DEMOCRACY FUNCTION FOR WAVELET BASES

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Abstract: For a quasi-Banach space $(\mathbb{B}, \| \ \|_{\mathbb{B}})$ with a countable basis $\mathcal{B} = \{e_k : k \in \mathbb{N}\}$, the left and right democracy functions are defined, for $N = 1, 2, 3, \ldots,$ by

$$h_l(N) = \inf_{\Gamma \subset \mathbb{N}; |\Gamma| = N} \left\| \sum_{k \in \Gamma} \frac{e_k}{\| e_k \|_{\mathbb{B}}} \right\|_{\mathbb{B}} \text{ and } h_r(N) = \sup_{\Gamma \subset \mathbb{N}; |\Gamma| = N} \left\| \sum_{k \in \Gamma} \frac{e_k}{\| e_k \|_{\mathbb{B}}} \right\|_{\mathbb{B}}.$$

When $h_l \approx h_r$, the basis $\mathcal{B}$ is said to be *democratic* in $\mathbb{B}$. The relevance of these functions is that they control the optimal embeddings between Approximation spaces $\mathcal{A}_q^{\alpha}(\mathcal{B}, \mathbb{B})$ and weighted discrete Lorentz spaces $\ell_{\eta}^{q}(\mathcal{B}, \mathbb{B})$, which, in the case of wavelet bases, can be identified with Besov spaces. Wavelet admissible bases are democratic in $L^p(\mathbb{R}^n), 1 < p < \infty,$ with $h(N) \approx N^{1/p}$ (also valid for Sobolev and Triebel-Lizorkin spaces). Democracy functions for other function spaces such as Orlicz $L^{\Phi}(\mathbb{R}^n)$ and Lorentz spaces $L^{p,q}(\mathbb{R}^n)$ will be given.