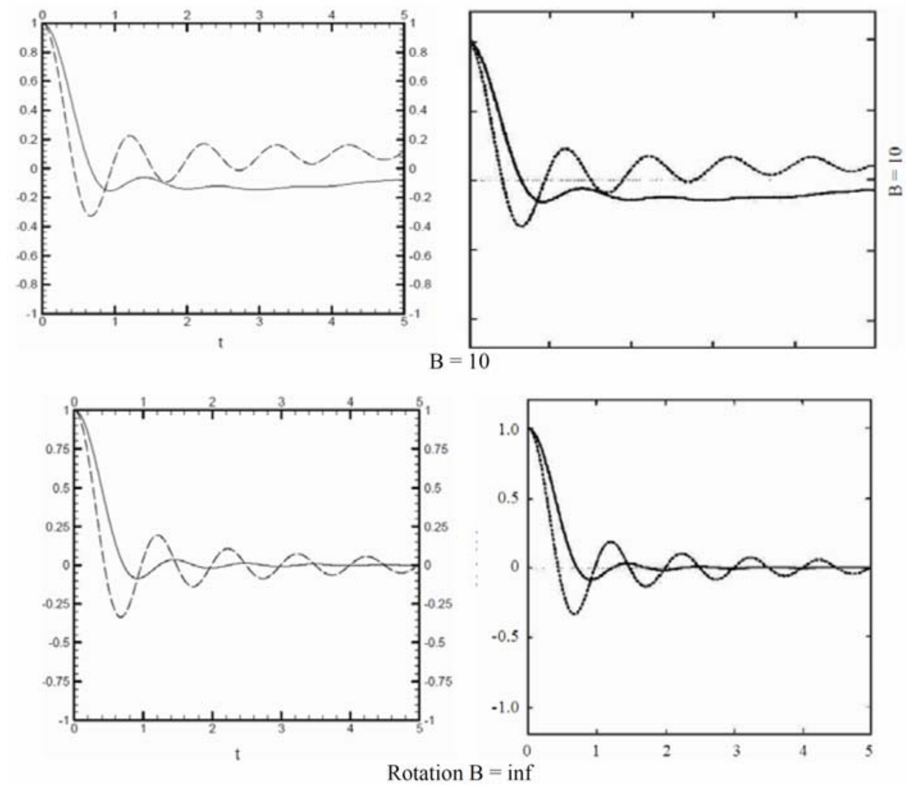


Figure 1. Evolution of spectra $\langle u_1(0)u_1(t)\rangle$, $\langle u_2(0)u_2(t)\rangle$ and $\langle u_3(0)u_3(t)\rangle$ for $Pr = 1$.

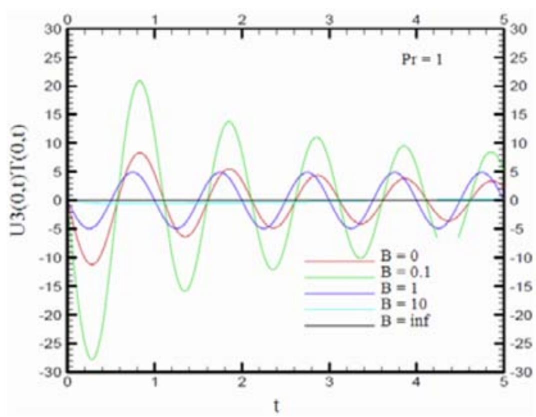


Figure 2. Evolution of the coefficient of vertical heat flux for $Pr = 1$.

Discussion

A numerical model was developed to predict the effects of rotation and stratification on turbulence. A linear analysis in spectral space provides access to the evolution of different physical quantities typical to such turbulence. These quantities are studied using dimensionless numbers namely Fr, Ro and $B = Fr/Ro.$ The results obtained using the developed model are relatively consistent with the those of Cambon et al. [3].

Conclusion

In this study, we have developed a code of calculation in order to numerically simulate the development of a stably stratified homogeneous turbulence in rotation, in different cases: a purely stratified flow, the flow evolves in the presence of a dominant stratification, dominant rotation flow, the flow change in the presence of a pure rotation and finally the flow corresponds to an equipartition between rotation and stratification. A linear analysis has been conducted and determined the evolution equation describing such turbulence. The resolution of these equations using the developed computer code has led to numerical results consistent with those available in the literature (Cambon et al. [3]).

Thus, we can determine the influence of different values of B associated with the turbulent heat flux vertical with $Pr = 1.$

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