



IMPORTANCE OF USING A METHOD TO SOLVE MATHEMATICAL PROBLEMS

Alejandra Lira Vázquez and Oscar Alfredo Cárdenas Rueda

Universidad Nacional Autónoma de México

CCH-Azcapotzalco

Área de Matemáticas

Avenida Aquiles Serdán 2060

Azcapotzalco, Ex-hacienda el Rosario

CDMX, C.P. 02020, Mexico

e-mail: alv@unam.mx

Colegio de Estudios de Posgrado de la Ciudad de México

Av. Morelos Oriente No. 110, Barrio de San Pedro

Municipio de Ixtlahuaca, Edo. de México

C.P. 50740, Mexico

e-mail: oscardcard1@hotmail.com

Abstract

This research presents results obtained in the classroom, by a group that uses a method for solving problems in the development of the unit of Mathematics I course in High School, and another group that does not use it. The difference in the two groups on the basis of answers of a questionnaire consisting of ten questions covering several important aspects of teaching-learning process can easily be evaluated. From the questionnaire responses, it could be concluded that students who used a method of solving problems acquire better skills which enabled them to solve problems easily.

Received: April 12, 2016; Revised: May 2, 2016; Accepted: May 25, 2016

Keywords and phrases: solving mathematical problem.

Communicated by K. K. Azad

Introduction

In this study, the importance of implementing a method of solving problems in Mathematics I in baccalaureate is established. For this, it was considered the third unit of this course and asked to solve problems to two groups, coursing Mathematics I, one of which did not apply any method; also they answered a questionnaire of ten questions to establish the percentages of various aspects of the teaching-learning process as well to detect some acquired skills.

With these results, the difference could be established in the two groups solving problems. These results were presented in a table. From the analysis of these results, it was determined that the group 1, that applied a method to solve problems, won a higher percentage in the acquisition of skills to solve problems and group 2, which did not apply any method, did not achieve to increase its percentage in the acquisition of skills throughout the unit. From these results, we can infer that it is necessary to teach students a general method for solving problems.

Problem Resolution

Overall problem solving is one of the most used strategies for learning mathematics, requiring the students make a proper application of mathematical concepts learned.

Most programs consider Math problem solving as the goal for students to demonstrate learning the concepts. In addition, the teaching of mathematics cannot be conceived without the demonstration that a student can solve problems in examinations.

The key to establish various solutions of the problems is the use of a method to obtain them clearly and concisely.

It is the task of teachers to implement strategies that allow them to show students the problems arising in the course of mathematics. These are interesting and easy to solve when a method is employed based on knowledge of the concepts learned in an orderly and structured way.

The effective resolution of the problems, which must establish a high school student in the subject of Mathematics I, depends on realization of the existence of mathematical concepts that may be useful in the process of problem solving.

Although some researchers have found that the activities for solving problems are not easy for the students, as they tend to address school problems focusing to achieve the answer in any way, and in the process ignore the understanding of the correct process (Pozo et al. [5]). Some try to solve problems mechanically and operationally to find the right formula (Mettes et al. [3]) arriving even sometimes at the correct solution without understanding what they have done (Gilbert [1]).

Application in the Classroom

According to the experience in the classroom, it has been found that if the teacher leads students to use a method of problem solving to avoid mechanization and memorization, creating interest in finding the unknown from mathematical concepts learnt, then a process that encourages the students can be evolved.

To establish the importance of applying a method for solving mathematical problems, monitoring was conducted in two groups Mathematics I of baccalaureate in Unit 3 of the course. Both groups had the same number of students with regular attendance. Being group 1 the pilot group working with a method of problem solving and group 2, the contrast group that did not use any method to solve problems.

To detect the gained skills 10 points were scored, which determined whether they met the questions marked in the central part of Table 1. This was done considering the problems solved by students during the second and last class of the unit. In the table is shown the percentage of students who were able to increase their capacity in certain skills while applying a method for solving problems.

Table 1. Shows the % of the pupils who achieved certain skills in groups 1 and 2

Acquired skills	2nd.Session of Unit III		Screening questions	Last session of unit III	
	Group 1	Group 2		Group 1	Group 2
Communication	42%	34%	1. During the session is a clear language used for the expression of their ideas?	98%	64%
	37%	25%	2. The exposition of the ideas was made concisely?	88%	58%
	41%	28%	3. The ideas raised were understood by the rest of the group?	89%	60%
Teamwork	38%	30%	4. During the session, did you maintain an attitude of collaboration with the rest of his teammates?	99%	62%
	30%	22%	5. The ideas were part of final consensus of the session?	83%	44%
Problem solving	34%	23%	6. Could you identify clues or relevant facts that allow you to start building or problem identification and establishment of the translation of the problem?	95%	45%
	29%	21%	7. Did you participate in the formulation of hypotheses or assumptions that would respond to the problems?	83%	35%
	27%	20%	8. Did your hypotheses or assumptions made provide answers to the problems?	62%	34%
Independent learning in problem solving	34%	25%	9. Did you present the information collected for solving a problem clearly and concisely?	86%	46%
	38%	19%	10. Did you apply the acquired knowledge to solve other problems?	61%	29%

Examples of Students Solving Mathematics Problems

Students in the group who are not following a method of problem solving, establish an outcome like guessing and just trying to see if the result obtained was correct, making failed attempts and most of them are not led to the correct solution. An example of this is presented below.

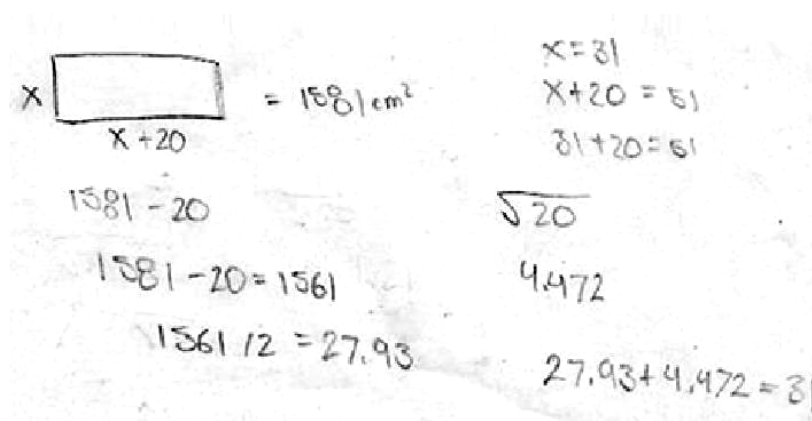
We proposed the problem for dimensions of rectangular terrain, which we enunciate with the following question:

What are the dimensions of a rectangular terrain if its length is 20 m more than its width and its area is 1581 m^2 ?

Here we present some answers of several students to the previously stated problem, first the case of two students that do not apply a method to properly solve the problem is exemplified.

Student 1

What are the dimensions of a rectangular terrain if its length is 20 m more than its width and its area is 1581 m^2 ?



Handwritten work for Student 1:

Diagram: A rectangle with width labeled X and length labeled $X+20$. To the right of the rectangle is the equation $= 1581 \text{ cm}^2$.

Calculations:

$$1581 - 20$$

$$1581 - 20 = 1561$$

$$1561 / 2 = 27.93$$

$$X = 81$$

$$X + 20 = 81$$

$$81 + 20 = 81$$

$$\sqrt{20}$$

$$4.472$$

$$27.93 + 4.472 = 81$$

Student 2

What are the dimensions of a rectangular terrain if its length is 20 m more than its width and its area is 1581 m^2 ?

$$\begin{aligned}
 x^2 + 20x - 1581 &= 0 \\
 x &= \frac{-20 \pm \sqrt{20^2 - 4(1)(-1581)}}{2(1)} \\
 \frac{-20 \pm \sqrt{400 + 6324}}{2} &= \frac{-20 \pm \sqrt{6724}}{2} = \frac{-20 \pm 82}{2} \\
 x_1 &= \frac{-20 + 82}{2} = 31 \\
 x_2 &= \frac{-20 - 82}{2} = -51
 \end{aligned}$$

$L = 51 \text{ m}$
 $A = 31 \text{ m}$

The two solutions show that these do not follow any method to solve the problem. Students guess the results and implement calculations. In the case of the first student it is only assuming or adjusting operations to obtain a result that seems to be the closest to the solution. In the case of the second student he is led to a quadratic equation, which is resolved by determining the second square root with the sign changed and assuming a solution adjusted with the square roots obtained. It is an incorrect procedure because of not using an appropriate method to solve this problem.

Now, two cases in which students try to apply a method such as Polya presented.

Student 3

What are the dimensions of a rectangular terrain if its length is 20 m more than its width and its area is 1581 m^2 ?

$$\begin{aligned}
 &\text{Diagram: } \square \text{ with width } x \text{ and length } 20+x \\
 &A(x) \cdot (x+20) = 1581 \\
 &x^2 + 20x = 1581 \\
 &x^2 + 20x - 1581 = 0 \\
 &a=1 \quad b=20 \quad c=-1581 \\
 &x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &x_{1,2} = \frac{-20 \pm \sqrt{400 + 4(1)(1581)}}{2(1)} \\
 &x_{1,2} = \frac{-20 \pm \sqrt{400 + 6324}}{2} \\
 &x_{1,2} = \frac{-20 \pm \sqrt{6724}}{2} \\
 &x_1 = \frac{-20 + 82}{2} = \frac{62}{2} = 31 \\
 &x_2 = \frac{-20 - 82}{2} = \frac{-102}{2} = -51 \\
 &\text{Dimensions: } 31 \text{ m by } 51 \text{ m}
 \end{aligned}$$

In the above solution, the student sets his model and relates the area with the dimensions of the rectangle, establishes a quadratic equation which is solved by the general formula, finding square roots, only missing some to justify the result.

Student 4

What are the dimensions of a rectangular terrain if its length 20 m more than its width and its area is 1581 m².

$x = \text{ancho}$
 $X = x$
 $20 + x(x) = 1581$
 $x(20 + x) = 1581$
 $20x + x^2 = 1581$
 $20 + 31(31) = 1581$
 $51(51) = 1581$
 $1581 = 1581$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $x = \frac{-20 \pm \sqrt{20^2 - 4(1)(-1581)}}{2(1)}$
 $x = \frac{-20 \pm \sqrt{400 + 6324}}{2}$
 $x = \frac{-20 \pm \sqrt{6724}}{2}$

$x_1 = \frac{-20 + 82}{2}$
 $x_1 = \frac{62}{2}$
 $x_1 = 31$

The solution sets the last student to establish the model and find results, check the same and justify it a little more.

Analysis

According to Table 1: Students of Group 1, applying a method to solve problems in communication increased from 42% to 98%, in the expression of their ideas being clearer, increasing the percentage of students 37% to 88% and in a concise manner so that these were understood by the rest of the group, increased from 31% to 89%. With regard to teamwork, it increased from 38% to 99%, the cooperative attitude of students and the percentage of students whose ideas were part of consensus increase of 30% to 83%. The resolution of problems increased from 34% to 95% of students who identified the clues to translate the problem.

Detecting Differences

In assessing their problems solved, we considered the following skills: Communication, Teamwork, Problem solving and independent learning in problem solving. These were detected for 10 questions that we framed. We met a certain percentage of students in each group. The percentages in Table 1 state that both progressed on increasing the skills considered in each group and establish that students in group 1 had better progress in these skills and resolved the problems better because they used a method to arrive at the solution.

Conclusions

Obtaining skills in group 1 compared to group 2 was higher, because it was the group that used a method for solving problems throughout the unit.

Therefore, from the percentages obtained in groups 1 and 2, at the beginning and end of the unit, it can be said that it is important for students to apply a solution method to solve problems, because this will help learn to learn, learn to seek and achieve independent learning ability.

Also, teachers could use problems and solve them as a method to evaluate strategy, this being a way of establishing a formative evaluation that would greatly help to facilitate the summative evaluation of the course.

Finally, we can say that the teaching of mathematics through problem solving is valuable because these can establish an appropriate method to solve problems. A teacher encourages the students to acquire knowledge structured and contextualized. Thus students are helped to assimilate and apply significant knowledge developing skill to explain different solutions of problems.

Acknowledgement

The authors thank the anonymous referees for their valuable suggestions which led to the improvement of the manuscript.

References

- [1] George L. Gilbert, How do I get the answer? *Journal of Chemical Education* 57 (1980), 79-81.
- [2] Alejandra Lira Vázquez, Solución de problemas de física moderna a nivel bachillerato, UNAM, México, 2006.
- [3] C. T. Mettes, A. Pilot, J. H. Rossink and H. Kramers-Pals, Teaching and learning problem solving in science, Part 1: A general strategy, *Journal of Chemical Education* 57 (1980), 882-885.
- [4] M. Selvaratnam, Student's mistakes in problem solving, *Education in Chemistry*, 1983, pp. 125-128.
- [5] Juan I. Pozo, Yolanda Postigo and Miguel A. Gómez, Aprendizaje de estrategias para la solución de problemas en ciencias, *Alambique, Didáctica de las Ciencias Experimentales* 5 (1995), 16-26.
- [6] M. Watts, *The Science of Problem Solving: A Practical Guide for Science Teachers*, Cassell, London, 1991.
- [7] G. Polya, *How to Solve it?* Princeton University Press, Princeton, 1957.